

**A Universal Prediction of Stall Onset for Airfoils  
at a Wide Range of Reynolds Number Flows**

By

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## ABSTRACT

The inception of leading-edge stall on two-dimensional, smooth, thin airfoils at various Reynolds number flows in the range  $O(10^3)$  to  $O(10^7)$  is investigated by an asymptotic approach and numerical simulations. The theory demonstrates that a subsonic flow about a thin airfoil can be described in terms of an outer region, around most of the airfoil chord, and an inner region, around the nose, that asymptotically match each other. The flow in the outer region is dominated by the classical thin airfoil theory. Scaled coordinates and a modified Reynolds number  $Re_M$ , both based on the nose radius of curvature, are used to account for the nonlinear behavior and extreme velocity changes in the nose region, where stagnation and high suction occur. It results in a reduced-order model problem of a uniform, compressible, viscous flow past a semi-infinite canonic parabola. The inner far-field is governed by a circulation parameter  $\tilde{A}$  that is related to the airfoil's angle of attack, nose radius of curvature, thickness ratio, camber, and flow Mach number. The model parabola problem is solved numerically for various  $Re_M$  and  $\tilde{A}$  using two methods. The first technique uses the steady Reynolds-Averaged Navier-Stokes (RANS) equations with the Spalart-Allmaras turbulence model for simulating moderate to high  $Re_M$  flows. The second method applies direct numerical simulation (DNS) of the unsteady and incompressible Navier-Stokes equations for low to moderate  $Re_M$  flows. In both methods, the critical value  $\tilde{A}_s$  is determined when a large separation zone first appears in the nose flow and the minimum pressure coefficient suddenly drops. The change of  $\tilde{A}_s$  with  $Re_M$  is determined and these values indicate the onset of stall on the airfoil. The DNS results show that  $\tilde{A}_s$  decreases with  $Re_M$  for  $Re_M < \sim 250$ , in agreement with Marginal Separation Theory (MST). However, calculations display the appearance of unsteady waves above a limiting value  $Re_{M_{crit}} \sim 250$ , where  $\tilde{A}_s$  reaches a minimum of  $\sim 1.55$ . For  $Re_M > 250$ , these waves delay in the mean flow the onset of large separation to values of  $\tilde{A}_s$  that increase with  $Re_M$ , in contrast to MST predictions, and overlap the  $\tilde{A}_s$  RANS predictions. The RANS computed  $\tilde{A}_s$  shows a continuing increase with  $Re_M$  and good agreement for a wide range of Reynolds number with much available experimental data for the stall of airfoils. The simulations also exhibit the existence of stall hysteresis loops between  $\tilde{A}_s$  and a reattachment value of  $\tilde{A}$ , which is greater than MST stall prediction for a given  $Re_M$ . The reduced-order model is also applied to predict

the onset of leading-edge sheet cavitation on thin, smooth, hydrofoils. The approach determines the minimum pressure coefficient and a universal criterion for the critical cavitation number as function of hydrofoil's geometry, Reynolds number, and fluid thermodynamic properties. Predictions show agreement with results from available experimental data.