# A STATISTICAL APPROACH FOR PRICE-OPTIMAL PLUGIN ELECTRIC VEHICLE CHARGING


By

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A Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

Major Subject: COMPUTER AND SYSTEMS ENGINEERING

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Rensselaer Polytechnic Institute
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March 2017
(For Graduation May 2017)

# CONTENTS

## LIST OF TABLES

# LIST OF FIGURES

# ABSTRACT

Systems that consume energy and have energy storage capacity can defer purchasing energy during high price periods and use their stored energy to "ride through" those periods without impacting their ability to carry-out their main objective. Utilizing this practice can save money for consumers and, depending on the degree to which supply network loading is accurately reflected in prices, has the potential to smooth that network's demand with time. This thesis investigates practical methodologies for best utilizing this practice for achieving cost savings by considering the specific application of Plugin Electric Vehicle (PEV) charging. Specifically, this thesis documents the evolutionary development of a practical algorithm for charging a fleet of PEVs within a Real Time Market (RTM) price environment. The final algorithm, termed "STAT2," is also designed to limit charging rates to avoid violating network constraints, and to accommodate the unique charging needs of multiple independent PEVs.

The STAT2 algorithm is designed around statistical price-energy distributions which use forecast data and historical statistics to anticipate prices and charging capacity through an entire charging period. Such a model is necessary because these parameters are not known with certainty until real time, yet anticipating them for the entire charging period is critical for successful optimization. The statistical price-energy distributions are used to generate vehicle-specific price setpoints which are compared, in real time, to the current RTM price to make a charging decision. The distributions are continually updated based on the latest charging and network conditions so that vehicle charging goals can be achieved adaptively at minimal cost.

The STAT2 algorithm was iteratively developed by using historical data to simulate charging a fleet of Tesla PEVs in the New York City price environment for a given developmental algorithm, assessing algorithm performance by comparison to the performance of simple and ideal (theoretical best) baseline algorithms, and then revising the developmental algorithm. Compared to the simple method of charging a PEV immediately after midnight, the STAT2 algorithm reduces costs by 16%, compared to an ideal cost savings of 23%. These cost savings are reduced as the combination of network constraints and increased PEV penetration reduces opportunities for deferring charging. However, the algorithm's ability to successfully meet nightly PEV charging

goals closely matches the best performance possible, even as charging capacity limitations become significant.

Most of the STAT2 algorithm's cost savings potential comes from the fact that it operates as a "bang-bang" controller operating on a volatile price signal. However, current RTM price signals do not reflect instantaneous load conditions, so this behavior improves cost savings while exacerbating, rather than reducing, load swings. These swings must be counterbalanced with additional regulation capacity. This thesis attempts to quantify the combined cost impact of using the STAT2 algorithm and the resulting increased need for regulation. Depending on the regulation cost model assumed, the cost savings potential of the STAT2 algorithm is either reduced or reversed.

# 1. INTRODUCTION

Systems that consume energy and have energy storage capacity can defer purchasing energy during high price periods and use their stored energy to "ride through" those periods without impacting their ability to carry-out their main objective. This chapter introduces the problem of practical, cost-optimal, Plugin Electric Vehicle (PEV) charging as a specific problem within that general context.

This chapter then frames the work of this thesis in the context of similar research, and presents a general outline of the work performed.

This chapter is broken into several parts:

1. Background and Motivation
2. Related Work and Unique Contributions
3. Thesis Outline

## 1.1 Background and Motivation

Society demands a reliable supply of useful energy at minimum cost.

One factor which drives a significant portion of energy costs is the time-varying nature of energy demand. Energy infrastructure and operational costs are, in part, driven by the need to adequately meet this time-varying demand. On a seasonal and daily timescale, infrequent periods of high demand drive the capacity requirements, and therefore capital costs, for many pieces of energy infrastructure. Also, the need to maintain system performance parameters within specifications (e.g., network voltages throughout electrical distribution networks) over a potentially wide range of load conditions drives the use of more capital-intensive equipment.

On a shorter (hourly or less) timescale, demand volatility drives costs. For example, electrical generating units which can be brought on-line and taken off-line quickly are employed. Use of these units comes at a premium, as the overhead associated with their infrequent use must be compensated. Similarly, volatility in demand on a minute-by-minute basis needs to be met with a correspondingly agile generating source. Such sources of regulation are also accompanied by a cost premium.

The various costs tied to the time-varying nature of energy demand are reflected in prices. As a result, reducing the discrepancy between peak demand and average demand, and smoothing out intra-hour demand volatility, can be key mechanisms for reducing energy costs. Likewise, if additional energy demand is only added during periods of low energy usage or if demand is added in a way that minimizes intra-hour load volatility, added costs can be minimized.

The ability of systems to store energy can play a role in reducing the discrepancy between peak and average energy demand and smoothing out intra-hour demand. Such systems can withdraw energy from supply networks during periods of lower demand and use it during periods of higher demand. The magnitude of the impact of such energy storage systems depends on the amount of energy they can store relative to the rate of energy exchange. Large ratios of storage-to-exchange imply greater deferability of electricity purchase, and such systems therefore have an ability to help balance loads across long time scales (such as throughout the day, or between seasons). Smaller ratios of storage-to-exchange may be insufficient to help balance loads across a daily timescale, but may be sufficient to help smooth intra-hour load volatility.

PEV's represent one technology for energy cost reduction in general. This is primarily due to fuel costs. The transportation cost per mile of PEVs is less than that of gasoline-powered vehicles. This trend continues to be true despite the recent collapse in gasoline prices resulting from hydraulic fracturing (the lowest national average post-"fracking" retail gasoline prices were $1.72/gal in February 2016, but the "eGallon" equivalent prices at that time were only $1.11 [1]-[3]). These fuel cost savings, combined with the improving range and performance characteristics and reduced initial-purchase costs, ensure that PEVs will continue to gain market share.

The cost savings potential of PEVs can be reduced as their use increases and greater demand for electricity drives up electricity prices. However, <u>the load-balancing attributes of PEVs can work to minimize these cost increases</u>. This is largely accomplished through the general trend of charging PEVs at night when demand is otherwise low, and prices therefore cheap. PEV charging cost increases can also be reduced by exercising deferability on an intra-hour time scale to selectively purchase intermittently available low-priced electricity.

This thesis explores the ability of PEV's to reduce charging costs by selectively purchasing electricity when Real-Time Market (RTM) prices are low while also achieving desired charging goals. Both the magnitude of potential cost savings, as well as the characteristics and performance of algorithms for practically extracting those potential savings, are investigated. These results are intended for application to the specific goal of reducing PEV charging costs as well as to the general goal of developing methodologies for leveraging systems with energy storage for energy cost savings.

It is noted that, notionally, any consumer cost savings from selective RTM purchasing of electricity would reflect the benefit to the electricity grid of smoothing out electricity demand. However, the majority of this thesis does not assume cost-optimization from a consumer standpoint will positively impact grid operation. Rather, the work of this thesis is framed from a consumer cost-optimization perspective, and grid performance impacts are largely beyond the scope of this thesis.

## 1.2 Related Work and Unique Contributions

There is a significant body of existing literature on PEV charging. This research is driven by the sense that large scale adoption of PEVs by the general public in the near future is inevitable, and it is just a matter of time before economic, environmental, and political factors force a drastic change in the populace's preferred transportation fuel source. Popular media stories such as [4] and [5], as well as large-scale studies by establishment technical organizations ([6], [7], and [8]) of the overall impact of large degrees of PEV charging, demonstrate this belief. From a technical perspective, research as early as [9] recognized that large scale PEV charging has both the potential to significantly increase peak demand and the potential to improve grid load factor without increasing peak demand, depending on how it is implemented. Decades of subsequent research has aimed to elaborate on this general theme and investigate mechanisms which can be implemented to accommodate large scale PEV charging while minimizing the need for significant infrastructure upgrades to support peak demand increases.

Studies show that, if PEV loads are well-managed, "large-scale deployment of PHEVs will have limited, if any, negative impacts on the electric power system in terms of additional generation requirements," [10]. References [6] and [7] reinforce these conclusions but highlight the fact that, if PEV charging is uncoordinated, additional PEV charging loads could become superimposed onto existing periods of peak demand, with attendant detrimental effects on grid performance. Reference [11] further emphasizes the difficulty of coordinating PEV charging, the potential impact of uncoordinated PEV charging, and that increased demand associated with large scale PEV use will drive up electricity costs even if peak demand impact is minimized. Thus, the sizable benefit of establishing systems which shift PEV charging to periods of low demand in a coordinated manner is established.

References [12] and [13] expand on the concerns brought up in [6], [7], [9], and [11], by highlighting the need to consider specific distribution system details when evaluating the impacts of high levels of PEV charging, and developing systems to manage that impact. Especially highlighted is the need to consider system losses, voltage regulation, and the capacity of distribution transformers when determining the extent to which additional PEV charging capacity can be accommodated.

Development of specific methodologies for managing PEV charging is seen mostly in academic literature. These studies generally consider the need to "optimize" PEV charging for a fleet of vehicles while taking into account realistic constraints. Although different research focuses on optimizing different objective functions, all optimizations share the need to select a mechanism for determining which PEVs will charge at what times.

References [14] and [15] focus on computation/communication-efficient algorithms for maximizing PEV owner satisfaction by charging as rapidly as possible without violating simple network constraints. Reference [16] also focuses on "urgent" charging, but prioritizes charging based on explicitly calculating the grid impact of the contending PEVs. PEVs which cause highest network losses are deprioritzed, and explicit network modeling is used to ensure that the resulting charging schedule does not drive network voltages beyond their limits. Furthermore, [16] recognizes that users' charging urgency is cost dependent, and creates three time zone schedule bins to accommodate the varying user cost sensitivities to charging urgency.

References [17]-[19] optimize PEV charging around cost rather than charging urgency. Such optimization benefits both user and grid operator costs. Periods of low demand drive electricity costs down to incentivize use of underutilized generation resources. Shifting PEV charging to such periods benefits PEV owners with lower energy costs and provides higher revenue for generators without requiring investments in additional capacity. Reference [18] considers that overall load profile is representative of cost profile, seeks to optimize load "valley filling," and explores distributed algorithms for achieving this end. Like Reference [16], Reference [19] makes use of a network model for its optimization. A general charging schedule is developed which minimizes network voltage droop. This has the desired valley filling effect while also ensuring that network voltage limits are not violated. It is noted that valley filling optimization typically works against the goal of charging urgency since charging must be deferred until it is time to fill the valley. Thus, cost-optimizing charging algorithms have the need to fully charge a PEV by the end of a charging period as a constraint, rather than having charging urgency optimized.

Reference [20] provides a general framework for PEV charging in which various appliances, including PEVs, get assigned a utility function. The system then prioritizes charging in a way that maximizes overall utility. This allows some PEVs to be charged based on charging urgency and others to be charged based on cost minimization.

References [21] and [22] together consider cost optimization, but within a broader context than others. Specifically, these references seek to integrate the financial rewards of providing regulation services to the grid (described in [23] and [24]) into cost-optimal PEV charging. To this end, the PEVs optimize across three markets; the Day-Ahead Market (DAM), RTM, and ancillary services market.

Like the PEV charging methodology research described above, this thesis focuses on PEV charging from the viewpoint of constrained optimization. The goal of cost-optimization is selected since the increased battery capacity of today's PEVs (such as is available in Tesla vehicles) greatly exceeds average daily driving requirements and thus eliminates the need to focus on "range anxiety." Like [21] and [22], this thesis focuses on the ability of PEVs' fast-responding characteristics to respond to the volatile nature of system imbalances and reduce costs. However, rather than interfacing directly with the regulation marketplace, this thesis seeks to exploit the properties of PEVs for cost savings while interfacing only with the RTM, which has less onerous participation requirements. Interfacing with the RTM alone avoids the complication of coordinating between three marketplaces. Additionally, this thesis centers attention on statistical price prediction rather than assuming deterministic prices. Also, unlike [21] and [22], this thesis models network capacity limitations and their effect on cost and charge completion performance.

Aside from the overall concept of cost-optimizing PEV charging by selectively charging based on RTM prices, this thesis makes several unique contributions with respect to algorithm development. A RTM statistical price forecast is developed for use as the centerpiece of the charge decision algorithm. Also, an iterative solution technique is developed to resolve anticipated future network constraint violations within that statistical framework.

Furthermore, the methodologies developed in this thesis were developed by maximum utilization of actual market data. This enabled algorithm tuning to enable cost savings from complex effects which would be difficult to consider in a first-principles based model. Also, the scope of simulations was extended to cover an entire year, rather than just a few sample days, in order to characterize aggregate performance throughout a wide range of market conditions.

## 1.3 Thesis Outline

This thesis is broken down into the following sections:

1. Chapter 1: Introduction - Introduces the problem of practical, cost-optimal, PEV charging as a specific problem within the general context of using systems with energy storage to smooth-out the time varying nature of energy demand.
2. Chapter 2: System Model and Formulation – Introduces the PEV charging system model and RTM price environment assumed for simulation studies in this thesis.

3. Chapter 3: Homogeneous Vehicle Charging Without Network Constraints – Begins algorithm development for price-selective PEV charging for the simplified case of charging one PEV without consideration of network constraints. A statistical price-model-based purchasing algorithm is developed and its cost savings potential is quantified through simulation studies.
4. Chapter 4: Homogeneous Vehicle Charging With Network Constraints – Adapts the algorithm developed in the Chapter 3 to address the additional complexity of charging a fleet of vehicles on a network with limited overall capacity. The work done in this chapter makes the simplification of treating all vehicles within the fleet identically. Simulation results quantify cost savings potential.
5. Chapter 5: Heterogeneous Vehicle Charging With Network Constraints – Reformulates the algorithms developed in previous chapters for a general case of charging diverse vehicles with independent and random arrival and departure times. Describes a simulation framework to assess algorithm performance (actual results tabulated in Appendix A, but discussed in Chapter 5)
6. Chapter 6: Impact of Regulation Costs – Investigates the overall cost impact that the need for additional regulation would have on the results of previous sections. Describes a simulation framework to assess algorithm performance (actual results tabulated in Appendix A, but discussed in Chapter 6)
7. Chapter 7: Conclusions and Future Work – Summarizes high-level insights developed in this thesis, and lists relevant future studies which could be performed to expand those insights.
8. Appendix A: Simulation Results – Presents simulation results for all scenarios previously considered, but now performed on a consistent framework. These results facilitate results comparison between different simulation scenarios (i.e., homogenous and heterogeneous charging scenarios, and constrained and non-constrained scenarios)

9. Appendix B: Homogeneous Charging Algorithm Description – Presents a detailed description of the general problem of price-optimal PEV charging of a heterogeneous collection of PEVs, and the statistical algorithm developed herein to solve it.
10. Appendix C: Representative Transient Results – Presents a representative set of transient results for various PEV charging algorithms for a single vehicle and day. These results show the minute-by-minute behavior of the statistical algorithm developed herein, as well as the ideal and simple algorithms used for performance comparisons.

# 2. SYSTEM MODEL

The goal of this chapter is to provide a high level description of the major entities assumed in the simulation studies of this thesis. This chapter is broken into the following parts:

1. General Model
2. PEV System Description
3. Price Environment Description

## 2.1 General Model

The overall goal of this thesis is to develop practical means to charge one or more PEVs in a way that minimizes cost by selecting low price times for charging. This problem inherently involves interaction between two entities; the Plugin Electric Vehicle (PEV) system (and associated charger) and the price environment. Additionally, a charging framework must exist to facilitate the interaction.

