PROPAGATION OF ULTRA-SHORT ELECTROMAGNETIC PULSES IN A MAXWELL-DUFFING MEDIUM

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ABSTRACT

In this thesis we present a theoretical and numerical study of propagation dynamics of extremely short electromagnetic pulses in spatially homogenous double-resonant media with cubic nonlinear electric response. The mathematical model we explore (the Maxwell-Duffing model) is designed to capture salient features of the double-resonant metamaterials for which the combined effect of the interaction of the electric and plasmonic fields with the LC circuit-like nano-inclusions leads to simultaneous strong electric and magnetic resonances. In particular, we study the new nonlinear phenomena introduced by the strong interaction of the components of the pulse spectrum that experience negative and positive index of refraction with one another due to the nonlinearity of the medium.

We find numerically a whole range of solitary traveling-wave solutions. They are represented by homoclinic orbits of the corresponding system of ordinary differential equations, with the velocity of propagation entering as an eigenvalue. We demonstrate that the Maxwell–Duffing medium supports a discrete spectrum of solitary waves of different velocities, sizes and geometric structures. The medium has a preferred velocity range which is populated by simple ”one-hump” solitary waves of a very robust nature. The more intricate structures (multi-hump waves) are sparsely distributed throughout the velocity spectrum. We prove rigorously that all the traveling wave solutions are neutrally linearly stable. Furthermore, our numerical simulations on the full system of evolutionary partial differential equations indicate that these solitary waves are nonlinearly stable as well.

We designed a numerical algorithm and conducted numerical simulations aimed at elucidating the role of traveling solitary waves in the pulse dynamics. Our numerical results show that Gaussian-shaped pulses break up into the solitary waves of near preferred velocity; they are followed by some continuous radiation in their wake. The interactions of the simple solitary waves are largely of elastic nature. The phase shift can be observed for all interactions.