MEAN VELOCITY OF A DEVELOPING TURBULENT WALL BOUNDARY LAYER AT A ZERO PRESSURE GRADIENT

By

Lei Xu

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Examining Committee:

Zvi Rusak, Thesis Adviser
Luciano Castillo, Thesis Adviser
Assad Oberai, Member

Rensselaer Polytechnic Institute
Troy, New York

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ABSTRACT

A novel model equation for computing the properties of an incompressible, zero-pressure-gradient turbulent boundary layer over a smooth, solid, flat wall is developed. The mean axial velocity inside the boundary layer is described by the Reynolds-averaged Navier-Stokes equation where the Reynolds turbulent stresses are given by an extended mixing-length model. In this model the non-dimensional mixing length is given by a polynomial function in terms of non-dimensional distance from the wall. The coefficients of the polynomial function use information from pipe flow data and are modified for the case of the flat wall flow by iterations to satisfy the wall and far-field conditions. A stream function approach is applied where at leading-order it is described by a similarity function. A numerical integration method of the model ordinary differential equation for the similarity function is developed. The computations provide the mean axial velocity profiles, the derivative of the velocity with vertical distance as an accuracy indicator, the profile of the Reynolds stresses along the developing boundary layer, the relationship between the Reynolds number based on momentum thickness and that based on the axial distance from the wall leading edge, the local and accumulated skin friction coefficients, and the boundary layer growth with axial distance. The model allows a natural transition of mean flow solution from laminar to turbulent. Results agree with much experimental data for a wide range of flows with Reynolds number from 600 up to 10^7 except for the transition region from laminar to turbulent flow. Furthermore, results shed light on the von Kármán constant as a function of Reynolds number, the possible scaling laws, the possible four-layer structure of the velocity profile at each section, and the range of non-dimensional distance from the wall and of Reynolds number where either the *log law* or the power law may be applicable.