The charging framework consists of a charging period in which one or more vehicles will interact with the price environment. The charging period is variable, depending on the nature of the simulation to be performed. However, the charging period is typically several hours long. The charging period is broken into many timesteps of equal duration. Use of timesteps allows the problem to be discretized so that the total number of states to be considered is proportional to the number of timesteps in the charging period. Typically, these timesteps have a duration of five minutes. Timesteps are selected so that each timestep is characterized by constant electricity price, as dictated by the price environment. The goal of each vehicle is to transition from an initial actual charge level ($L_{init}$) to a desired final charge level ($L_{final}$) over the course of the charging period by purchasing electricity from a subset of timesteps which have low prices. The PEV charging cost for a given charging period is simply

$$Cost = \sum_{i=1}^{ni} \left( q(i) * p(i) \right) \tag{2-1}$$

where 'q(i)' is the quantity of energy purchased at timestep 'i', 'p(i)' is the price of energy at each timestep, and 'ni' is the total number of timesteps in the charging period.

Evaluations consist of simulating charging for multiple charging windows so that results are based on aggregate performance over a wide range of conditions.

The PEV system and price environment are introduced in the following two sections. However, much additional detail for how the system and environment interact is left for technical discussions within later chapters.

## 2.2 PEV System Description

PEV charging is an ideal system for studying the general ability of systems with energy storage to reduce costs through selective purchasing of electricity. PEV charging systems have many of the attributes of other systems with energy storage without being overly complex. For PEVs, charging and usage occur at different times and can be examined independently. For this study, modeling energy usage is essentially neglected and daily energy usage is assumed fixed. The remaining charging goal is simple; purchase a given quantity of electricity over a fixed time at lowest cost. By contrast, consideration of building heating and cooling would require simultaneous consideration of energy purchasing and energy use to maintain building temperature within a desired range. A heat loss model and external environment would need to be developed. Additionally, energy storage in a PEV is essentially lossless, so there is no cost penalty associated with buying energy early in a period and storing it for a longer duration. Again, this contrasts with a building system, where energy stored by increasing the temperature of the storage medium drives a higher energy loss rate.

The given PEV system consists of two time-invariant attributes; the energy storage capacity of a battery [kWhr] and the rated battery charging rate [kW]. Additionally, there are attributes dictated by the context of the charging scenario, such as initial actual and final desired charge levels [kWhr], and charging period duration [hr]. These five parameters are selected to perform simulations which 1) approximate a realistic charging scenario and 2) define a charging scenario with sufficient deferability so there is reasonable potential for cost savings.

Context-specific attributes are defined for specific simulation studies within subsequent chapters. However, time-invariant attributes are identical throughout this thesis and are discussed here. The fixed attributes are based on a Tesla PEV with an 85 kWhr battery. The charging rate for the PEV can span a wide range of values depending on the rating of the on-board charging system and the outlet/connector rating (see Table 2.1). In most cases, the rating of the connector is limiting. A 9.6kW charging rate is assumed since users which use their PEV extensively would tend to install the highest capacity charging system which could reasonably be installed at their home. When charging, the 9.6kW charger will purchase 0.80 kWhr of electricity during each 5-minute period during which the Real Time Market (RTM) price persists.

**Table 2.1 – Summary of Charging Systems for Tesla PEV. [25]**

| **Environment** | **Charger Type [kW]** | **Outlet/Connector Type** | **Voltage Rating [Vac]** | **Current Rating [A]** | **Power Rating [kW]** |
|---|---|---|---|---|---|
| Household | 10 | NEMA 5-15 | 110 | 12 | 1.4 |
| Household | 10 | NEMA 5-20 | 110 | 15 | 1.8 |
| Household | 10 | NEMA 10-30 | 240 | 24 | 5.8 |
| Household | 10 | NEMA 14-50 | 240 | 40 | 9.6 |
| Public | 10 | J1772 | 110 | 16 | 1.92 |
| Public | 20 | J1772 | 240 | 80 | 19.2 |

Since the simulations of this thesis discretize charging periods into timesteps for convenience, the charging rate is reduced into a parameter which represents the maximum amount of energy which can be charged during a single five-minute timestep. This parameter is the vehicle charging constraint ($c_v$ = 0.00080 MWhr). Similarly, battery levels are resized to units of MWhr for compatibility with the units of the price environment. Thus, the rated battery capacity is listed as ($L_{max}$ = 0.085 MWhr).

Some simulated charging characteristics may not be prototypical of actual charger behavior. In some cases, vehicle specific charging rate is reduced so that the combined charging rate across multiple vehicles can be lowered below a constrained level. The assumed vehicle chargers do not provide such a provision. However, it is assumed that a charge controller can be employed which can effectively implement a reduced charging rate by limiting the duration of charging to a subinterval of a timestep and overlay charging from multiple vehicles so that the price environment observes a consistent charge rate throughout the entire timestep. Similarly, some vehicle chargers vary charge rate depending on level to minimize battery wear. That level of complexity is not modeled in these simulations.

## 2.3 Price Environment Description

Selecting the price environment with which the energy system interacts is also an important decision in framing the nature of this thesis. The PEV charging system is assumed to purchase electricity from the RTM. RTM prices vary approximately every five minutes depending on electricity supply and demand. This periodicity is a good match for the level of deferability inherent in PEV charging. Over an eight hour charging period, there are 96 timesteps over which to make purchasing decisions. This contrasts with eight decisions required which would be required if interacting with Day-Ahead Market (DAM) prices for the same period. Price-optimal charging with DAM prices would be relatively straight forward, while charging with RTM prices allows for studying interactions with a more dynamic and uncertain price environment. Interaction with the RTM pushes understanding of real-time demand response characteristics which is becoming increasingly important with the adoption of smart grid technology.

The RTM price environment used throughout this thesis is based on historic data from New York City. This location is assumed because the relatively isolated location with respect to the rest of New York State (NYS) drives electricity prices higher. Specifically, electricity transmission bottlenecks sometimes limit electricity imports to the New York City area, so higher priced local generation sets the market price. This same phenomenon is also expected to increase price volatility, a desirable attribute for this study.

Two other significant assumptions/simplifications should be noted. First, the use of wholesale electricity prices for a simulation of charging a single or small group of PEVs is not prototypical. The residential users who would charge their PEV nightly are not large enough customers to purchase electricity from the wholesale market. Rather, they would purchase electricity from a middleman who would impose their own price structure on the individual consumer. The retail level price structure would likely be significantly different from the wholesale structure. However, the simulation in this study evaluates this retail phenomenon to determine whether selective purchasing strategies can yield significant savings to aggregator middlemen if the strategy were expanded to aggregate the charging of many PEVs.

Secondly, if the selective purchasing strategy were expanded for large numbers of PEV's, the strategy would affect the RTM prices that this simulation uses as input. Such "loading effects" could lower potential savings. This thesis assumes prices are not affected by use of selective purchasing strategies.

### 2.3.1 Additional Background – Market Mechanics [26], [27]

Electricity marketplaces are focused on establishing electricity prices by matching electricity supply with electricity demand. The RTM and DAM perform this same function in different ways and on different time scales.

In the DAM, producers and consumers both provide bids on the price and quantities of electricity they want to trade. Electricity producers provide minimum prices for which they will sell a quantity of electricity, and consumers provide maximum prices for which they will purchase a quantity of electricity. Thus, each MWhr of electricity which is bid to be sold has a minimum price, and each MWhr of electricity which is bid to be purchased has a maximum price. The market outcome is the price at which there are equal quantities of electricity above the minimum "sell" price and below the maximum "buy" price. Producers and consumers of winning bids are expected to provide or consume the agreed upon quantity of electricity at the designated hour, and will get paid or charged the agreed upon price. For the DAM in NYS, the market outcome is determined by simultaneously considering the bids of all buyers and sellers. This is performed for each hour of a given day with results determined by 11am of day-ahead. Thus, the DAM price for any given hour is known at least 13 hours ahead of time.

In the RTM, conditions change too rapidly for organized matching of producer and consumer bids in a manner equivalent to the DAM. Instead, the quantities of electricity which a producer sells and the associated prices are dictated by curves which represent the prices they demand (i.e., $/MWhr) as a function of load they provide. The load is unknown until real time. Rather than buying electricity by negotiating a deal, consumers make purchasing decisions merely by using a quantity of electricity which is metered. The RTM price of electricity is determined by matching supply (in the form of aggregated producer curves) with demand (in the form of metered actual electricity use). The money owed is settled after the fact based on metered data.

The timing of the RTM is as follows. Producer curves are selected for, and remain in effect through, a given hour. The selected curves are based on producer bids which can be submitted up to 75 minutes before the applicable hour, with results posted 45 minutes before the applicable hour. Within each hour, the selected curves and metered data are used to determine prices which remain in effect for five minutes. After five minutes, a new price is established based on more recent load data. Since the process of determining prices based on curves and metered data can take as long as five minutes, the prices in effect at any given moment reflect system loading at least five minutes earlier.

The quantity of electricity traded in the RTM almost exclusively reflects loads that were not forecast, so the quantities traded can be considered random. The random nature of these quantities drives the volatility of RTM prices.

Additionally, it should be noted that regulation services are used to help balance supply and demand on a time scale shorter than that of the RTM. Select generators act to continually (every 6 seconds) balance demand changes. Although the price of regulation is continually revised based on grid conditions, regulation acts as an overhead service and load serving entities are not charged according to the degree of regulation which they require.

# 3. HOMOGENEOUS VEHICLE CHARGING WITHOUT NETWORK CONSTRAINTS

The goal of this chapter is to develop and investigate practical algorithms for cost efficient Real-Time Market (RTM) electricity purchasing for Plugin Electric Vehicles (PEVs) which have the ability to selectively purchase electricity during low-price periods. Various algorithms are studied and their relative effectiveness is quantified through simulation results using historic actual prices from the New York City market. The best performing algorithms are further developed for more complex and realistic scenarios in subsequent chapters.

This chapter is broken into several parts:

1. Historic Price Analysis
2. Simulation Framework Summary
3. Description of Algorithms and Their Development
4. Performance Results Summary and Conclusions

## 3.1 Historic Price Analysis

An analysis of historical RTM prices was performed to better understand the price environment with which the algorithms will interact. Insights from this analysis helped inform algorithm development. Since the price data which is analyzed is also used to evaluate algorithm performance, the insights from this section provide context for later discussion of algorithm mechanics and performance.

Price data trends were developed for the years 2006-2014 using historic actual wholesale RTM prices for the New York City area (obtained from NYISO, [28]). This eight year period was considered long enough to show general behavior which is invariant of year-specific aberrations (e.g., abnormally cold winter, several years of abnormally high fuel prices). However, data processing was required to refine the vast quantity of data (multiple years' worth of data changing with five minute periodicity) into a more tractable set of quantities. This data processing was performed as follows to clearly show overall time-of-year and time-of-day trends:

1. Prices were refined to strictly conform to 5-minute price updates. Although RTM prices are generally updated every 5 minutes, there are a few

exceptional times where prices are updated more frequently on an emergent basis (i.e., Real-Time Dispatch, Corrective Action Mode). For ease of analysis, an aggregation algorithm was used to process the price data to ensure that all price data changes at only 5 minute intervals (time-weighted average prices are used to represent periods where prices change more frequently than every 5 minutes).

2. Statistics were generated for each hour's worth of data. These include hourly average RTM prices ($\mu_{RTM}$) and hourly average RTM prices normalized to the Day-Ahead Market (DAM) price ($\mu_{RTM/DAM}$) for each hour. Also, the standard deviation of normalized prices about their hourly average ($\sigma_{RTM/DAM}$) was calculated.
3. Hourly statistical data was aggregated into a 24x12 matrix of bins, each bin representing one of the 24 hours of the day and one of the 12 months of the year.

These aggregated statistics are shown in Table 3.1, Table 3.2, and Table 3.3, with colored shading used to highlight trends. These tables are presented and described below.

The first trend discussed is the overall RTM trend of electricity prices in units of dollars (Table 3.1).

**Table 3.1 – Aggregate Average Real Time Market Prices ($\mu_{RTM}$): 2006-2014.**

| RTM Price [$/MWhr] - Multi-Year-Average | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hour | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Hour Aggregate |
| 0 | 62.4 | 59.4 | 50.5 | 43.1 | 40.3 | 49.0 | 57.5 | 48.2 | 36.5 | 32.1 | 39.0 | 49.0 | 47.2 |
| 1 | 59.3 | 56.7 | 46.6 | 41.6 | 38.0 | 43.4 | 50.5 | 39.9 | 31.9 | 32.7 | 36.5 | 43.6 | 43.4 |
| 2 | 58.0 | 55.2 | 44.6 | 37.5 | 36.1 | 37.9 | 43.1 | 36.9 | 27.3 | 29.1 | 34.3 | 39.9 | 40.0 |
| 3 | 55.9 | 54.9 | 43.9 | 36.3 | 36.1 | 35.7 | 41.7 | 35.4 | 27.9 | 28.5 | 33.9 | 39.0 | 39.1 |
| 4 | 55.8 | 52.5 | 44.7 | 38.4 | 37.1 | 34.7 | 40.1 | 35.5 | 26.9 | 28.3 | 34.4 | 39.3 | 39.0 |
| 5 | 61.1 | 57.2 | 50.1 | 40.7 | 37.1 | 34.1 | 39.4 | 39.7 | 31.8 | 31.6 | 38.4 | 41.4 | 41.9 |
| 6 | 82.1 | 73.7 | 65.4 | 48.5 | 41.5 | 37.9 | 42.0 | 40.9 | 38.8 | 42.8 | 48.4 | 51.9 | 51.2 |
| 7 | 88.1 | 74.4 | 70.4 | 55.6 | 48.9 | 45.6 | 51.2 | 43.1 | 40.2 | 46.5 | 50.5 | 56.6 | 55.9 |
| 8 | 88.8 | 78.6 | 72.7 | 63.3 | 53.6 | 52.6 | 58.8 | 49.2 | 45.3 | 47.6 | 52.0 | 56.7 | 59.9 |
| 9 | 104.1 | 81.7 | 76.6 | 67.8 | 61.3 | 58.5 | 63.9 | 54.1 | 51.8 | 52.9 | 54.8 | 58.8 | 65.5 |
| 10 | 105.4 | 83.5 | 74.1 | 69.0 | 66.9 | 65.1 | 70.8 | 59.5 | 53.1 | 52.1 | 54.2 | 61.9 | 68.0 |
| 11 | 95.9 | 79.9 | 70.6 | 68.7 | 67.9 | 71.0 | 75.8 | 65.5 | 55.6 | 52.9 | 53.8 | 63.1 | 68.4 |
| 12 | 87.2 | 77.6 | 66.9 | 68.0 | 66.7 | 77.5 | 86.1 | 71.9 | 61.7 | 54.0 | 52.8 | 61.8 | 69.3 |
| 13 | 88.0 | 75.2 | 63.7 | 68.4 | 72.5 | 88.8 | 101.8 | 80.1 | 62.2 | 52.7 | 51.5 | 58.2 | 71.9 |
| 14 | 81.5 | 72.8 | 60.4 | 65.3 | 73.3 | 96.3 | 120.1 | 85.5 | 64.6 | 52.7 | 50.5 | 53.9 | 73.1 |
| 15 | 82.2 | 68.3 | 60.1 | 65.8 | 78.2 | 105.4 | 114.8 | 96.9 | 67.7 | 52.7 | 49.7 | 53.6 | 74.6 |
| 16 | 92.1 | 72.9 | 62.6 | 63.6 | 79.4 | 101.2 | 125.5 | 96.3 | 71.5 | 52.9 | 60.1 | 78.0 | 79.7 |
| 17 | 134.5 | 93.7 | 65.6 | 61.7 | 77.7 | 89.2 | 112.5 | 83.2 | 66.0 | 53.6 | 78.1 | 92.0 | 84.0 |
| 18 | 106.5 | 106.7 | 76.6 | 58.1 | 65.7 | 71.2 | 83.3 | 70.9 | 54.5 | 67.9 | 67.0 | 81.4 | 75.8 |
| 19 | 101.8 | 86.3 | 84.8 | 67.5 | 61.7 | 67.5 | 75.6 | 66.1 | 70.3 | 67.1 | 61.0 | 77.1 | 73.9 |
| 20 | 88.0 | 81.3 | 73.0 | 77.7 | 70.8 | 69.4 | 74.1 | 71.4 | 61.3 | 52.4 | 55.6 | 69.7 | 70.4 |
| 21 | 81.3 | 76.6 | 64.0 | 60.6 | 58.5 | 65.8 | 72.9 | 63.0 | 50.9 | 49.3 | 49.7 | 60.0 | 62.7 |
| 22 | 70.1 | 69.6 | 60.1 | 54.8 | 51.4 | 59.4 | 66.8 | 54.8 | 43.9 | 43.7 | 43.1 | 55.5 | 56.1 |
| 23 | 64.3 | 61.0 | 51.4 | 47.0 | 46.5 | 51.3 | 63.1 | 50.6 | 39.3 | 38.1 | 40.1 | 47.6 | 50.0 |
| Month Aggregate | 83.1 | 72.9 | 62.5 | 57.0 | 57.0 | 62.9 | 72.1 | 60.0 | 49.2 | 46.4 | 49.6 | 57.9 | 60.9 |

