CREEPING CHANNEL FLOW WITH FLEXIBLE OBSTRUCTIONS

By

Adam Boucher

An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: MATHEMATICS

The original of the complete thesis is on file
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Examining Committee:

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Zvi Rusak, Member

Rensselaer Polytechnic Institute
Troy, New York

August 2008
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ABSTRACT

Existence results are obtained for a model free boundary problem coupling the Stokes equations of creeping, viscous flow in a two-dimensional, bounded domain to a one dimensional, linear Kirchoff model of a thin beam undergoing small, elastic deflection. Here functional analysis, control theory and shape optimization are applied to formulate the combined problem as a 1-dimensional elasticity problem with non-linear forcing. By formulating the problem as a non-linear elasticity problem, the Schauder fixed point theorem is applied to obtain the existence of at least one weak solution.

First, the flow problem is formulated in a bounded region with a fixed fiber position. This formulation is followed by a demonstration of the existence of velocity and pressure fields which obey certain regularity requirements with the fiber position fixed. After obtaining the existence of a solution to the flow problem for any fixed fiber position, the problem is reformulated to allow the fiber to change shape in response to the flow. To achieve this the hydrodynamic drag is defined to provide a nonlinear forcing along the length of the fiber. To solve the simultaneous flow-fiber problem the problem is formulated in terms of a fixed point for the fiber mapping. The existence of a fixed point is demonstrated through the following procedure.

A mapping which takes a fixed fiber shape and returns another fiber shape in the same regularity class is defined. This mapping is shown to be continuous and to map closed sets to relatively compact sets. The Schauder fixed point theorem is applied to establish the existence of at least one fixed point for any fibers with sufficiently high flexural rigidity. By the construction of the mapping, this fixed point defines a steady state solution to the coupled flow-fiber problem, and defines a feasible shape for the free boundary.