

**OPTICAL PULSE DYNAMICS IN ACTIVE MEDIA  
WITH NEGATIVE REFRACTIVE INDEX**

By

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## ABSTRACT

Metamaterials exhibit electromagnetic properties unattainable in conventional materials. These materials can be manufactured to have negative electric permittivity and negative magnetic permeability, and thus negative refractive index ( $n_i < 0$ ) by the periodic inclusion of small metallic elements. The negative values of the electric permittivity and magnetic permeability occur when certain frequencies of electromagnetic waves cause resonant currents to be induced in the nano-inclusions of the metamaterial. Thus, a metamaterial will have negative refractive index only for a limited bandwidth of electromagnetic waves — outside of this bandwidth it has a positive refractive index. A goal of this thesis is to study the interaction of two waves at the interface of this bandwidth, that is, when one is in the positive and the other in the negative index regime. The interaction of these waves is induced through an appropriate nonlinearity. This thesis explores doping of the metamaterial by active atoms in the  $\Lambda$  Configuration, which, will provide the desired nonlinearity. Under the proper conditions, this nonlinearity will cause the light propagating in the positive index regime to switch into light propagating in the negative index regime. This switching phenomenon is simulated in numerical experiments following the derivation of the equations describing the physical situation.

The set of Maxwell-Bloch partial differential equations that model the light-matter interaction under investigation is derived from Maxwell's Equations, the Lorentz model, and Schrodinger's Equation. After non-dimensionalizing these equations, the method of multiple scales is utilized to derive the appropriate envelope equations for the electric field and material variables. The resulting system of differential equations is solved based on the characteristics of the equations using a second order implicit numerical scheme. Numerical simulations of one input pulse in the positive index regime, intended to switch to light the negative index regime, were run with a variety of boundary and initial conditions. The results show the input pulse transform to continuous radiation when its amplitude is small and its switching into another pulse propagating in the opposite direction when its

amplitude is higher.