RTM prices are generally lowest after midnight and highest in the late afternoon and early evening. Seasonally, prices are high in the winter and summer.

Table 3.2 shows average RTM prices normalized to DAM prices ($\mu_{RTM/DAM}$). Although no obvious trends are apparent, it is noted that the overall aggregated value of $\mu_{RTM/DAM}$ over the entire period of 2006-2014 is 0.985 (value in lower right corner of Table 3.2). This demonstrates the general equivalency of RTM and DAM prices (they are within 1.5% of each other); presumably because virtual trading acts to make RTM and DAM prices converge in the aggregate over longer periods of time. Since RTM prices are generally slightly lower than their equivalent DAM prices, the potential cost savings found in this study for selective RTM purchasing above average or late-night

RTM purchasing are also assumed to conservatively represent potential savings of selective RTM purchasing above average or late-night DAM purchasing.

**Table 3.2 – Aggregate Average Normalized Real Time Market Prices ($\mu_{RTM/DAM}$): 2006-2014.**

| RTM/DAM Price - Multi-Year-Average | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hour | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Hour Aggregate |
| 0 | 0.972 | 1.008 | 1.004 | 1.000 | 0.979 | 0.989 | 0.979 | 0.981 | 0.952 | 0.901 | 0.980 | 1.012 | 0.980 |
| 1 | 0.997 | 1.023 | 1.017 | 1.056 | 1.026 | 1.011 | 0.999 | 0.960 | 0.966 | 1.044 | 1.009 | 1.003 | 1.009 |
| 2 | 1.004 | 1.051 | 1.018 | 1.011 | 1.017 | 0.990 | 0.941 | 0.984 | 0.940 | 1.012 | 1.009 | 0.970 | 0.996 |
| 3 | 1.003 | 1.051 | 0.994 | 1.016 | 1.063 | 0.974 | 1.009 | 0.989 | 1.010 | 1.024 | 1.010 | 0.979 | 1.010 |
| 4 | 0.973 | 0.984 | 0.989 | 1.038 | 1.061 | 0.980 | 0.958 | 1.016 | 0.958 | 1.000 | 1.006 | 0.971 | 0.994 |
| 5 | 0.951 | 0.986 | 0.992 | 0.992 | 0.980 | 0.890 | 0.937 | 1.049 | 0.978 | 0.963 | 0.984 | 0.911 | 0.968 |
| 6 | 1.011 | 1.011 | 1.036 | 0.952 | 0.932 | 0.851 | 0.906 | 0.986 | 0.982 | 0.950 | 1.004 | 0.934 | 0.963 |
| 7 | 0.970 | 0.942 | 1.019 | 0.994 | 0.948 | 0.912 | 0.924 | 0.926 | 0.962 | 0.958 | 0.963 | 0.937 | 0.955 |
| 8 | 0.947 | 0.957 | 1.064 | 1.052 | 0.919 | 0.907 | 0.939 | 0.936 | 0.989 | 0.960 | 0.977 | 0.916 | 0.963 |
| 9 | 1.063 | 0.939 | 1.079 | 1.043 | 0.969 | 0.906 | 0.924 | 0.934 | 1.029 | 0.994 | 0.977 | 0.919 | 0.981 |
| 10 | 1.033 | 0.934 | 1.010 | 1.030 | 1.036 | 0.938 | 0.921 | 0.942 | 0.998 | 0.946 | 0.958 | 0.951 | 0.975 |
| 11 | 1.004 | 0.949 | 0.994 | 1.010 | 1.009 | 0.944 | 0.920 | 0.947 | 0.992 | 0.953 | 0.961 | 0.971 | 0.971 |
| 12 | 0.981 | 0.949 | 0.989 | 1.028 | 0.978 | 1.016 | 0.948 | 0.942 | 1.050 | 0.979 | 0.970 | 0.990 | 0.985 |
| 13 | 1.007 | 0.959 | 0.961 | 1.034 | 1.022 | 1.064 | 1.031 | 0.968 | 1.010 | 0.958 | 0.970 | 0.969 | 0.996 |
| 14 | 0.982 | 0.956 | 0.948 | 1.012 | 1.038 | 1.107 | 1.108 | 0.972 | 1.034 | 0.977 | 0.967 | 0.939 | 1.003 |
| 15 | 0.970 | 0.920 | 0.951 | 1.030 | 1.087 | 1.114 | 1.012 | 1.044 | 1.032 | 0.974 | 0.952 | 0.911 | 1.000 |
| 16 | 0.981 | 0.920 | 0.952 | 1.000 | 1.092 | 1.040 | 1.083 | 1.024 | 1.050 | 0.941 | 0.993 | 1.067 | 1.012 |
| 17 | 1.131 | 0.964 | 0.934 | 0.976 | 1.094 | 0.983 | 1.081 | 0.954 | 1.051 | 0.943 | 1.051 | 1.021 | 1.015 |
| 18 | 0.954 | 1.019 | 0.953 | 0.937 | 1.007 | 0.904 | 0.932 | 0.938 | 0.952 | 1.079 | 1.001 | 1.029 | 0.975 |
| 19 | 0.992 | 0.929 | 1.054 | 0.993 | 0.959 | 0.920 | 0.927 | 0.966 | 1.157 | 1.033 | 0.981 | 1.011 | 0.994 |
| 20 | 0.928 | 0.947 | 0.976 | 1.077 | 1.021 | 0.948 | 0.926 | 1.028 | 1.022 | 0.941 | 0.973 | 0.988 | 0.981 |
| 21 | 0.944 | 0.984 | 0.976 | 0.999 | 0.950 | 0.946 | 0.954 | 0.971 | 0.976 | 0.990 | 0.979 | 0.966 | 0.970 |
| 22 | 0.946 | 1.018 | 1.056 | 1.051 | 0.983 | 0.964 | 0.932 | 0.933 | 0.962 | 0.989 | 0.957 | 0.991 | 0.982 |
| 23 | 0.948 | 0.980 | 1.006 | 1.019 | 1.000 | 0.949 | 0.963 | 0.949 | 0.972 | 0.987 | 0.964 | 0.928 | 0.972 |
| Month Aggregate | 0.987 | 0.974 | 0.999 | 1.015 | 1.007 | 0.969 | 0.969 | 0.972 | 1.001 | 0.979 | 0.983 | 0.970 | 0.985 |

Additionally, although there are no apparent normalized price trends, it is noted that the values of the 24x12 matrix are used in some investigated algorithms as part of a method to better anticipate RTM prices in the immediate future. For a given day, the already determined DAM prices can be used to anticipate the yet-to-be-determined RTM prices. Knowing the historical bias between RTM and DAM price for a given hour can allow for better prediction of RTM prices.

Table 3.3 shows trends for standard deviation of normalized RTM prices (i.e., $\sigma_{RTM/DAM}$). Generally, $\sigma_{RTM/DAM}$ is a measure of price volatility.

**Table 3.3 – Aggregate Standard Deviation of Normalized Real Time Market Prices ($\sigma_{RTM/DAM}$): 2006-2014.**

| RTM Stdev (Normalized to DAM) - Multi-Year-Average | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hour | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Hour Aggregate |
| 0 | 0.437 | 0.389 | 0.288 | 0.451 | 0.424 | 0.313 | 0.203 | 0.226 | 0.280 | 0.474 | 0.336 | 0.483 | 0.359 |
| 1 | 0.296 | 0.286 | 0.259 | 0.264 | 0.356 | 0.287 | 0.239 | 0.278 | 0.327 | 0.310 | 0.210 | 0.386 | 0.291 |
| 2 | 0.232 | 0.224 | 0.234 | 0.188 | 0.321 | 0.249 | 0.303 | 0.192 | 0.273 | 0.304 | 0.188 | 0.246 | 0.246 |
| 3 | 0.214 | 0.213 | 0.131 | 0.177 | 0.252 | 0.230 | 0.172 | 0.157 | 0.219 | 0.190 | 0.129 | 0.196 | 0.190 |
| 4 | 0.486 | 0.477 | 0.360 | 0.363 | 0.364 | 0.263 | 0.266 | 0.204 | 0.366 | 0.374 | 0.413 | 0.468 | 0.367 |
| 5 | 0.540 | 0.468 | 0.459 | 0.386 | 0.470 | 0.600 | 0.411 | 0.263 | 0.450 | 0.479 | 0.447 | 0.569 | 0.462 |
| 6 | 0.456 | 0.443 | 0.453 | 0.364 | 0.434 | 0.547 | 0.450 | 0.266 | 0.350 | 0.348 | 0.353 | 0.414 | 0.407 |
| 7 | 0.370 | 0.248 | 0.313 | 0.349 | 0.351 | 0.316 | 0.343 | 0.308 | 0.263 | 0.190 | 0.271 | 0.273 | 0.300 |
| 8 | 0.298 | 0.220 | 0.317 | 0.298 | 0.241 | 0.243 | 0.191 | 0.161 | 0.199 | 0.159 | 0.249 | 0.276 | 0.238 |
| 9 | 0.367 | 0.197 | 0.347 | 0.286 | 0.271 | 0.172 | 0.190 | 0.134 | 0.223 | 0.216 | 0.224 | 0.172 | 0.233 |
| 10 | 0.288 | 0.190 | 0.250 | 0.239 | 0.264 | 0.173 | 0.133 | 0.141 | 0.183 | 0.138 | 0.148 | 0.189 | 0.195 |
| 11 | 0.229 | 0.208 | 0.221 | 0.232 | 0.206 | 0.173 | 0.134 | 0.149 | 0.209 | 0.107 | 0.139 | 0.207 | 0.184 |
| 12 | 0.197 | 0.159 | 0.177 | 0.218 | 0.187 | 0.270 | 0.161 | 0.150 | 0.256 | 0.154 | 0.147 | 0.217 | 0.191 |
| 13 | 0.199 | 0.180 | 0.137 | 0.222 | 0.221 | 0.260 | 0.264 | 0.194 | 0.223 | 0.119 | 0.122 | 0.172 | 0.193 |
| 14 | 0.197 | 0.158 | 0.143 | 0.209 | 0.248 | 0.280 | 0.300 | 0.200 | 0.230 | 0.134 | 0.128 | 0.152 | 0.198 |
| 15 | 0.284 | 0.171 | 0.142 | 0.199 | 0.238 | 0.302 | 0.257 | 0.291 | 0.226 | 0.153 | 0.212 | 0.336 | 0.234 |
| 16 | 0.492 | 0.318 | 0.188 | 0.186 | 0.272 | 0.244 | 0.326 | 0.256 | 0.191 | 0.144 | 0.508 | 0.753 | 0.323 |
| 17 | 0.557 | 0.473 | 0.282 | 0.166 | 0.259 | 0.229 | 0.312 | 0.196 | 0.240 | 0.190 | 0.468 | 0.398 | 0.314 |
| 18 | 0.318 | 0.404 | 0.316 | 0.181 | 0.211 | 0.157 | 0.190 | 0.182 | 0.158 | 0.474 | 0.383 | 0.377 | 0.279 |
| 19 | 0.286 | 0.217 | 0.492 | 0.341 | 0.178 | 0.136 | 0.153 | 0.164 | 0.419 | 0.353 | 0.329 | 0.366 | 0.286 |
| 20 | 0.294 | 0.277 | 0.352 | 0.427 | 0.326 | 0.154 | 0.120 | 0.244 | 0.237 | 0.261 | 0.270 | 0.340 | 0.275 |
| 21 | 0.419 | 0.336 | 0.318 | 0.308 | 0.273 | 0.200 | 0.181 | 0.246 | 0.193 | 0.214 | 0.282 | 0.313 | 0.274 |
| 22 | 0.360 | 0.272 | 0.384 | 0.319 | 0.308 | 0.248 | 0.169 | 0.190 | 0.186 | 0.204 | 0.228 | 0.350 | 0.268 |
| 23 | 0.578 | 0.486 | 0.471 | 0.477 | 0.541 | 0.374 | 0.258 | 0.222 | 0.253 | 0.242 | 0.304 | 0.426 | 0.386 |
| Month Aggregate | 0.350 | 0.292 | 0.293 | 0.285 | 0.301 | 0.268 | 0.239 | 0.209 | 0.256 | 0.247 | 0.270 | 0.337 | 0.279 |

From the perspective of volatility, the following is observed:

1. The 4am to 8am period is consistently volatile throughout the year. This seems reasonable because this period is dominated by a general "ramp-up" of loads as people and businesses start-up for the day. Performing simulation using prices from this period can characterize purchasing algorithm performance for volatile price environments in general.
2. The 8am to 4pm period is generally the least volatile period throughout the year. This seems reasonable because, once people "settle in" for the day, their energy usage is relatively consistent. Performing simulation using prices from this period can characterize purchasing algorithm performance for such periods of low volatility.

3. Price behavior for the 4pm to 8pm period varies throughout the year. This is observed by a green, convex “up”, trend in Table 3.3, with a parallel orange trend beneath. These trends are driven by the variation of daylight throughout the year. Longer days have prolonged periods of low price volatility. Shorter days see an earlier and more pronounced change from low-volatility daytime behavior to higher-volatility “sunset” behavior.
4. The 8pm to 4am period does not exhibit especially strong price behavior. However, this is the preferred time for PEV charging due to generally low prices (see Table 3.1) and consumer convenience.

