STATISTICAL EQUILIBRIA OF THE COUPLED BAROTROPIC FLOW AND SHALLOW WATER FLOW ON A ROTATING SPHERE

By

Xueru Ding

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Examining Committee:

Chjan C. Lim, Thesis Adviser
Isom Herron, Member
David Isaacson, Member
György Korniss , Member
Fengyan Li , Member

Rensselaer Polytechnic Institute
Troy, New York

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ABSTRACT

The motivation of this research is to build equilibrium statistical models that can apply to explain two enigmatic phenomena in the atmospheres of the solar system's planets: (1) the super-rotation of the atmospheres of slowly-rotating terrestrial planets - namely Venus and Titan, and (2) the persistent anticyclonic large vortex storms on the gas giants, such as the Great Red Spot (GRS) on Jupiter.

My thesis is composed of two main parts: the first part focuses on the statistical equilibrium of the coupled barotropic vorticity flow (non-divergent) on a rotating sphere; the other one has to do with the divergent shallow water flow rotating sphere system. The statistical equilibria of these two systems are simulated in a wide range of parameter space by Monte Carlo methods based on recent energy-relative enstrophy theory and extended energy-relative enstrophy theory. These kind of models remove the low temperatures defect in the old classical doubly canonical energy-enstrophy theory which cannot support any phase transitions.

The other big difference of our research from previous work is that we work on the coupled fluid-sphere system, which consists of a rotating high density rigid sphere, enveloped by a thin shell of fluid. The sphere is considered to have infinite mass and angular momentum; therefore, it can serve as a reservoir of angular momentum. Unlike the fluid sphere system itself, the coupled fluid sphere system allows for the exchange of angular momentum between the atmosphere and the solid planet. This exchange is the key point in any model that is expected to capture coherent structures such as the super-rotation and GRS - like vortices problems in planetary atmospheres.

We discovered that slowly-rotating planets can have super-rotation at high energy state. All known slowly-rotating cases in the solar system - Venus and Titan - have super-rotation. Moreover, we showed that the anticyclonicity in the GRS - like structures is closely associated with the relatively low mechanical energy to enstrophy ratios and a rapidly rotating sphere.