UNIQUENESS AND SOLUTION OF TIME-HARMONIC INVERSE VISCOELASTICITY PROBLEMS WITH INTERIOR DATA

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ABSTRACT

In elastography, the displacement field in the interior of tissue in response to an excitation is measured using either ultrasound or magnetic resonance imaging (MRI). The research is focused on solving the subsequent inverse problem of determining the spatial distribution of the viscoelastic parameters of the tissue given the knowledge of the displacement fields in its interior. In particular, the goal is to create maps of the complex-valued shear modulus for an incompressible linear viscoelastic material undergoing infinitesimal time-harmonic deformation. This problem is motivated by applications in biomechanical imaging, where the material modulus distributions are used to detect and/or diagnose cancerous tumors.

Our approach to analyzing and solving the inverse viscoelastic problem is based on recognizing that the measured displacement fields and the reconstructed material properties satisfy the appropriate equations of motion. In the most general case these are the equations of conservation of momentum for the time-harmonic response of an incompressible, isotropic material in three dimensions. Often, approximations are introduced leading to simplified models that include the scalar Helmholtz equation, anti-plane shear, plane stress and plane strain. We consider each of these models as well as the original three dimensional time-harmonic viscoelastic equations. In each case we analyze the uniqueness of the inverse problem given single or multiple measurements. We also develop and implement a unified variational method for solving all these problems.

With regards to the uniqueness of these problems we make the following observations: (1) the problem of plane stress with a single measurement is identical to that of anti-plane shear with two measurements; (2) the problem of plane strain and the 3D problem share the same uniqueness properties, and that these problems are more ill-posed than those of plane stress and anti-plane strain; (3) in every case, including more measurements helps considerably in reducing the space of possible solutions, thus makes the solution to the problem closer to being unique.

We propose a unified direct variational approach to solve these inverse problems. This approach can accommodate multiple measurements and multiple unknowns (the shear modulus and the pressure) simultaneously with relative case. It is derived by weighting the original partial differential equation for the shear modulus by the adjoint operator acting on the complex-conjugate of the weighting functions. For this reason we refer to it as the complex adjoint weighted equation (CAWE). We consider a straightforward finite element discretization of these equations and test its performance with synthetically generated and experimentally measured data. We also append to the CAWE formulation the total variation diminishing regularization to improve its performance in the presence of noise. We conclude that the CAWE method is accurate and represents a viable approach for determining the viscoelastic properties of tissue.