In addition to time-of-day, and time-of-year trends, there are trends between years (supporting data not included herein). Each of the years used for simulation in this study (i.e., 2012-2014) has similar price behavior for May through December, although prices during the summer of 2013 are higher than for the other two years. However, prices for the winter months vary greatly between years. 2014 winter prices are extremely high due to the “polar vortex” weather phenomenon. By contrast 2012 winter prices are relatively low, and 2013 winter prices lie between 2012 and 2014 prices. Volatility follows high prices.

## 3.2 Simulation Framework Summary

To understand the performance of purchasing algorithms under different conditions, the same algorithm may be simulated through various distinct charging scenarios; each with its own attributes. Other attributes are common to all scenarios in this chapter to ensure useful performance comparisons can readily be made. The common and distinct attributes assumed in this chapter are summarized in Table 3.4 below (formatted consistently with those in other chapters). Rationale follows.

All scenarios in this chapter consist of a year’s worth of daily charging for a single vehicle at wholesale RTM rates in New York City, with the instantaneous charging rate being limited only by the vehicle’s charger capacity (i.e., 9.6kW). Subsequent chapters assume that limitations in network capacity may require temporarily reducing an individual vehicle's charging rate below the maximum. However, in this chapter, the simplification of assuming no network constraints allows single-vehicle charging results to be scaled up by simply multiplying charging costs and quantities by the number of vehicles. Results for charging costs per unit energy are therefore invariant of the number of vehicles being charged in this chapter.

**Table 3.4 – Simulation Framework for Homogeneous Vehicle Charging Without Network Constraints.**

| | |
|---|---|
| Network Characteristics | Prices: Real Time Market, New York City<br>Network Charging Rate Constraint: None |
| Vehicle Characteristics | Number: Single Vehicle<br>Charger Maximum Rate: 9.6kW (0.80 kWhr per five-minute timestep)<br>Daily Energy Usage (per Vehicle):<br>4 hour periods – 21.25 of 85 kWhr (25%), equivalent to 62.5 miles/day usage<br>8 hour periods – 42.50 of 85 kWhr (50%), equivalent to 125 miles/day usage |
| Charging Period Characteristics | AM – Connection Time: 4am to 8am (4 hours, or 48 five minutes steps)<br>Charging Time: 2.21 hours (55.3%, or 26.56 of 48 timesteps)<br>DAY – Connection Time: 8am to 4pm (8 hours, or 96 five minutes steps)<br>Charging Time: 4.43 hours (55.4%, or 53.125 of 96 timesteps)<br>PM – Connection Time: 4pm to 8pm (4 hours, or 48 five minutes steps)<br>Charging Time: 2.21 hours (55.3%, or 26.56 of 48 timesteps)<br>NIGHT – Connection Time: 8pm to 4am (8 hours, or 96 five minutes steps)<br>Charging Time: 4.43 hours (55.4%, or 53.125 of 96 timesteps) |
| Years Simulated | 2012 – all days except Jan 1, Feb 29, Mar 10, Nov 4, and Dec 31<br>2013 – all days except Jan 1, Mar 9, Nov 3, and Dec 31<br>2014 – all days except Jan 1, Mar 8, Nov 2, and Dec 31<br><br>Days within a year are simulated independently (initial charge is always as specified above). |

Performing year-long simulations allows overall performance to be judged across a variety of weather and market conditions. Since general weather and market trends may differ from year to year (e.g, the "polar vortex" weather phenomenon of 2014 dominated the price environment for a significant portion of 2014, but similar behavior did not exist in 2012), three simulation years are considered (2012-2014).

Four daily charging periods are studied in order to observe how algorithm performance varies depending on market characteristics. As described in the previous section, market behavior varies for each of the four charging periods. For example, the AM period consistently has the greatest price volatility.

An important attribute of all simulations in this chapter is that days are treated independently. This is achieved by assuming the vehicle uses a consistent amount of energy each day, and ensuring that all algorithms are designed to ensure a full charge is attained at the end of each charging period, regardless of cost. Thus, initial and final charge levels are consistent from day to day. Although this may hinder the goal of minimizing charging cost over the entire year, it greatly simplifies analysis by allowing days to be analyzed independently of each other. For example, algorithms which do not end each charging period with a fully-charged state may result in the unacceptable outcome of an insufficiently charged battery following several days. This is undesirable from a PEV owner point of view; whereas completely charging a PEV by the end of every night reduces PEV owner “range anxiety.” Also, “one-day” packaging simplifies simulation to understand long-term performance; anomalous single days can be discarded without corrupting the entire long-term simulation. Thus, ensuring identical daily “boundary conditions” avoids complexity.

The assumed daily energy consumed is selected so that the vehicle must charge at the maximum rate for approximately half of the charging period in order to be fully charged by the end of the period. This ensures all simulations are performed with a similar level of assumed deferability.

## 3.3 Description of Algorithms and Their Development

This section describes several PEV charging algorithms whose cost-saving performance has been investigated. These algorithms fall into various classes:

1. Simple
2. Ideal (Theoretical Best)
3. Dynamic

The simple and ideal classes are developed and simulated as points of comparison for assessing the performance of the dynamic algorithms which are to be a product of this thesis. The simple and ideal classes serve as "bookends" which bound the range of performance, and the goal is to develop a practically realizable dynamic algorithm which approaches the performance of the ideal case.

The algorithms are discussed as follows, with simple and ideal algorithms discussed first. Discussion of dynamic algorithms follows, and their order of presentation is dictated by the order in which they were developed. Dynamic algorithms of increasing sophistication and performance evolved from earlier algorithms, and that evolution is preserved through the order in which they are discussed. Specific performance results of these algorithms are discussed in Section 3.4

### 3.3.1 Average Price Purchasing (Simple – AVG)

The first simple algorithm considers the business-as-usual case of purchasing electricity at the average price throughout the charging period. This algorithm is a useful point of comparison because it represents the case of being completely unselective. From a practical point of view, this case could be realized by scaling down the PEV charging rate so the PEV must be charged continuously at a this rate to obtain a full charge at the end of the charging period. Alternatively, this strategy could be realized by charging at the full charging rate for only a fixed fraction of all 5-minute periods.

Mathematically, the charging cost of this case is:

$$Cost = \sum_{i=1}^{ni}\left(q_{avg} * p_{RTM}(i)\right) = q_{avg} * \left[n_1^i\right]^T * \left[p_{RTM}^i\right] \quad \textbf{(3-1)}$$

$$q_{avg} = \frac{q_{des}}{ni} = \left(\frac{L_{final} - L_{init}}{ni}\right) \quad \textbf{(3-2)}$$

Where:

1. $p_{RTM}$ is the RTM price for each five-minute timestep [$/MW-hr]
2. $q_{avg}$ is the quantity of energy purchased for each five-minute timestep if the charging period average price is to be obtained [MW-hr]

3. $q_{des}$ is the desired quantity of energy purchased over the entire charging period [MW-hr]
4. $L_{final}$ and $L_{init}$ are the final and initial vehicle charge levels, respectively [MW-hr]
5. $[n_1^i]$ is a vector of ones of length $ni$
6. $[p_{RTM}^i]$ is the vector (length $ni$) of RTM prices

### 3.3.2 Late Night Purchasing (Simple – LATE)

An alternative business-as-usual baseline case exists and is used for the NIGHT charging period only; late-night charging. In this case, charging is performed at the full charging rate for only the latest 5-minute timesteps of the charging period. The rationale for this is that in the 8pm to 4am period, prices generally decrease with time. Therefore, a simple and effective baseline algorithm (which does not use selectivity to exploit price variations arising from volatility) already exists which lowers electricity purchasing costs. Mathematically, the charging cost for this case is:

$$Cost = \sum_{i=ni-nf}^{ni} \left(q_{max} * p_{RTM}(i)\right) = \left[q_{act}^i\right]^T * \left[p_{RTM}^i\right] \quad \textbf{(3-3)}$$

$$nf = \frac{q_{des}}{q_{max}} \quad \textbf{(3-4)}$$

Where the parameters not previously defined include:

1. $q_{max}$ is the quantity of energy associated with charging for five minutes at the maximum charging rate [MW-hr]
2. $nf$ is the number of five-minute timesteps needed to charge the vehicle at the maximum charging rate
3. $[q_{act}^i]$ is a vector (length $ni$) of purchased energy quantities [MW-hr] whose first $(nf - ni)$ elements are zero, and whose final $nf$ elements are $q_{max}$

### 3.3.3 Linear Programming (Ideal – LP)

A third baseline case represents the case of optimal selectivity, and is used to benchmark how well an algorithm performs when compared to the best possible performance. This algorithm represents the case where electricity is purchased only during the lowest priced 5-minute periods. Although a dynamic programming algorithm was actually used for simulation, equivalent functionality is best described via a linear programming formulation:

Find $[q_{act}^{i}]$, which minimizes

$$Cost = [q_{act}^{i}]^{T} * [p_{RTM}^{i}] \quad \textbf{(3-5)}$$

Subject to the constraint:

$$0 \leq q_{act}(i) \leq q_{max} \quad \textbf{(3-6)}$$

It is noted that this algorithm is not actually realizable. Purchasing electricity only during the cheapest portions of the charging cycle requires knowledge of prices throughout the entire charging period prior to the beginning of the charging period. Advance price knowledge allows the algorithm to determine whether the current price compares favorably with alternative, future, candidate purchase periods. However, RTM prices are established based on market conditions immediately before they go into effect; hence the term “real-time”. Therefore, this base case represents the unrealizable upper limit of purchase algorithm performance.

### 3.3.4 Naïve Setpoint Purchasing (Dynamic – BUYSET)

The first set of practical algorithms developed for selectively purchasing electricity are based on purchasing electricity whenever the price falls below a pre-established threshold (or setpoint). This general algorithm is described in the pseudocode of Figure 3.1:

For i = 1 to ni

    if ($p_{RTM}(i)$ < Setpoint)

        $q_{act}(i) = q_{max}$

    else

        $q_{act}(i) = 0$

    end

end

$$Cost = \sum_{i=1}^{ni} \left( q_{act}(i) * p_{RTM}(i) \right) = \left[ q_{act}^{i} \right]^{T} * \left[ p_{RTM}^{i} \right]$$

**Figure 3.1 – General Pseudocode for Naïve Setpoint Purchasing.**

The difficulty that arises is establishing a setpoint value that will result in completely charging the PEV by the end of the charging period at minimum price. Some insight into resolving this difficulty can be gleaned by considering the case where the RTM prices are normally distributed about a mean price throughout a charging period (See Figure 3.2).

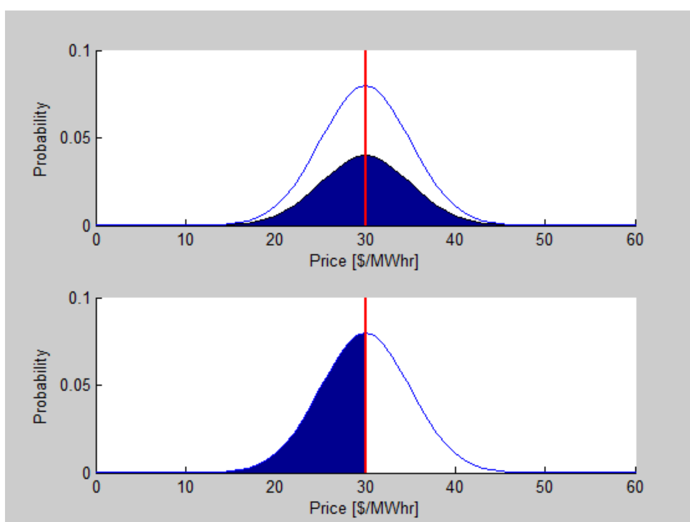


**Figure 3.2 – Comparison of Average Price Buying (Top) and Optimal Buying (Bottom).**

In the figure, the Gaussian curve can be assumed to bound the total quantity of electricity available for purchase in the charging period. If the ability to defer purchasing exists, then only part of the area (energy available) under the curve needs to be purchased. The shading in Figure 3.2 represents the case where only half of the available energy needs to be purchased to fulfill PEV charging needs. The shaded area in the top subplot shows the price distribution of the energy purchased for an average charging policy (which results in an average cost). By contrast, the shaded area in the bottom subplot shows the price distribution of the energy purchased for an optimal charging policy. In this case, all electricity is purchased at below the average price and costs are reduced. For a Gaussian distribution, the cost savings is driven by the standard deviation, so greater price volatility will result in greater cost savings.

Clearly, for the case where only half of available energy needs to be purchased, the optimal setpoint is the average or median price. Since the mean price is not known with certainty until the charging period is complete, a useful starting point for setpoint determination then appears to be the DAM price. This is because RTM and DAM prices are proven to be roughly equivalent in aggregate due to arbitrage via virtual trading. Therefore, the DAM price is likely to be close to the median RTM price, and the individual 5-minute RTM prices can then be assumed to be symmetrically distributed about that median price. Additionally, DAM prices are listed at least 13 hours prior to the hour during which they are applied, so they can be used to anticipate RTM prices throughout the charging period.

For the case of purchasing half of the available energy during a charging period, this pricing model suggests the following modification to the setpoint purchasing algorithm:

$$\text{if}\,(p_{RTM}(i) < p_{DAM}(i)) \quad \rightarrow \quad q_{act}(i) = q_{max} \qquad \textbf{(3-7)}$$

While use of the DAM price as a purchasing setpoint results in an optimal purchasing policy for the special case where half of all available energy is to be purchased and the DAM price represents the median RTM price, additional considerations need to be taken into account for a practical purchasing algorithm. These considerations include:

1. Level of Deferability – Using DAM prices as a purchase setpoint assumes that twice as much charging time is available than is needed to fully charge the battery. However, in some scenarios charging is required for a higher fraction of the overall charging period and use of the DAM price will result in falling short of the charging goal. Conversely, if charging is required for less than half of the overall charging period, a greater cost savings potential exists but will be lost. Energy will be purchased at higher than optimal prices, resulting in full charge prior to the end of the time period.
2. Average Price Estimation Error – The DAM price of electricity is not always an accurate predictor of the average RTM price for a given hour. If the average RTM price is lower than the DAM-based setpoint, charging will be selected for a larger-than-expected number of 5-minute periods, resulting in premature charging at a higher than optimal cost. Conversely, if the average RTM price is higher than the DAM-based setpoint, charging will be selected less often than is needed. This will result in an incomplete charge, albeit at an average price that is lower than anticipated.

Although the above complications need to be resolved in order to produce a general purpose PEV charging algorithm, use of a DAM price based setpoint was judged sufficiently practical and potentially effective to warrant further study via simulation. To gain additional insight to potential solutions, a sensitivity study was performed for the NIGHT charging period using several variations of the DAM price based algorithm. Each algorithm uses a DAM price based setpoint adjusted by a scaling factor:

$$\text{if}\,(p_{RTM}(i) \ < \ f * \ p_{DAM}(i)\ ) \ \rightarrow \qquad q_{act}(i) \ = q_{max} \qquad \textbf{(3-8)}$$

Where $f$ is one of the following 9 fractions: 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, or 0.90

The above algorithm, however, was augmented by a “late” purchasing feature. As explained in the framework summary section, all daily simulations are to achieve the goal of obtaining a full battery charge at the end of the charging period. The setpoint purchasing algorithm alone cannot guarantee that outcome. Therefore, regardless of cost, electricity is also to be purchased as late in the charging period as possible if necessary to meet the daily charging objective:

$$\text{If}\ (\ L_{act}(i)\ <\ L_{\max rate}(M-(ni-i))\ )\qquad \rightarrow \qquad q_{act}(i)\ = q_{max} \qquad \textbf{(3-9)}$$

Where $L_{\max rate}$ is a profile of minimum allowable charge level verses time, and $L_{\max rate}$ varies linearly between $L_{\max rate}$ (0) = 0 and $L_{\max rate}$ (M) = $L_{final}$.

