

# NONLINEAR INTERNAL WAVE EFFECTS ON ACOUSTIC PROPAGATION AND SCATTERING

By

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## ABSTRACT

Experimental observations and theoretical studies show that nonlinear internal waves occur widely in shallow water and cause acoustic propagation effects including ducting and mode coupling. Horizontal ducting results when acoustic modes travel between internal wave fronts that form waveguide boundaries. For small grazing angles between a mode trajectory and a front, an interference pattern may arise that is a horizontal Lloyd mirror pattern. An analytic description for this feature is provided, along with comparisons between results from the formulated model predicting a horizontal Lloyd mirror pattern and an adiabatic mode parabolic equation. Different waveguide models are considered, including boxcar and jump sound speed profiles where change in sound speed is assumed 12 m/s. Modifications to the model are made to include multiple and moving fronts. The focus of this analysis is on different front locations relative to the source, as well as on the number of fronts and their curvatures and speeds. Curvature influences mode incidence angles and thereby changes the interference patterns. For sources oriented so that the front appears concave, the areas with interference patterns shrink as curvature increases, while convexly oriented fronts cause patterns to expand.

Curvature also influence how energy is distributed in the internal wave duct. For certain curvatures and duct widths energy forms a whispering gallery or becomes fully ducted. Angular constraints which indicate when to expect these phenomena are presented. Results are compared to propagation calculations and were found to agree in most examples.

In some cases trailing internal waves are present in the duct and disturb horizontal propagation. This type of propagation is characterized as a scattering process as a result of broken internal wave fronts between the lead waves. Traditionally this is handled in regimes where adiabatic normal modes are valid using sound speed perturbations to describe energy propagation along horizontal rays. An alternate is a radiative transport method in which acoustic vertical modes are assumed to carry energy and a 2-D transport equation describes horizontal propagation. This model

has parameters which are related to properties in the duct and are found using physical internal wave features. The two approaches are compared and contrasted. To incorporate significant curvature in observed fronts an additional parameter is used in calculations and specified for data from the Shallow Water 06 experiment.