# CLASSIFICATION OF CONTACT QUADRATICS


By

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# ABSTRACT

Quadratic functions possess several inherent properties that are useful within many branches of mathematics, and in this particular effort, Applied Geometry. By considering quadratics $f(x) = a_1x^2 + b_1x + c_1$ and $g(x) = a_2x^2 + b_2x + c_2$ in the Euclidean plane and given points $(s, f(s))$ and $(t, g(t))$, respectively, a third quadratic, say $h(x) = ax^2 + bx + c$, can be constructed by developing a relationship between $s$ and $t$. This third quadratic, $h(x)$, referred to as a contact quadratic, joins $f(x)$ and $g(x)$ at $(s, f(s))$ and $(t, g(t))$ smoothly, provided there is a relationship between $t$ and $s$. By analyzing and classifying the behavior between $s$ and $t$ and how this applies to the contact quadratic $h(x)$, it is shown how the general concavity of $h(x)$ is determined from the quadratics $f(x)$ and $g(x)$. For instance, by treating the coefficient $a$ as a function of $t$ and supposing the denominator is zero at two real values of $t$ and with $a_1$ and $a_2$ each of the same sign, it is shown that it is always true the numerator is zero for two values of $t$. This type of classification, as well as other cases, are worked out illustrating the algebraic and geometric significance. Thus, these classifications provide the framework for further investigative efforts within Applied Geometry.