### 3.3.5 Statistical Purchasing (Dynamic – STAT)

The results from the naïve setpoint purchasing algorithm inform the need for a series of algorithm enhancements. These enhancements yielded an additional class of PEV charging algorithms, discussed here.

The results of the setpoint purchasing study (see results section for quantitative results) indicate that the optimal purchasing setpoint is approximately 75% of the DAM price. This result was not expected; when only half of the available energy is to be purchased, the ideal setpoint was expected to be 100% of the DAM price. The reason for the unanticipated results is that the lower-than-expected setpoint results in under-purchasing energy early in the charging period and therefore forces additional purchasing of electricity in the later part of the charging period. This trend is to be expected for the NIGHT period as prices tend to decrease through the charging period. Thus, <u>the fundamental problem with the setpoint purchasing algorithm is that it seeks to optimize purchasing for each hourly period (where a specific DAM price is in effect) independently rather than optimizing purchasing over the entire charging period</u>. An overall optimization thus requires simultaneous consideration of prices throughout the entire period.

The statistical purchasing algorithm described here uses a combination of DAM prices and statistics from historical price data to construct a model of anticipated RTM prices for the entire charging period. The price model is then used to estimate an optimal purchasing setpoint valid for the entire charging period which should result in achieving the charging goal. This section describes a general statistical model based algorithm with several variants. Each variant employs different mechanisms for estimating future-price statistics to populate the model.

The statistical model represents RTM prices for each hour of the charging period as a Gaussian distribution. The distributions are anchored to DAM prices, which have already been established for the entire charging period. Statistics are then used to construct the distribution about the anchor. A normalized mean ($\mu_{RTM/DAM}$) is used to place the center of the distribution with respect to the DAM price (i.e., $\mu_{RTM} = p_{DAM} * \mu_{RTM/DAM}$), and a normalized standard deviation defines the broadness of the distribution with respect to DAM price (i.e., $\sigma_{RTM} = p_{DAM} * \sigma_{RTM/DAM}$). Various methodologies are used to predict appropriate future values of $\mu_{RTM/DAM}$ and $\sigma_{RTM/DAM}$, however, those methodologies are discussed later.

Once the statistical price model is developed, the model is used to determine the appropriate purchasing setpoint. The employed strategy is best described by the example of charging overnight starting January 20$^{th}$, 2013 at 8pm (see Figure 3.3). Prices for each hour of the charging period are shown as a separate distribution "row". For each row, the already published DAM prices are used as anchor points (shown as vertical red bars). The Gaussian price distribution are then drawn about that anchor point using $p_{DAM}$, $\mu_{RTM/DAM}$, and $\sigma_{RTM/DAM}$ as discussed above and shown in the figure. The area under each curve represents a potential charging duration of one hour. Setting a price setpoint results in the purchase of electricity whenever prices are below that setpoint; the fraction of the hour during which charging will take place is then represented by sum of Cumulative Distribution Functions (i.e., CDF(Setpoint)). This is represented by the shaded area under each curve.

To determine an optimal price threshold, a price setpoint value is guessed and that value is iterated until the estimated charging time for the entire charging period is equal to that required to fully charge the system (See pseudocode in Figure 3.4). Figure 3.3 illustrates a price threshold of $28. Note that, when compared to the naïve purchasing algorithm discussed earlier, this strategy increases buying during hours where the overall price of electricity is low, and decreases buying during hours where the price is high.

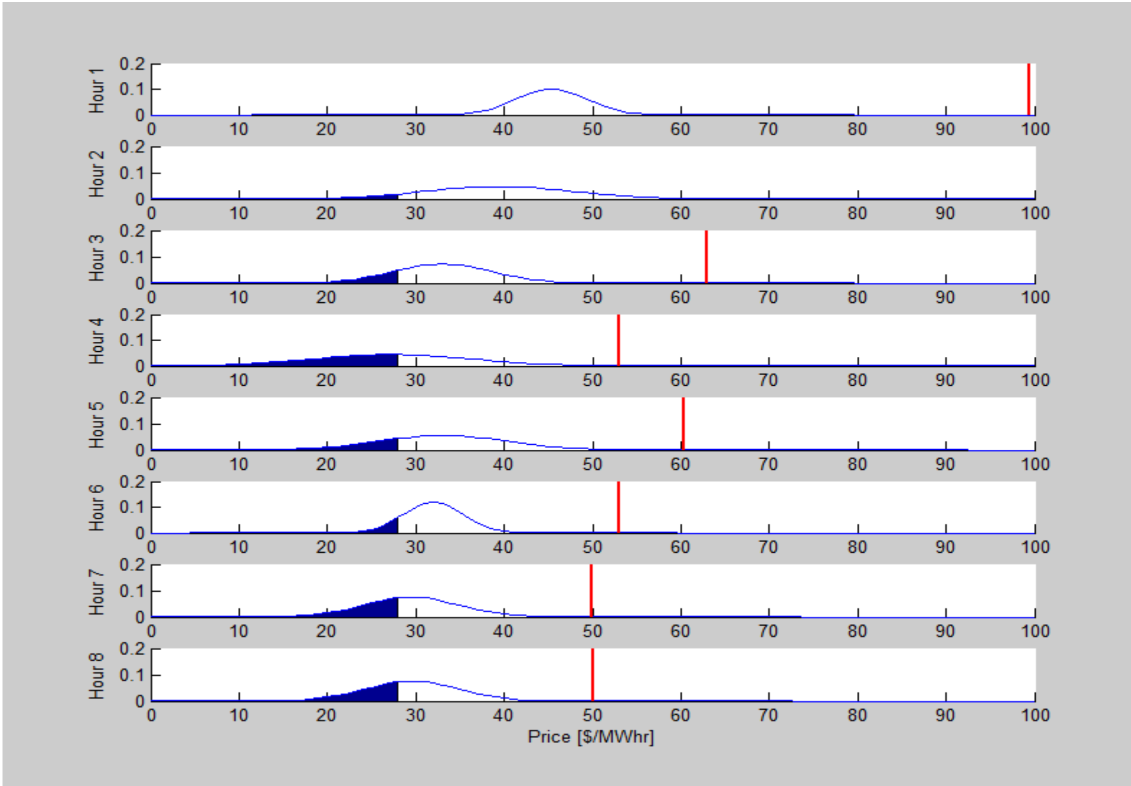


**Figure 3.3 – Diagram of Statistical RTM Price Model for January 20th, 2013: Hours are 8pm to 4am (January 21st). Red bar is DAM price, and blue curves represent RTM price distribution using the actual statistics of that time. An arbitrary setpoint of $28 is selected.**

```
SetpointMin = 0
SetpointMax = 200
SetpointMid = 0.5*( SetpointMax - SetpointMin)
fTarget = (# required charging steps / # remaining charging steps)
while ((SetpointMax - SetpointMin) > 0.0001)
        fCalculated = 0;
        for each hour (h) in charging period
                if (hour has passed),          scale = 0
                elseif (hour has not started),  scale = 1
                else,                          scale = remaining fraction of hour
                z = (SetpointMid – pDAM(h) * μRTM/DAM) / (pDAM(h) * σRTM/DAM);
                portion = (scale * CDF(z));
                fCalculated = fCalculated + (portion / #hours remaining);
        if (fCalculated > fTarget)
                SetpointMax = SetpointMid
                SetpointMid = 0.5*( SetpointMin + SetpointMax)
        elsif (fCalculated <= fTarget)
                SetpointMin = SetpointMid
                SetpointMid = 0.5*( SetpointMin + SetpointMax)
Setpoint = SetpointMid
```

**Figure 3.4 – Pseudocode for Establishing a RTM Purchasing Setpoint based on a Statistical Price Model.**

Several variants of the statistical purchasing algorithm exist depending on how the statistical values for $\mu_{RTM/DAM}$ and $\sigma_{RTM/DAM}$ are determined. These are as follows:

1. <u>Perfect</u> – “Perfect” price statistics were used to establish baseline performance for the statistical purchasing algorithm in general. The statistics used are based on the actual price data for the simulated charging period. This method is not actually realizable since such statistics could only be generated by knowledge of prices that are yet to be determined.

2. Long-Time Aggregate (24x12) – Price statistics are based on aggregation of historic data from 2006 through 2014 “binned” into a 24 (hours of day) by 12 (months of year) matrix (i.e., the values in Table 3.2 and Table 3.3). Use of these statistics assumes that the best predictor of RTM price variability is the historical average.

3. Continuous – Price statistics are based on the most recent data only. This assumes that RTM price variability is dominated by unique, unpredictable, short-term phenomena. The price characteristics of this phenomenon are best determined by the most recent data, and the statistics should be applied to decisions while the phenomenon persists. The specific methodology simulated makes use of a rolling 1-hour window, consisting of the 60 minutes prior to the purchasing decision about to be made. Values of $\mu_{RTM/DAM}$ and $\sigma_{RTM/DAM}$ are calculated based on the prices within that rolling window. These statistics are then applied to the statistical model of all hours remaining in the charging period.

   This algorithm recomputes a setpoint based on the algorithm in Figure 3.4 after each 5-minute time period. To do this, the value of $f_{Target}$ is continually recomputed based on the latest data of the number charging timesteps required for a full charge, and the number of timesteps remaining in the overall charging period. Also, computation of $f_{Calculated}$ must be adjusted to account for the fact that hours which have already passed cannot contribute to new charging, and only part of the current hour can contribute to new charging.

   It is noted that continually recalculating the buy setpoint based on the most recent charge status inherently implements the late-in-period purchasing algorithm initially implemented in the naïve setpoint simulation. If the PEV is insufficiently charged near the end of the charging period, the algorithm will recognize that electricity must be purchased for the majority or entire remainder of the charging period. This will increase the value of $f_{Target}$, and the resulting setpoint will be raised. Visually, this corresponds to a setpoint traversing (to the right) across a Gaussian price distribution as the required probability of purchasing electricity approaches unity. In short, increased charging urgency drives the setpoint higher.

4. Continuous Hybrid – Price statistics are based on the "Continuous" methodology for a persistence horizon, and then based on the "Long-Time Aggregate (24x12)" methodology beyond that horizon. The rationale for this algorithm is that using statistics from the most recent data is good, but valuable for only a short duration. Beyond that duration, the best data for statistics is from 24 hours previously, since the drivers of price behavior are likely to repeat for the same times in consecutive days. Simulations of this methodology were performed for several "persistence horizons" within a charging period. That is, for an 8 hour charging period, 7 versions of the algorithm were simulated with a persistence horizon ranging from 0 to 6 hours. These variants are termed STAT0 through STAT6.

   Simulation results indicate an optimal persistence horizon of two hours. This is corroborated by viewing the results of an autocorrelation calculation on RTM price data. The results indicate that prices are most similar to each other (i.e., price persistence exists) within 2 hours. However, beyond 2 hours, the magnitude of results are lower until the 24 hour mark. This indicates that prices from 24 hours earlier are better predictors than prices 3-8 hours earlier.

## 3.4 Performance Results Summary and Conclusions

Simulation study results indicate that the potential savings of selective PEV charging are significant enough to warrant further investigation of implementable purchasing algorithms. The statistical-price-model based algorithms developed in this chapter indicate that it is possible to practically extract a significant portion of these potential savings. A statistical algorithm (STAT2) which uses a combination DAM prices, price statistics from the previous day, and price statistics from a rolling one hour window immediately preceding real time show the best performance from a realizable algorithm. These conclusions are discussed in more detail below and detailed numerical results for all simulations are tabulated in Table 3.7 (NIGHT), Table 3.8 (AM), Table 3.9 (PM), and Table 3.10 (DAY). In each table, results for the best performing algorithms are highlighted and in bold font.

Simulation studies of ideal algorithms indicate potential cost savings in the 20-25% range with respect to the AVG base case for the DAY and NIGHT charging periods. Greater potential savings are observed for the more volatile AM and PM time periods; exceeding 50% for the most volatile (2013-AM) period. Generally, more than half of this cost savings potential can be extracted with the realizable algorithms developed in this chapter. Numerical results to support these conclusions are summarized in Table 3.5.

**Table 3.5 – Savings Potential of Selective Purchase of Electricity in the RTM: Average Cost Baseline.**

| | 2012 | | 2013 | | 2014 | |
|---|---|---|---|---|---|---|
| Period | Max Possible | Realizable | Max Possible | Realizable | Max Possible | Realizable |
| AM | 20.5% | 14.3% | 56.8% | 52.3% | 46.7% | 38.1% |
| DAY | 19.0% | 16.8% | 23.8% | 16.9% | 23.7% | 14.6% |
| PM | 26.1% | 19.5% | 28.6% | 20.7% | 22.0% | 11.3% |
| NIGHT | 22.8% | 10.7% | 22.8% | 19.2% | 22.7% | 17.2% |

The NIGHT charging period is of especial interest since it offers the absolute lowest prices of any period and is a consistently convenient period for consumers to charge their PEVs. However, it should be noted that the relative savings potential for the NIGHT period are undercut because an effective and simple alternative charging methodology exists (i.e., LATE). Comparison of statistical algorithms results to LATE baseline results are summarized in Table 3.6. Despite the reduced relative cost savings potential in the NIGHT period, potential savings are still considered significant enough to warrant further exploration.

**Table 3.6 – Savings Potential of Selective Purchase of Electricity in the RTM: Late-Night Baseline.**

| | 2012 | | 2013 | | 2014 | |
|---|---|---|---|---|---|---|
| Period | Max Possible | Realizable | Max Possible | Realizable | Max Possible | Realizable |
| NIGHT | 6.2% | 2.9% | 12.1% | 6.9% | 13.8% | 6.5% |

The overall performance of the various statistical algorithms is superb. The "Perfect" statistical case shows performance approaching the results from the Dynamic Programming method. Practical selection of statistics diminishes performance from the "Perfect" case. However, the STAT2 method minimizes this diminishment. This is because the type of phenomena which drive the difference between RTM prices and DAM prices are believed to persist approximately two hours. RTM deviation from DAM prices beyond two hours is better predicted by similar behavior 24 hours earlier.

