CHARACTERIZATION OF THERMALIZED FERMI-PASTA-ULAM CHAINS

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ABSTRACT

The Fermi-Pasta-Ulam (FPU) chains of particles in *thermal equilibrium* are studied from both wave-interaction and particle-interaction points of view. It is shown that, even in a strongly nonlinear regime, the chain in thermal equilibrium can be effectively described by a system of weakly interacting *renormalized* nonlinear waves. These waves possess (i) the Rayleigh-Jeans distribution and (ii) zero correlations between waves, just as noninteracting free waves would. This renormalization is achieved through a set of canonical transformations. The renormalized linear dispersion of these renormalized waves is obtained and shown to be in excellent agreement with numerical experiments. Moreover, a dynamical interpretation of the renormalization of the dispersion relation is provided via a self-consistency, mean-field argument. It turns out that this renormalization arises mainly from the trivial resonant wave interactions, i.e., interactions with no momentum exchange. Furthermore, using a multiple time-scale, statistical averaging method, we show that the interactions of near-resonant waves give rise to the broadening of the resonance peaks in the frequency spectrum of renormalized modes. The theoretical prediction for the resonance width for the thermalized β-FPU chain is found to be in very good agreement with its numerically measured value. Moreover, we show that the dynamical scenario for thermalized β-FPU chains is spatially highly localized discrete breathers riding chaotically on spatially extended, renormalized waves. We present numerical evidence of existence of discrete breathers in thermal equilibrium.