HIGH CAPABILITY PARABOLIC EQUATIONS FOR ELASTIC MEDIA PROPAGATION

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ABSTRACT

Parabolic equation techniques are very efficient and can provide accurate solutions for range-dependent problems, which involve environments that change in the direction of wave propagation. A traditional approach used to handle range dependence is to approximate the medium as a series of range-independent segments in which the factorization leading to the parabolic equation is exact. At the vertical interfaces between these range-independent regions, specific conditions must be applied to accurately march the solution from one region into the next. For elastic media a single-scattering correction [E. T. Küsel et al., J. Acoust. Soc. Am. 121, 808 (2007)] accurately treats solid-solid interfaces, unlike other techniques that have proven successful for fluid media. This thesis is concerned with investigating and improving the accuracy of the single-scattering correction, extending the technique to elastic environments that contain anisotropy, and obtaining new parabolic equation formulations for poro-elastic media.

The parabolic equation method with a single-scattering correction allows for accurate modeling of range-dependent environments in elastic layered media. For problems with large contrasts, accuracy and efficiency are gained by subdividing vertical interfaces into a series of two or more single-scattering problems. This approach generates several computational parameters, such as the number of interface slices, an iteration convergence parameter τ, and the number of iterations n for convergence. Using a narrow-angle approximation, the choices of $n = 1$ and $\tau = 2$ give accurate solutions. Analogous results from the narrow-angle approximation extend to environments with larger variations when slices are used as needed at vertical interfaces. The approach is applied to a generic ocean waveguide that includes the generation of a Rayleigh interface wave. This example is presented in both the frequency and time domains.

A parabolic equation for calculating propagation in range-dependent, heterogeneous anisotropic media is developed and benchmarked. Recent progress in elastic parabolic equation development, specifically in the treatment of range dependence and heterogeneous layers, is extended to transversely isotropic (TI) elastic media. Range dependence is incorporated by the single-scattering correction. Depth dependence is treated through appropriate heterogeneous operators in the TI equations of motion. Local relationships for elastic moduli are obtained from the p and s wave speeds at specific angles [A. J. Fredricks et. al., Wave Motion 31, 139 (2000)]. The approach is applied to example media consisting of TI and isotropic layers. Results show that isotropic approximations to TI layers break down when shear effects are strong. In addition, homogeneous approximations to heterogeneous layers are shown to be applicable only when the layer is too thin or when heterogeneous gradients are sufficiently small. Otherwise, the depth heterogeneity must be returned for accurate calculations.

Improvements of parabolic equations for fluid and elastic media are extended to formulate new versions for poro-elastic media, including models for shallow-water sediments. A previous parabolic equation solution for poro-elastic media [M. D. Collins, et al., J. Acoust. Soc. Am. 98, 1645 (1995)] does not produce accurate solutions for environments with two or more poro-elastic layers. One variable formulation developed for elastic media is generalized for a more accurate and capable poro-elastic parabolic equation. Another variable formulation is introduced with horizontal interface conditions that contain no depth derivatives higher than first-order. This characteristic should aid in treating range dependence, since discretized first-order derivatives allow convenient matching across vertical interfaces, in contrast to second-order depth derivatives which do not. Considering range-independent problems, both new formulations are superior to the original for environments containing layered poro-elastic media.