**Table 3.7 – Cost Performance of Various Charging Algorithms - NIGHT Charging.**

| NIGHT Charging 8pm to 4am | 2012 Dollars | 2012 Frac of AVG Price | 2012 Frac of LATE Price | 2013 Dollars | 2013 Frac of AVG Price | 2013 Frac of LATE Price | 2014 Dollars | 2014 Frac of AVG Price | 2014 Frac of LATE Price |
|---|---|---|---|---|---|---|---|---|---|
| AVG | 489.50 | 1.000 | 1.158 | 657.20 | 1.000 | 1.153 | 747.72 | 1.000 | 1.130 |
| LATE | 422.85 | 0.864 | 1.000 | 570.09 | 0.867 | 1.000 | 661.88 | 0.885 | 1.000 |
| BUYSET (f = 0.10) | 420.12 | 0.858 | 0.994 | 567.55 | 0.864 | 0.996 | 660.63 | 0.884 | 0.998 |
| BUYSET (f = 0.20) | 419.79 | 0.858 | 0.993 | 566.63 | 0.862 | 0.994 | 660.21 | 0.883 | 0.997 |
| BUYSET (f = 0.30) | 419.30 | 0.857 | 0.992 | 564.49 | 0.859 | 0.990 | 654.31 | 0.875 | 0.989 |
| BUYSET (f = 0.40) | 418.70 | 0.855 | 0.990 | 555.95 | 0.846 | 0.975 | 648.69 | 0.868 | 0.980 |
| BUYSET (f = 0.50) | 417.91 | 0.854 | 0.988 | 550.59 | 0.838 | 0.966 | 643.85 | 0.861 | 0.973 |
| BUYSET (f = 0.60) | 417.47 | 0.853 | 0.987 | 549.65 | 0.836 | 0.964 | 641.99 | 0.859 | 0.970 |
| BUYSET (f = 0.70) | 417.16 | 0.852 | 0.987 | 547.29 | 0.833 | 0.960 | 640.92 | 0.857 | 0.968 |
| BUYSET (f = 0.80) | 422.46 | 0.863 | 0.999 | 552.78 | 0.841 | 0.970 | 647.83 | 0.866 | 0.979 |
| BUYSET (f = 0.90) | 435.95 | 0.891 | 1.031 | 562.91 | 0.857 | 0.987 | 657.99 | 0.880 | 0.994 |
| STAT (24x12) | 413.49 | 0.845 | 0.978 | 532.44 | 0.810 | 0.934 | 619.95 | 0.829 | 0.937 |
| STAT0 | 414.21 | 0.846 | 0.980 | 536.49 | 0.816 | 0.941 | 625.02 | 0.836 | 0.944 |
| STAT1 | 410.68 | 0.839 | 0.971 | 531.07 | 0.808 | 0.932 | 620.91 | 0.830 | 0.938 |
| **STAT2** | **410.46** | **0.839** | **0.971** | **530.92** | **0.808** | **0.931** | **619.01** | **0.828** | **0.935** |
| STAT3 | 411.86 | 0.841 | 0.974 | 533.25 | 0.811 | 0.935 | 619.67 | 0.829 | 0.936 |
| STAT4 | 414.85 | 0.847 | 0.981 | 536.19 | 0.816 | 0.941 | 621.18 | 0.831 | 0.939 |
| STAT5 | 416.61 | 0.851 | 0.985 | 538.77 | 0.820 | 0.945 | 623.59 | 0.834 | 0.942 |
| STAT6 | 417.46 | 0.853 | 0.987 | 540.72 | 0.823 | 0.948 | 624.72 | 0.836 | 0.944 |
| STAT (Cont.) | 417.89 | 0.854 | 0.988 | 541.61 | 0.824 | 0.950 | 624.63 | 0.835 | 0.944 |
| STAT (Perfect) | 395.37 | 0.808 | 0.935 | 501.71 | 0.763 | 0.880 | 583.16 | 0.780 | 0.881 |
| LP | 396.55 | 0.810 | 0.938 | 500.82 | 0.762 | 0.879 | 570.41 | 0.763 | 0.862 |
| *Excluded Dates:* | *2012 - Jan 1, Feb 28-29, Mar 10-11, Nov 3-4, Dec 31* | | | | | | | | |
| | *2013 - Jan 1, Mar 9-10, Nov 2-3, Dec 31* | | | | | | | | |
| | *2014 - Jan 1, Mar 8-9, Nov 1-2, Dec 31* | | | | | | | | |

**Table 3.8 – Cost Performance of Various Charging Algorithms – AM Charging.**

| AM Charging 4am to 8am | 2012 Dollars | 2012 Fraction of AVG Price | 2013 Dollars | 2013 Fraction of AVG Price | 2014 Dollars | 2014 Fraction of AVG Price |
|---|---|---|---|---|---|---|
| AVG | 222.33 | 1.000 | 540.70 | 1.000 | 534.26 | 1.000 |
| STAT (24x12) | **189.14** | **0.851** | 431.03 | 0.797 | 466.97 | 0.874 |
| STAT0 | 189.86 | 0.854 | 255.82 | 0.473 | 334.73 | 0.627 |
| STAT1 | 188.55 | 0.848 | **253.83** | **0.469** | **328.41** | **0.615** |
| STAT2 | 190.59 | 0.857 | 257.78 | 0.477 | 330.68 | 0.619 |
| STAT3 | 191.19 | 0.860 | 259.64 | 0.480 | 331.93 | 0.621 |
| STAT4 | 191.19 | 0.860 | 259.64 | 0.480 | 331.93 | 0.621 |
| STAT (Cont.) | 189.49 | 0.852 | 258.42 | 0.478 | 330.39 | 0.618 |
| STAT (Perfect) | 179.42 | 0.807 | 239.81 | 0.444 | 313.85 | 0.587 |
| LP | 176.72 | 0.795 | 233.50 | 0.432 | 285.00 | 0.533 |
| *Excluded Dates:* | *2012 - Jan 1, Feb 29, Mar 10, Nov 4, Dec 31* | | | | | |
| | *2013 - Jan 1, Mar 9, Nov 3, Dec 31* | | | | | |
| | *2014 - Jan 1, Mar 8, Nov 2, Dec 31* | | | | | |

**Table 3.9 – Cost Performance of Various Charging Algorithms – PM Charging.**

| PM Charging | 2012 | | 2013 | | 2014 | |
|---|---|---|---|---|---|---|
| 4pm to 8pm | Dollars | Fraction of AVG Price | Dollars | Fraction of AVG Price | Dollars | Fraction of AVG Price |
| AVG | 386.23 | 1.000 | 540.70 | 1.000 | 534.26 | 1.000 |
| STAT (24x12) | **304.36** | **0.788** | 431.03 | 0.797 | **466.97** | **0.874** |
| STAT0 | 308.40 | 0.798 | 433.97 | 0.803 | 474.82 | 0.889 |
| STAT1 | 308.74 | 0.799 | 428.99 | 0.793 | 472.16 | 0.884 |
| STAT2 | 311.05 | 0.805 | **428.53** | **0.793** | 473.98 | 0.887 |
| STAT3 | 312.89 | 0.810 | 429.90 | 0.795 | 476.59 | 0.892 |
| STAT4 | 312.89 | 0.810 | 429.90 | 0.795 | 476.59 | 0.892 |
| STAT (Cont.) | 313.10 | 0.811 | 428.67 | 0.793 | 475.62 | 0.890 |
| STAT (Perfect) | 287.85 | 0.745 | 402.10 | 0.744 | 449.76 | 0.842 |
| LP | 285.60 | 0.739 | 386.31 | 0.714 | 416.78 | 0.780 |
| *Excluded Dates:* | *2012 - Jan 1, Feb 29, Mar 10, Nov 4, Dec 31* | | | | | |
| | *2013 - Jan 1, Mar 9, Nov 3, Dec 31* | | | | | |
| | *2014 - Jan 1, Mar 8, Nov 2, Dec 31* | | | | | |

**Table 3.10 – Cost Performance of Various Charging Algorithms – DAY Charging.**

| DAY Charging | 2012 | | 2013 | | 2014 | |
|---|---|---|---|---|---|---|
| 8am to 4pm | Dollars | Fraction of AVG Price | Dollars | Fraction of AVG Price | Dollars | Fraction of AVG Price |
| AVG | 678.83 | 1.000 | 898.63 | 1.000 | 1008.73 | 1.000 |
| STAT (24x12) | **560.75** | **0.826** | **745.22** | **0.829** | 871.46 | 0.864 |
| STAT0 | 584.80 | 0.861 | 751.32 | 0.836 | 870.38 | 0.863 |
| STAT1 | 574.13 | 0.846 | 747.10 | 0.831 | 861.95 | 0.854 |
| STAT2 | 564.89 | 0.832 | 746.35 | 0.831 | **861.18** | **0.854** |
| STAT3 | 562.96 | 0.829 | 747.82 | 0.832 | 862.27 | 0.855 |
| STAT4 | 563.22 | 0.830 | 749.90 | 0.834 | 864.93 | 0.857 |
| STAT5 | 563.59 | 0.830 | 751.96 | 0.837 | 867.99 | 0.860 |
| STAT6 | 564.27 | 0.831 | 754.36 | 0.839 | 871.10 | 0.864 |
| STAT (Cont.) | 561.89 | 0.828 | 753.53 | 0.839 | 868.89 | 0.861 |
| STAT (Perfect) | 531.24 | 0.783 | 702.83 | 0.782 | 817.14 | 0.810 |
| LP | 523.78 | 0.772 | 693.82 | 0.772 | 780.16 | 0.773 |
| *Excluded Dates:* | *2012 - Jan 1, Feb 29, Mar 10, Nov 4, Dec 31* | | | | | |
| | *2013 - Jan 1, Mar 9, Nov 3, Dec 31* | | | | | |
| | *2014 - Jan 1, Mar 8, Nov 2, Dec 31* | | | | | |

# 4. HOMOGENEOUS VEHICLE CHARGING WITH NETWORK CONSTRAINTS

The goal of this chapter is to adapt the Plugin Electric Vehicle (PEV) charging algorithms developed in Chapter 3 for more realistic charging scenarios and determine the extent to which the cost savings potential of those algorithms persist. Specifically, this chapter considers the additional complication where a network constraint limits the aggregate charging rate available to a collection of PEVs. This chapter considers the homogeneous case where the entire collection of PEVs initiates and concludes the charging period at the same time, has identical initial and final charge levels, and has identical charging capacities. The algorithms will be further adapted for the additional complication of a heterogeneous collection of PEVs having distinct arrival and departure times in the following chapter.

This chapter is broken into several parts:

1. Simulation Framework Summary
2. Description of Algorithms Modifications
3. Performance Results Summary and Conclusions

## 4.1 Simulation Framework Summary

The simulation studies performed in this chapter are characterized in the following table. Rationale follows.

**Table 4.1 – Simulation Framework for Homogeneous Vehicle Charging With Network Constraints.**

| | |
|---|---|
| Network Characteristics | Prices: Real Time Market, New York City<br>Network Charging Rate Constraint: PEV Load ≤ 1.44 – 1.2*(Ld/Ldmax) [MW]<br>Ld is the actual load of New York City (from NYISO)<br>$Ld_{max}$ is yearly maximum load in New York City (11,500 MW) |
| Vehicle Characteristics | Number: 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150 (Single Vehicle Equiv.)<br>Charger Maximum Rate: 9.6kW (0.80 kWhr per five-minute timestep)<br>Daily Energy Usage (per Vehicle):<br>4 hour periods – Not Simulated<br>8 hour periods – 42.50 of 85 kWhr (50%), equivalent to 125 miles/day usage |
| Charging Period Characteristics | AM - Not Simulated<br>DAY - Not Simulated<br>PM – Not Simulated<br>NIGHT – Connection Time: 8pm to 4am (8 hours, or 96 five minutes steps)<br>Charging Time: 4.43 hours (55.4%, or 53.125 of 96 timesteps) |
| Years Simulated | 2012 – all days except Jan 1, Feb 29, Mar 10, Nov 4, and Dec 31<br>2013 – all days except Jan 1, Mar 9, Nov 3, and Dec 31<br>2014 – all days except Jan 1, Mar 8, Nov 2, and Dec 31<br><br>Days within a year are simulated interdependently (initial charge is 42.5kWhr less than the final charge of the previous night). |

As stated previously, the primary modification for the work in this chapter is the addition of a network constraint. This constraint represents the fact that a significant increase in the number of PEVs in a neighborhood could result in periods where overall neighborhood electricity demand exceeds the electrical network's ability to provide that electricity (e.g., the rating of the transformer feeding the neighborhood could be too low to support peak household electrical load and the simultaneous charging of all PEVs in the neighborhood). This constraint is studied because it is a realistic challenge to the ability of the PEV charging algorithm to provide a full charge to all PEVs on a daily basis. Additionally, capacity limitations necessitate purchasing electricity at times that would otherwise not be optimal. The work of this chapter seeks to quantify the reduction in savings potential associated with capacity limitations.

In all cases, the network constraint is modeled by scaling actual load data for New York City (at times corresponding to the price data used) to a modeled neighborhood. The neighborhood is comprised of 300 homes (each with a maximum potential household load of 4W), and a variable number of PEVs, ranging from 50 to 150. The network is assumed capable of supporting 120% of the maximum household load (i.e., 1.2 * 300 homes * 4kW = 1.44MW). The hourly load data is scaled similarly. First, the load is normalized by dividing by the maximum load seen throughout the three years considered; 11500MW. Then, the normalized load is scaled to the model neighborhood (multiply by 300 homes and 4kW). The difference between the 1.44MW of available capacity and the time-varying neighborhood load can be used for PEV charging. Implementation details for how the PEV charging algorithms use this data is discussed further in the next section.

Addition of a network capacity constraint drives another change in the overall simulation framework for this chapter. The constraint introduces the potential for incomplete charging at the end of the charging period. For the case where there is no network constraint, there is complete certainty on the maximum available charging rate. Therefore, the moment when a deferability-based charging scheme must transition to a maximum-charging based scheme is known precisely. The transition can be effected to ensure charging goals are met. By contrast, the presence of a network constraint means that future charging capacity is not precisely known. The transition from a deferability-based charging scheme to a maximum-charging based scheme may be effected too late, and the charging goals may not be achieved. As a result, this study models PEV charging so that a charging shortfall in one day is manifested in the next day's initial charge level. This decision provides the ability to gain insight into capacity shortfalls and how they may impact drivers' range needs.

The impact of a network charging constraint will increase with the number of PEVs assumed in the neighborhood. Therefore, this chapter performs equivalent simulations for a variety of assumed PEVs to understand how cost and charge completion performance change with the level of PEV penetration.

Since this chapter is focused on the impact of added network constraints, attention is diverted away from the time-of-day performance impacts. Therefore, only the NIGHT charging period is considered. The night charging period is retained because it represents the most likely period for PEV charging. The night period is the most convenient time to charge for most people and offers the lowest overall electricity prices (despite not offing the greatest savings potential from deferability alone). Simulating three case years (2012-2014) is retained, however, to allow some comparison of how the effects of adding network constraints can vary depending on the price environment.

## 4.2 Description of Algorithm Modifications

Only two general algorithms are considered in this chapter, a dynamic algorithm based on the statistical algorithm developed in the previous chapter, and an ideal algorithm. Testing these algorithms will show how cost savings performance changes as PEV penetration increases and the impact of the network constraint becomes more significant. Each of these algorithms is described below.

