$2D$ LOG-ELASTOGRAPHIC METHODS FOR TISSUE SHEAR STIFFNESS RECONSTRUCTION USING A $2D$ PLANE STRAIN ELASTIC SYSTEM

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An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Mathematics

The original of the complete thesis is on file
in the Rensselaer Polytechnic Institute Library

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August 2009
(For Graduation December 2009)

ABSTRACT

As a promising medical imaging technique for disease diagnostics, elastography targets reconstructing and imaging elastic parameters in biological tissue, due to the fact that abnormal tissue and healthy tissue exhibit distinct elastic properties. In all the experiments developed so far, tissue is mechanically excited and the interior displacement of the propagating wave is measured using ultrasound or magnetic resonance imaging (MRI) in order to recover the elastic properties of the tissue.

This thesis concerns the reconstruction of shear stiffness biomechanical parameters. With given single frequency $2D$ elastographic displacement data, the mathematical model is a first order partial differential equation system derived from a $2D$ plane strain elastic system. A nonlinear $2D$ Log-Elastographic algorithm is developed to recover the shear modulus, together with the hydrostatic pressure, a term neglected in common practice when biomechanical parameters are imaged. In many previous works, the elastic system is reduced to either a Helmholtz equation or a single first order p.d.e., while in this work we utilize the $2D$ plane strain elastic system without neglecting the hydrostatic pressure term as the basis for our first order p.d.e. system.

The main advantage of the $2D$ Log-Elastographic method we develop here for solving the first order p.d.e. system is that it effectively controls possible exponential error growth without using a very fine discretization, a restriction needed by standard numerical methods. Stability and first order convergence are established.

Numerical examples with synthetic data show that the $2D$ Log-Elastographic algorithm improves the quality of the recovered images compared with the images obtained from the Direct Inversion method, the acoustic Log-Elastographic algorithm and the standard upwind algorithm. We also demonstrate that: (1) neglecting the hydrostatic pressure term can cause significant undershooting in the biomechanical image; and (2) we obtain agreement between the hydrostatic pressure obtained from this new algorithm and the hydrostatic pressure calculated with our forward algorithm.

Finally, images of stiffness variations in a diseased human liver are obtained by applying this $2D$ Log-Elastographic algorithm with *in-vivo* data from Richard Ehman's laboratory at Mayo Clinic. In this case, an additional filtering step is added.