APPLICATIONS OF MODAL APPROXIMATIONS TO ATTENUATION PROPERTIES IN SANDY-SILTY SEDIMENTS

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ABSTRACT

The parametric dependence of acoustic modes in shallow water ocean waveguides is examined with the goal of understanding their influences on modal and overall energy loss. The environments considered have sandy-silty sediments with a fast bottom and include water sound speed profiles with isospeed layers above and below a middle thermocline layer of decreasing sound speed.

Concise and convenient modal approximation formulas valid in specific regimes are developed to show parametric dependence of the modal phase speed v_n, mode amplitude at the water-sediment interface $\psi_n(H)$, mode cutoff frequency, mode values in the sediment, and number of propagating modes. Two types of approximations are derived for higher modes for which v_n is greater than the maximum water sound speed c_h. One type is obtained from perturbation methods and the generalized Green's function, and the other is developed using a modified form of the Wentzel-Kramers-Brillouin-Jefferys (WKBJ) method. For lower modes characterized by $v_n < c_h$ approximation formulas are derived and expressed in terms of Airy functions. Comparisons with numerical calculations are used to illustrate the accuracy of these approximations over a range of frequencies and mode numbers.

Expressions for $\psi_n(H)$ obtained from modal approximations are used to derive expressions for modal attenuation coefficients (MACs) which demonstrate their dependence on frequency, mode number, channel depth, and water column and bottom sound speeds. For environments with a linear downward refracting gradient at the water-sediment interface, the MACs for modes with $v_n < c_h$ are proportional to the product of the sound speed gradient and f^{m-1} where f is the frequency and m is the power-law exponent of the sediment attenuation. For environments with a positive or upward refracting gradient at the water-sediment interface, the MACs are shown to decrease exponentially with increasing frequency. For the case of a lower isovelocity layer in the water, MACs for lower order modes are proportional to the product of the square of the mode number and f^{m-3}.

Results are applied to explain how the MACs of the propagating modes control overall loss behavior. An expression for reduced averaged transmission loss $\overline{TL}$ is derived for stratified shallow ocean channels and is used to devise a method for determining the number of modes that contribute effectively to the loss field. For a Pekeris waveguide, the number of contributing modes is shown to have the same parameter dependence at higher frequencies as a known expression from ray theory. In this waveguide and parameter regime, convenient $\overline{TL}$ expressions are developed and compared with another averaged loss formula derived previously using ray theory arguments. The loss expressions are shown to be identical in the important situations characterized by mode stripping and transition to single mode propagation.