RECENT APPLICATIONS OF NASH EQUILIBRIA

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ABSTRACT

The problem of how to make decisions in a competitive environment is very common and important in many fields. The Nash equilibrium makes a fundamental contribution to formulating and analyzing the rational decision-making in such a problem. This thesis concentrates on the following recent applications of the Nash equilibrium.

The first application considers a simple synchronized supply-assembly system. This part of the thesis analyzes generalized Nash equilibrium decisions in a simple assembly supply system with common joint performance constraints. While they lead to a much more realistic model of a stochastic supply chain system, the performances constraints derived from queuing analysis are highly nonlinear. As a result, it significantly complicates the game models and imposes challenges for numerical analysis. Quasi-variational inequalities are used to serve as the main theoretical framework for constructive equilibrium analysis. This work makes considerable contributions to supply chain and game equilibria analysis.

The second application pertains to emissions allowance allocation systems. This part of the thesis addresses the pressing issues of how emissions allowances should be initially allocated in electric power markets, via the development of game-theoretic nonlinear complementarity models. A significant difficulty in the existence of an equilibrium in the models is that the resulting variational inequality (VI) is not monotone. Therefore, the classical theories and numerical methods of VI do not apply to such models. This challenge is overcome by applying a degree-theoretic result to the VI formulation. The game-theoretical models explicitly represent the interaction between the emissions and electricity markets, which provides us with valuable insights toward the inefficiency of different allocation rules.

The last application deals with the dynamic traffic flow assignment model of single bottleneck on a link and with heterogeneous commuters classes. A linear complementarity problem is developed to find the dynamic user equilibrium of the discrete-time single bottleneck model. The complementarity formulation provides

a formal framework for rigorous mathematical analysis of the dynamic equilibrium problem, and offers a provably convergent algorithm for computing a solution. The uniqueness of equilibrium is established in the homogeneous case. The departure patterns are investigated in the heterogeneous case.