### 4.2.1 Statistical Purchasing (Dynamic – STAT2)

The best-performing statistical PEV charging algorithm developed in the previous chapter is adapted for consideration of network constraints. That algorithm uses "hybrid" statistics for the price model; price statistics for the next two hours are based on price data from the immediately previous hour, while price statistics for times beyond the next two hours are based on statistics from 24 hours earlier than those times.

```
SetpointMin = 0
SetpointMax = 200
SetpointMid = 0.5*( SetpointMax - SetpointMin)
fTarget = (# required charging steps / # remaining charging steps)
while ((SetpointMax - SetpointMin) > 0.0001)
        fCalculated = 0;
        for each hour (h) in charging period
                Constraint = min(Constraintntwk, Constraintvehicle)/ Constraintvehicle
                if (hour has passed),           scale = 0
                elseif (hour has not started),  scale = 1
                else,                           scale = remaining fraction of hour
                z = (SetpointMid – pDAM(h) * μRTM/DAM) / (pDAM(h) * σRTM/DAM);
                portion = (scale * Constraint * CDF(z));
                fCalculated = fCalculated + (portion / #hours remaining);
        if (fCalculated > fTarget)
                SetpointMax = SetpointMid
                SetpointMid = 0.5*( SetpointMin + SetpointMax)
        elsif (fCalculated <= fTarget)
                SetpointMin = SetpointMid
                SetpointMid = 0.5*( SetpointMin + SetpointMax)
Setpoint = SetpointMid
```

**Figure 4.1 – Pseudocode for Establishing a RTM Purchasing Setpoint Based on a Statistical Price Model when Network Constraints Exist.**

The algorithm will now need to consider a variable number of vehicles. Since a single-vehicle-equivalent case is assumed, the algorithm acts in the same manner as a single-vehicle simulation except that the size of the battery, the charge levels, and the maximum charging capacity are scaled by the number of vehicles.

The statistical model is adapted for network constraints by scaling each of the hourly Gaussian price distributions by the normalized maximum charge capacity available for that hour. That is, if the aggregate capacity of all vehicle chargers is 100kW, and the network capacity available for PEV charging is 50kW, the Gaussian price distribution for that hour will by scaled by 0.5. By doing this, the algorithm anticipates future capacity shortages and reacts by raising the purchasing setpoint to obtain energy at otherwise less favorable times. This algorithm modification is show in the pseudocode of Figure 4.1, where the bold text indicates changes from the pseudocode of the previous chapter.

The calculated setpoint is compared to the Real-Time Market (RTM) price at each timestep and a purchasing decision is made. If the decision is made to purchase, the purchased quantity of electricity is dictated by the lesser of the network constraint or the aggregate PEV charger capacity. That is:

$$q_{act}(i) = \min(q_{max}, c_{ntwk} - ld(i)) \quad \textbf{(4-1)}$$

Where ld(i) represents the neighborhood load at timestep 'i'.

### 4.2.2 Linear Programming (Ideal – LP)

A linear programming (LP) algorithm is used as a point of comparison to the constrained statistical case. Given several input parameters, the LP routine yields the price-optimal vector of electricity quantities $[q_{act}^{i}]$ to be purchased at each timestep. The input parameters are as follows:

1. Objective function – is a mathematical expression whose resulting value is to be minimized. For PEV charging, the objective function is the overall charging cost:

$$Cost = [q_{act}^{i}]^{T} * [p_{RTM}^{i}] \quad \textbf{(4-2)}$$

   Since the quantity vector $[q_{act}^{i}]$ is to be solved for, only the price vector $[p_{RTM}^{i}]$ needs to be explicitly stated to define the objective function.

2. Equality Constraint – expresses a constraint which must be satisfied. For the PEV charging application, the equality constraint states that the sum of all electricity purchased must equal the desired amount of electricity to be purchased:

$$q_{des} = [n_{1}^{i}]^{T} * [q_{act}^{i}] \quad \textbf{(4-3)}$$

3. Timestep-Specific Boundaries – expresses upper and lower limits on the quantity to be purchased at each timestep. For PEV charging, such constraints limit the minimum amount of charging at each timestep to zero, and the maximum amount of charging at each timestep to that dictated by the vehicle charger:

$$0 \leq q_{act}(i) \leq nv * q_{max} \qquad \textbf{(4-4)}$$

It should be noted that limiting the minimum allowable charge to zero prevents cost optimization by "selling" back into the grid at a high price.

Another, independent, timestep specific boundary is imposed; the network constraint. As stated earlier, the maximum charging for each timestep must be within the spare network capacity. This is expressed as:

$$q_{act}(i) \leq c_{ntwk} - ld(i) \qquad \textbf{(4-5)}$$

## 4.3 Performance Results Summary and Conclusions

Simulation study results indicate that deferability-driven PEV cost savings potential in systems with network constraints decreases as vehicle penetration levels increase. This is best observed in Figure 4.2, which shows effective yearly charging prices as a function of vehicle penetration for both the STAT2 and LP method. For low penetration levels, price is unaffected by the network constraint as the aggregate load of all PEVs charging is not significant enough to be affected by the network constraint. However, as penetration levels increase, aggregate demand must be reduced for a larger number of low priced timesteps to prevent violating network constraints. This reduces deferability in the system and therefore reduces potential cost savings. Effective prices are seen to increase, and the STAT2 and LP methods converge. The behavior visually demonstrated in Figure 4.2 is numerically summarized for select PEV penetration levels in Table 4.2. Maximum savings potential drops from 20-25% to 5-10% as penetration level increases from 50 to 150 vehicles when considering an AVG baseline. For a LATE baseline, savings potential decreases from 7-15% to 5-13% as penetration level increases from 50 to 100 vehicles (note, use of AVG and LATE baselines from the unconstrained studies of the previous chapter limit the usefulness of those baselines for high penetration levels since savings becomes negative).

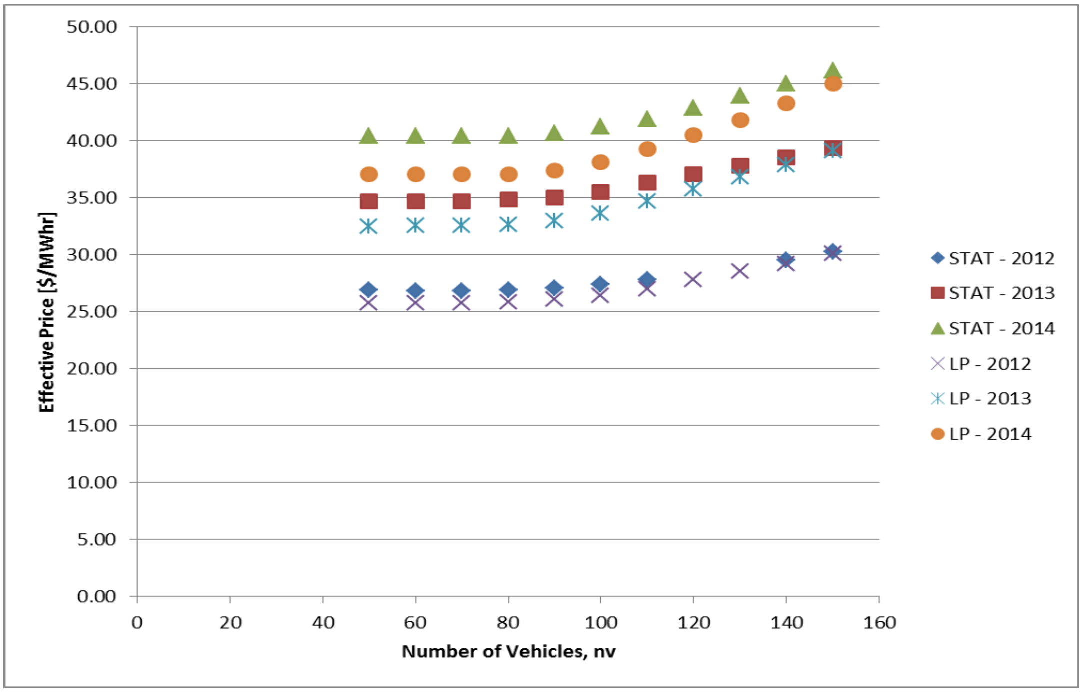


**Figure 4.2 – Cost Performance Results of PEV Charging with Network Constraints.**

**Table 4.2 – Savings Potential of PEV Charging Algorithm for Select Vehicle Penetration Levels: AVG and LATE Baseline.**

| | | 2012 | | | 2013 | | | 2014 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Max Possible | Realizable | Realizable/ Possible | Max Possible | Realizable | Realizable/ Possible | Max Possible | Realizable | Realizable/ Possible |
| AVG | nv50 | 20.0% | 16.4% | 0.821 | 24.6% | 19.6% | 0.797 | 24.5% | 6.9% | 0.281 |
| AVG | nv100 | 18.0% | 14.8% | 0.820 | 22.0% | 17.5% | 0.797 | 22.3% | 5.0% | 0.226 |
| AVG | nv150 | 6.4% | 6.0% | 0.930 | 9.2% | 8.7% | 0.944 | 8.3% | -6.3% | NA |
| LATE | nv50 | 7.4% | 3.2% | 0.438 | 13.1% | 7.3% | 0.561 | 14.7% | 6.9% | 0.468 |
| LATE | nv100 | 5.1% | 1.4% | 0.266 | 10.1% | 4.9% | 0.489 | 12.2% | 5.0% | 0.414 |
| LATE | nv150 | -8.3% | -8.8% | NA | -4.6% | -5.2% | NA | -3.6% | -6.3% | NA |

It should be noted that the performance of the STAT2 algorithm relative to the LP baseline remains high for many cases with high penetration levels (e.g., over 90% of possible savings can be realized when considering an AVG baseline for 150 vehicles in 2012 and 2013.) Comprehensive cost performance results are included at the end of this chapter; in Table 4.3 (which presents results normalized to baseline cases) as well as

Table 4.4, Table 4.5, and Table 4.6 (which display both cost performance and charging level performance, separated by year).

Simulation results also show the impact of a network constraint on the ability to completely charge a PEV by the end of the charging period. As vehicle penetration level increases, the ability of PEVs to completely charge is impacted. These results are shown in Table 4.4 (2012), Table 4.5 (2013), and Table 4.6 (2014), which indicate the number of days where final charging levels are below 95% and the number of days when final charging levels are below 50%. In general, for high levels of penetration, there are days with insufficient charging capacity available to fully charge all vehicles by the end of the charging period.

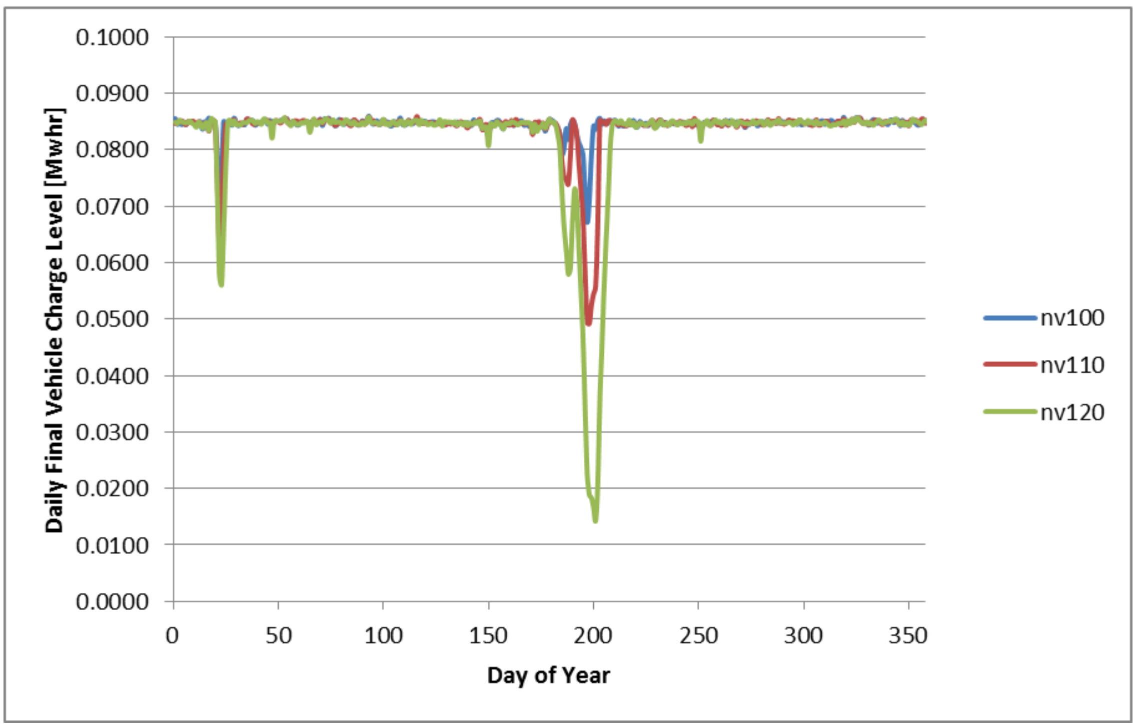


**Figure 4.3 – Level Performance Results for PEV Charging with Constraints (2013 Only).**

This can be observed in Figure 4.3; as penetration level increases, final charge level is impacted for parts of the year where overall demand is high. For PEV penetration levels above 120 vehicles, charge levels drop to zero, which is unacceptable, so that data is omitted in Figure 4.3 and shaded in Table 4.4, Table 4.5, and Table 4.6.

Despite the inability to guarantee a complete charge for high penetration levels, the performance characteristics of the STAT2 algorithm were shown to be adequate for charging PEVs with network constraints. Additional modeling within the framework of subsequent chapters is used to confirm performance when network constraints exist without the shortcomings of negative charge levels.

