POWER CONTROL FOR MULTIUSER COMMUNICATION SYSTEMS AND COMPUTATION OF GENERALIZED NASH EQUILIBRIA

By

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ABSTRACT

This thesis considers the problem of power control for a multiuser communication system in a frequency selective environment where users share a common spectrum and can interference with each other. To ensure the prescribed quality of service, we propose a non-cooperative generalized Nash game in which each user competes against the others by choosing his power allocation that attains the desired rate with the minimum transmit power. We reformulate the generalized Nash game as a mixed complementary problem and obtain a sufficient condition for the nonemptiness and boundedness of the solution set of the problem. By reformulating the game again as a variational inequality, we obtain a sufficient condition for uniqueness of the Nash equilibrium. To compute the solution of the game, we develop a distributed iterative water-filling algorithm where each user locally measures the interference-plus-noise and independently solves his own optimization problem. We prove that the condition for the uniqueness of the Nash equilibrium is sufficient for the convergence of the algorithm.

To reach a social optimum of the multiuser communication system, we consider the maximization of the sum of achievable rates of all users in the system. With the sum-rate being a nonconcave function, we develop a branch-and-bound procedure to compute an approximate global optimal solution. The computational results show that the algorithm can achieve noticeably larger sum-rate than the existing online solvers and the iterative water-filling algorithm in a reasonable amount of time.

Finally we derive a sufficient condition in matrix form for the convergence of the distributed algorithm for solving standard Nash equilibrium problems (NEPs). We also compare the distributed algorithm with the existing centralized relaxation algorithm. For generalized Nash equilibrium problems (GNEPs) a penalty method exists where a sequence of penalized NEPs are solved iteratively. The convergence has been proved assuming that the Nash subproblems are solves exactly. We propose an inexact version of the penalty method by solving each subproblem in a distributed way and establish the convergence.