APPLICATIONS OF BAYESIAN SPECTRUM REPRESENTATION IN ACOUSTICS

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ABSTRACT

This dissertation utilizes a Bayesian inference framework to enhance the solution of inverse problems where the forward model maps to acoustic spectra. A Bayesian solution to filter design inverts a acoustic spectra to pole-zero locations of a discrete-time filter model. Spatial sound field analysis with a spherical microphone array is a data analysis problem that requires inversion of spatio-temporal spectra to directions of arrival. As with many inverse problems, a probabilistic analysis results in richer solutions than can be achieved with ad-hoc methods. In the filter design problem, the Bayesian inversion results in globally optimal coefficient estimates as well as an estimate the most concise filter capable of representing the given spectrum, within a single framework. This approach is demonstrated on synthetic spectra, head-related transfer function spectra, and measured acoustic reflection spectra. The Bayesian model-based analysis of spatial room impulse responses is presented as an analogous problem with equally rich solution. The model selection mechanism provides an estimate of the number of arrivals, which is necessary to properly infer the directions of simultaneous arrivals. Although, spectrum inversion problems are fairly ubiquitous, the scope of this dissertation has been limited to these two and derivative problems.

The Bayesian approach to filter design is demonstrated on an artificial spectrum to illustrate the model comparison mechanism and then on measured head-related transfer functions to show the potential range of application. Coupled with sampling methods, the Bayesian approach is shown to outperform least-squares filter design methods commonly used in commercial software, confirming the need for a global search of the parameter space. The resulting designs are shown to be comparable to those that result from global optimization methods, but the Bayesian approach has the added advantage of a filter length estimate within the same unified framework. The application to reflection data is useful for representing frequency-dependent impedance boundaries in finite difference acoustic simulations. Furthermore, since the filter transfer function is a parametric model, it can be modified to

incorporate arbitrary frequency weighting and account for the band-limited nature of measured reflection spectra. Finally, the model is modified to compensate for dispersive error in the finite difference simulation, from the filter design process.

Stemming from the filter boundary problem, the implementation of pressure sources in finite difference simulation is addressed in order to assure that schemes properly converge. A class of parameterized source functions is proposed and shown to offer straightforward control of residual error in the simulation. Guided by the notion that the solution to be approximated affects the approximation error, sources are designed which reduce residual dispersive error to the size of round-off errors.

The early part of a room impulse response can be characterized by a series of isolated plane waves. Measured with an array of microphones, plane waves map to a directional response of the array or spatial intensity map. Probabilistic inversion of this response results in estimates of the number and directions of image source arrivals. The model-based inversion is shown to avoid ambiguities associated with peak-finding or inspection of the spatial intensity map. For this problem, determining the number of arrivals in a given frame is critical for properly inferring the state of the sound field. This analysis is effectively compression of the spatial room response, which is useful for analysis or encoding of the spatial sound field.

Parametric, model-based formulations of these problems enhance the solution in all cases, and a Bayesian interpretation provides a principled approach to model comparison and parameter estimation.