IMAGING FROM SPARSE MEASUREMENTS

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ABSTRACT

In this dissertation we consider the inverse problem for the scalar wave equation with sparse and non-equally spaced sources and receivers. We develop a method to weight different parts of the data differently to compensate for nonuniform sampling.

We use the single-scattering (Born) approximation and an inversion formula based on a filtered version of the adjoint operator of the forward model. We study the point-spread function to determine the resolution of the reconstruction. For sparsely positioned sources and receivers, the point-spread function can be approximated by a weighted sum of oscillatory functions. A regularized least-squares method can be formulated to determine weights that make the point-spread function as close as possible to the Dirac delta function. Once the weights are determined, the same set of weights can be applied to form an image from measured data.

We test our minimization scheme with different regularization parameters. The sensitivity of the reconstruction with respect to noise and positioning error is tested. We can choose regularization parameters properly to improve resolution and gain stability at the same time.

This method also applies to the regular grid problem. We show examples of point-spread functions constructed with weights corresponding to three different types of source-receiver geometry with different frequency bands. These results not only show that using the right weights improves the resolution relative to reconstructions with constant weights, but also illustrate the relation between resolution and the source-receiver geometry and bandwidth.