PARALLEL ALGEBRAIC MULTIGRID FOR PRESSURE POISSON EQUATION IN A FINITE ELEMENT NAVIER-STOKES SOLVER

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ABSTRACT

The focus of this thesis is on a parallel Algebraic Multigrid (AMG) algorithm and its application within the parallel Navier-Stokes finite element (FEM) solver, *PHASTA*. Specifically we focus on solving the Pressure Poisson Equation (PPE) arising from the finite element discretization of the incompressible Navier-Stokes equations using parallel AMG as the preconditioner to a Conjugate Gradient (CG) solver.

First a review of recent serial and parallel AMG algorithms is provided to set the stage for a detailed description of our state-of-the-art multilevel serial AMG implementation. Classical Ruge-Stüben AMG is set as the base algorithm. It is then altered in several aspects to better fit for our flow problems. Also, recent smoother advancements like polynomial smoothing are tested and implemented with different options. A recently developed external accelerator for multigrid, the General Global Basis (GGB) method, is studied and implemented. In the serial AMG study, a dramatic reduction (usually above 90% in our test problems) in the number of iteration vectors is observed.

The parallelization of the PPE solver is then studied in great detail. The difficulties for AMG are carefully studied. These difficulties include the matrix-matrix product form of PPE from multiplicand matrices that are locally incomplete, the special parallel data structures designed for a simple Krylov solver, and the mesh-based partitioning scheme. These difficulties make it impractical to construct a parallel AMG cycle that is identical to the serial case. A new AMG algorithm for parallelization that works close to the serial one but is easy to parallelize, is then proposed. This new parallel AMG algorithm separates the smoothing process for different levels of AMG, and makes use of the PPE construction at the finest level to apply a complete smoothing operation only at that level. The implementation of this new algorithm is carefully matched within the framework of the current parallel FEM software. A serial validation of the algorithm is also carried out.

The algorithms are also contrasted to assess efficiency. Techniques like freezing setups for multiple solves are studied to increase the efficiency for long time runs

in the numerical examples. For several examples tested, significant reduction in the number of iteration vectors does not degrade as the number of processors is increased. The scaling of the algorithm is tested on IBM Bluegene supercomputer, and the excellent computational time scalability of the original FEM plus CG solve is maintained with each algorithm improvement up to 16k processors.