PARAMETRIZATION OF ENHANCED TRANSPORT IN MESO-SCALE OCEANIC TURBULENCE THROUGH NON-GAUSSIAN STOCHASTIC FLOW MODELS

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ABSTRACT

The ocean component of current coupled atmosphere-ocean climate models cannot resolve meso-scales below about 100km whereas much of ocean's turbulent energy lies in this range of scales inducing fluxes that affect all aspects of ocean circulation and hence the associated climate models. In this thesis, we focus on the study of the problem of investigating and parameterizing the effective large-scale transport properties of vortex dominated sub-grid scale flow structures in meso-scale oceanic turbulence.

We develop a methodology based on homogenization theory toward representing the effects of meso-scale coherent structures on large-scale transport in the ocean. Our systematic parametrization strategy relies on constructing a hierarchy of deterministic and random flow models and on coupling the results from numerical simulations of these models and the asymptotic analysis with respect to key non-dimensional physical parameters such as Péclet and Strouhal numbers of the cell problems arising from homogenization theory and of the model equations.

In our hierarchy of flow models, we first construct the prototype compactly supported smoothly decaying vortex, the modified Rankine vortex, used in constructing the kinematic vortex dominated deterministic and random flows designed in this thesis. Next, in increasing order of complexity, we investigate the transport properties of first a simple deterministic flow, a Rankine vortex flow, in which a periodic array of compactly supported smoothly decaying vortices is superposed with a constant mean flow, second of a steady non-Gaussian random flow, a Poisson blob velocity model consisting of a superposition of randomly distributed compactly supported smoothly decaying vortices, and third of two time-dependent non-Gaussian random flows, the stochastically advected Poisson blob flow, in which a random number of uniformly randomly distributed modified Rankine vortices move around the domain according to independent Brownian motions, and the stochastic blinking vortex flow in which at any time a random number of uniformly randomly distributed modified Rankine vortices having exponential lifetimes exist in the domain. In the

sequel, we derive analytically the second order covariance structures of the time-dependent random velocity fields using probabilistic formulations of these velocity fields, and the universal defining properties such as the Eulerian root-mean-squared velocity, the Eulerian correlation length, and the Eulerian correlation time of these time-dependent random velocity fields using their covariance structures.

We show that in all our kinematic models, it is possible to parametrize the large-scale effective transport properties of the investigated idealized vortex dominated flows by simple functions of key universal non-dimensional flow parameters after identifying the data collapses in the parameter spaces. In all our models, in the asymptotic regimes of high and low Péclet number, we establish the relationships between the effective transport properties of our models and the underlying model specific parameters, such as the vortex radius, the vortex velocity and the amount of vortex overlap in the domain, as well as the relationships between the effective transport properties of our models and the universal defining flow characteristics such as the Eulerian root-mean-squared velocity, the Eulerian correlation length, and the Eulerian correlation time of the random velocity fields used in these models. Using arguments based on the mixing length theory and the Taylor's formula, we derive the underlying intrinsic Lagrangian correlation length and time scales of our random models in these asymptotic regimes. In our time-dependent random models, we show that at moderate to high Péclet numbers and for large enough Strouhal number, the effective transport properties are sensitive to the changes in Strouhal number. We account for this behaviour by proving formally through a stochastic averaging principle that in the limit of high Strouhal number the effective transport properties of our time-dependent random models can be described by Kubo's formula. At present, we find a substantial discrepancy between the results of our numerical computations for the enhancement of diffusivity and low Péclet number asymptotic calculations for the stochastic blinking vortex model and aim to resolve this in future work.

After extraction of spatial and temporal vortex statistics from a vortex dominated quasi-geostrophic simulation and calibration of our time-dependent models in accordance with these statistics, we compare the results of our calibrated models with the results obtained from the vortex dominated quasi-geostrophic simulation of the meso-scale oceanic turbulence.