APPLICATIONS OF 3−CONVEXITY

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An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Mathematics

The original of the complete thesis is on file
in the Rensselaer Polytechnic Institute Library

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December 2009
(For Graduation December 2009)

ABSTRACT

An interesting shape aspect can be illustrated by using Taylor Polynomials to approximate a function at several points and show a nesting relation between the Taylor Polynomials, in the sense that one lies below the other. Creating a nested family of functions is a visually aesthetic shape characteristic which naturally segues into constructing nice curves and surfaces. The nesting property is related to $n-$convexity, which for smooth functions means that the n^{th} derivative is positive. The approximation theory literature has studied $n-$convexity from a theoretical standpoint , but only $1-$convexity and $2-$convexity have been widely used in applications. This is due to the difficult nonlinear problems that arise when applying higher orders. We develop a theoretical framework for $3-$convexity to allow for its application to shape preserving curve interpolation and approximation. The algorithms are extended to create piecewise $2-$convex and $3-$convex functions. The ideas from the nested family of functions and construction of $3-$convex curves provide a novel method for constructing a surface patch such that the properties of $3-$convexity gives insight to local surface properties. The substantial improvement of $3-$convexity over $2-$convexity then lends itself to other uses such as derivative estimation for surfaces.