**Table 4.3 – Baseline-Normalized Cost Performance of Algorithms for PEV Charging with Network Constraints for a Variety of Vehicle Penetration Levels.**

| Night Charging 8pm to 4am | 2012 Effective Price [$/MWhr] | 2012 Frac of AVG Price | 2012 Frac of LATE Price | 2013 Effective Price [$/MWhr] | 2013 Frac of AVG Price | 2013 Frac of LATE Price | 2014 Effective Price [$/MWhr] | 2014 Frac of AVG Price | 2014 Frac of LATE Price |
|---|---|---|---|---|---|---|---|---|---|
| AVG: nv = 1 | 32.17 | 1.000 | 1.158 | 43.07 | 1.000 | 1.153 | 49.01 | 1.000 | 1.130 |
| LATE: nv = 1 | 27.79 | 0.864 | 1.000 | 37.36 | 0.867 | 1.000 | 43.38 | 0.885 | 1.000 |
| STAT: nv = 50 | 26.89 | 0.836 | 0.968 | 34.63 | 0.804 | 0.927 | 40.40 | 0.824 | 0.931 |
| STAT: nv = 60 | 26.83 | 0.834 | 0.966 | 34.64 | 0.804 | 0.927 | 40.39 | 0.824 | 0.931 |
| STAT: nv = 70 | 26.83 | 0.834 | 0.965 | 34.67 | 0.805 | 0.928 | 40.40 | 0.824 | 0.931 |
| STAT: nv = 80 | 26.89 | 0.836 | 0.968 | 34.78 | 0.808 | 0.931 | 40.40 | 0.824 | 0.931 |
| STAT: nv = 90 | 27.06 | 0.841 | 0.974 | 35.03 | 0.813 | 0.937 | 40.62 | 0.829 | 0.936 |
| STAT: nv = 100 | 27.41 | 0.852 | 0.986 | 35.52 | 0.825 | 0.951 | 41.19 | 0.841 | 0.950 |
| STAT: nv = 110 | 27.82 | 0.865 | 1.001 | 36.29 | 0.842 | 0.971 | 41.89 | 0.855 | 0.966 |
| STAT: nv = 120 | NA | NA | NA | 37.06 | 0.860 | 0.992 | 42.85 | 0.874 | 0.988 |
| STAT: nv = 130 | NA | NA | NA | 37.77 | 0.877 | 1.011 | 43.91 | 0.896 | 1.012 |
| STAT: nv = 140 | 29.49 | 0.917 | 1.061 | 38.53 | 0.894 | 1.031 | 44.98 | 0.918 | 1.037 |
| STAT: nv = 150 | 30.25 | 0.940 | 1.088 | 39.32 | 0.913 | 1.052 | 46.13 | 0.941 | 1.063 |
| LP: nv = 50 | 25.74 | 0.800 | 0.926 | 32.48 | 0.754 | 0.869 | 37.01 | 0.755 | 0.853 |
| LP: nv = 60 | 25.76 | 0.801 | 0.927 | 32.50 | 0.755 | 0.870 | 37.01 | 0.755 | 0.853 |
| LP: nv = 70 | 25.77 | 0.801 | 0.927 | 32.53 | 0.755 | 0.871 | 37.01 | 0.755 | 0.853 |
| LP: nv = 80 | 25.84 | 0.803 | 0.930 | 32.64 | 0.758 | 0.874 | 37.06 | 0.756 | 0.854 |
| LP: nv = 90 | 26.03 | 0.809 | 0.937 | 32.94 | 0.765 | 0.882 | 37.33 | 0.762 | 0.861 |
| LP: nv = 100 | 26.37 | 0.820 | 0.949 | 33.59 | 0.780 | 0.899 | 38.09 | 0.777 | 0.878 |
| LP: nv = 110 | 26.94 | 0.837 | 0.969 | 34.67 | 0.805 | 0.928 | 39.25 | 0.801 | 0.905 |
| LP: nv = 120 | 27.75 | 0.863 | 0.999 | 35.70 | 0.829 | 0.955 | 40.50 | 0.826 | 0.934 |
| LP: nv = 130 | 28.55 | 0.887 | 1.027 | 36.76 | 0.853 | 0.984 | 41.80 | 0.853 | 0.964 |
| LP: nv = 140 | 29.21 | 0.908 | 1.051 | 37.88 | 0.879 | 1.014 | 43.26 | 0.883 | 0.997 |
| LP: nv = 150 | 30.10 | 0.936 | 1.083 | 39.10 | 0.908 | 1.046 | 44.95 | 0.917 | 1.036 |
| *Excluded Dates:* | *2012 - Jan 1, Feb 28-29, Mar 10-11, Nov 3-4, Dec 31* | | | | | | | | |
| | *2013 - Jan 1, Mar 9-10, Nov 2-3, Dec 31* | | | | | | | | |
| | *2014 - Jan 1, Mar 8-9, Nov 1-2, Dec 31* | | | | | | | | |

**Table 4.4 – Performance Results for PEV Charging with Network Constraints – 2012.**

| | STAT (Hybrid - 2 hour) | | | LP | | |
|---|---|---|---|---|---|---|
| nv | Effective Price [$/MWhr] | Days < 95% | Days < 50% | Effective Price [$/MWhr] | Days < 95% | Days < 50% |
| 50 | 26.89 | 0 | 0 | 25.74 | 0 | 0 |
| 60 | 26.83 | 0 | 0 | 25.76 | 0 | 0 |
| 70 | 26.83 | 0 | 0 | 25.77 | 0 | 0 |
| 80 | 26.89 | 0 | 0 | 25.84 | 0 | 0 |
| 90 | 27.06 | 1 | 0 | 26.03 | 0 | 0 |
| 100 | 27.41 | 2 | 0 | 26.37 | 2 | 0 |
| 110 | 27.82 | 6 | 0 | 26.94 | 7 | 0 |
| 120 | NA | NA | NA | 27.75 | 32 | 0 |
| 130 | NA | NA | NA | 28.55 | 81 | 47 |
| 140 | 29.49 | 115 | 97 | 29.21 | 115 | 96 |
| 150 | 30.25 | 161 | 141 | 30.10 | 159 | 138 |
| Includes all days in 2012 except: Jan 1, Feb 28-29, Mar 10-11, Nov 3-4, Dec 31 | | | | | | |

**Table 4.5 – Performance Results for PEV Charging with Network Constraints – 2013.**

| | STAT (Hybrid - 2 hour) | | | LP | | |
|---|---|---|---|---|---|---|
| NV | Effective Price [$/MWhr] | Days < 95% | Days < 50% | Effective Price [$/MWhr] | Days < 95% | Days < 50% |
| 50 | 34.63 | 2 | 0 | 32.48 | 0 | 0 |
| 60 | 34.64 | 2 | 0 | 32.50 | 0 | 0 |
| 70 | 34.67 | 2 | 0 | 32.53 | 0 | 0 |
| 80 | 34.78 | 2 | 0 | 32.64 | 0 | 0 |
| 90 | 35.03 | 3 | 0 | 32.94 | 2 | 0 |
| 100 | 35.52 | 11 | 0 | 33.59 | 7 | 0 |
| 110 | 36.29 | 19 | 0 | 34.67 | 14 | 0 |
| 120 | 37.06 | 30 | 8 | 35.70 | 28 | 7 |
| 130 | 37.77 | 61 | 32 | 36.76 | 57 | 24 |
| 140 | 38.53 | 106 | 80 | 37.88 | 99 | 76 |
| 150 | 39.32 | 164 | 131 | 39.10 | 147 | 124 |
| Includes all days in 2013 except: Jan 1, Mar 9-10, Nov 2-3, Dec 31 | | | | | | |

**Table 4.6 – Performance Results for PEV Charging with Network Constraints – 2014.**

| | STAT (Hybrid - 2 hour) | | | LP | | |
|---|---|---|---|---|---|---|
| NV | Effective Price [$/MWhr] | Days < 95% | Days < 50% | Effective Price [$/MWhr] | Days < 95% | Days < 50% |
| 50 | 40.40 | 0 | 0 | 37.01 | 0 | 0 |
| 60 | 40.39 | 0 | 0 | 37.01 | 0 | 0 |
| 70 | 40.40 | 0 | 0 | 37.01 | 0 | 0 |
| 80 | 40.40 | 0 | 0 | 37.06 | 0 | 0 |
| 90 | 40.62 | 0 | 0 | 37.33 | 0 | 0 |
| 100 | 41.19 | 0 | 0 | 38.09 | 0 | 0 |
| 110 | 41.89 | 0 | 0 | 39.25 | 0 | 0 |
| 120 | 42.85 | 1 | 0 | 40.50 | 2 | 0 |
| 130 | 43.91 | 10 | 0 | 41.80 | 20 | 0 |
| 140 | 44.98 | 37 | 0 | 43.26 | 54 | 0 |
| 150 | 46.13 | 112 | 85 | 44.95 | 113 | 86 |
| Includes all days in 2014 except: Jan 1, Mar 8-9, Nov 1-2, Dec 31 | | | | | | |

# 5. HETEROGENEOUS VEHICLE CHARGING WITH NETWORK CONSTRAINTS

The goal of this chapter is to adapt the Plugin Electric Vehicle (PEV) charging algorithm developed in earlier chapters to work effectively for general purpose cases and evaluate the performance of the resulting algorithm.

Although this chapter is rooted in the specific need to address the additional complication of charging multiple PEVs with unique arrival and departure times (heterogeneous case), an entirely new, generic, problem formulation and solution is developed.

This chapter is broken into several parts:

1. Algorithm Development
    a. General Problem Definition
    b. General Solution Approach
    c. Simulation Framework Summary
    d. Comparison Algorithm Development
2. Performance Results Summary and Conclusions

## 5.1 Algorithm Development

The STAT2 algorithm developed in previous chapters has demonstrated the ability to markedly reduce PEV charging costs for the special case where a collection of identical vehicles with identical charge levels arrives in a neighborhood for charging on an identical schedule. A real-world application of such an algorithm must function properly for a collection of diverse PEVs. In such cases, the PEV arrival times, departure times, initial charge levels, desired final charge levels, and charging characteristics can be both diverse and uncertain. Also, the collection of vehicles can be connected throughout a network where there are location-specific constraints which affect the charging capacity available for subsets of the overall collection of vehicles. This section develops a problem formulation and solution method for such general cases. Then, the algorithm is refined for simulation studies that are less general in nature.

### 5.1.1 General Problem Definition

The use of a general purpose Linear Programming (LP) routine shows the usefulness of breaking an optimization problem down into a general form, and then having a solution method which works on any problem of that general form. Here, the PEV charging problem is generalized. A later section will develop and describe the corresponding solution method.

The PEV charging problem considered in this thesis can verbally be described in the following general way:

Consider a system where multiple independent consumers must accumulate a resource with a time-varying (but statistically forecast) cost. The rate of resource accumulation is limited by the availability of the consumers, as well as by maximum accumulation rates (both at the consumer level as and by the network which distributes the resource). Also, various system parameters (e.g., consumer arrival times and departure times, consumer desired level of accumulation, consumer-independent network load, and resource prices) are uncertain (forecast) at the beginning of the simulation period, but become discovered in real time. Find a setpoint rule for making purchasing decisions in real time which aims to accumulate the desired quantity of the resource at minimum cost.

This problem can be symbolically expressed as follows:

<u>Minimize:</u>

Objective function

$$Cost = \left[p_{act}^{i}\right]^{T} * \left[Q_{act}^{iv}\right] * \left[n_1^{v}\right] \tag{5-1}$$

$$(\boldsymbol{where}\ [\boldsymbol{p_{stat}^{i}}] \to [\boldsymbol{p_{act}^{i}}]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

<u>Subject to</u>:

Consumer Accumulation (Goal) Constraint:

$$\left[Q_{act}^{iv}\right]^{T} * \left[n_1^{i}\right] = \left[q_{des,act}^{v}\right] = \left[L_{des,final}^{v}\right] - \left[L_{act,init}^{v}\right] \tag{5-2}$$

$$(\boldsymbol{where}\ [\boldsymbol{L_{fc,init}^{v}}] \to [\boldsymbol{L_{act,init}^{v}}]\ \boldsymbol{on}\ '\boldsymbol{i}'\ \boldsymbol{at\ which\ vehicle}\ '\boldsymbol{v}'\ \boldsymbol{arrives})$$

Consumer Rate Constraint:

$$0 * \left[n_1^{i}\right]\left[n_1^{v}\right]^{T} \le \left[Q_{act}^{iv}\right] \le \left[A_{act}^{iv}\right].* \left(\left[n_1^{i}\right]\left[c_v^{v}\right]^{T}\right) \tag{5-3}$$

$$(\boldsymbol{where}\ [A_{fc}^{iv}] \rightarrow [A_{act}^{iv}]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

System Rate Constraints:

$$[Q_{act}^{iv}] * [a_a^v] \leq c_a * [n_1^i] - [ld_{a,act}^i] \tag{5-4a}$$

$$[Q_{act}^{iv}] * [a_b^v] \leq c_b * [n_1^i] - [ld_{b,act}^i] \tag{5-4b}$$

...

$$[Q_{act}^{iv}] * [a_x^v] \leq c_x * [n_1^i] - [ld_{x,act}^i] \tag{5-4c}$$

...

$$[Q_{act}^{iv}] * [a_{nc}^v] \leq c_{nc} * [n_1^i] - [ld_{nc,act}^i] \tag{5-4d}$$

$$(\boldsymbol{where}\ [\boldsymbol{ld}_{x,fc}^{i}] \rightarrow [\boldsymbol{ld}_{x,act}^{i}]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

Where the variables are defined as follows:

| **Name** | **Dimension** | **Description** |
|---|---|---|
| A→B on 'i' | Operator | Represents transformation from one vector or matrix (A) to another (B), updated as timesteps 'i' increment. |
| Cost | Scalar | Aggregate Cost for all Consumers |
| i | Scalar | Index for timesteps (maximum is 'ni') |
| v | Scalar | Index for consumer (maximum is 'nv') |
| c | Scalar | Index for network constraints (maximum is 'nc') |
| $c_x$ | Scalar | Magnitude of network accumulation constraint 'x' |
| $[c_v^v]$ | Vector (v x 1) | Consumer-specific accumulation constraint, for each consumer 'v' |
| $[n_1^i]$ | Vector (i x 1) | Vector of ones for each timestep 'i' |
| $[n_1^v]$ | Vector (v x 1) | Vector of ones for each timestep 'v' |
| $[p_{act}^i]$ | Vector (i x 1) | Actual resource price at each timestep 'i' |
| $[p_{stat}^i]$ | Vector (i x 1) | Forecast resource price (statistically described) at each timestep 'i'.<br>Statistical description at each timestep includes:<br>mean expected price (μ)<br>price standard deviation (σ) |
| $[Q_{act}^{iv}]$ | Matrix (i x v) | Actual resource quantity purchased at each timestep 'i', and for each consumer 'v' |

| | | |
|---|---|---|
| $[q^v_{des,act}]$ | Vector (v x 1) | Actual desired resource quantity to be purchased, for each consumer 'v' |
| $[L^v_{des,final}]$ | Vector (v x 1) | Actual desired final resource level, for each consumer 'v' |
| $[L^v_{act,init}]$ | Vector (v x 1) | Actual initial resource level, for each consumer 'v' |
| $[L^v_{fc,init}]$ | Vector (v x 1) | Forecast initial resource level, for each consumer 'v' |
| $[A^{iv}_{act}]$ | Matrix (i x v) | Actual availability of each consumer for resource accumulation at each timestep 'i', for each consumer 'v' |
| $[A^{iv}_{fc}]$ | Matrix (i x v) | Forecast availability of each consumer for resource accumulation at each timestep 'i', for each consumer 'v' |
| $[a^v_x]$ | Vector (v x 1) | Applicability of network accumulation constraint 'x' to consumers |
| $[ld^i_{x,act}]$ | Vector (i x 1) | Actual consumer-independent load at network point 'x', for each timestep 'i' |
| $[ld^i_{x,fc}]$ | Vector (i x 1) | Forecast consumer-independent load at network point 'x', for each timestep 'i' |

It is noted that this problem is framed identically to a LP problem with the exception of the information included in bold font. The fundamental difference in the above problem is that many of the parameters in the above problem are not know with certainty at the time the algorithm must be executed. Therefore, these parameter values must be guessed. That is, forecast data must be substituted for actual data until the actual data becomes available. Since the optimization is based on cost, a statistical forecast is used for resource price.

This fundamental difference drives the form of the solution. In the case of LP, parameter values are known precisely at each timestep. Therefore, it is possible for the form of the solution to be a schedule for how much resource each consumer is to accumulate at each timestep. However, since the parameters for the real-world problem are not known precisely, a more robust solution form must be found. A price setpoint for real-time purchasing decisions is used. Since the nature of prices is statistical, the setpoint ensures that the "correct" low price resources are purchased without assuming those prices occur in a particular sequence.

### 5.1.2 General Solution Approach

The general solution method for the above-stated problem is derived from that developed for previous chapters. However, expanding the scope of the problem to include multiple diverse vehicles requires that a key architectural decision be made; whether the optimization should be system-focused or individual-vehicle focused. In the previous chapters, a single-vehicle equivalent model could be employed for simulating any number of vehicles, so the system-level and vehicle-level optimization schemes are equivalent. In the heterogeneous case here, that simplification is no longer possible and one of the following must be selected:

1. System Optimization – Determine a single optimal purchase strategy based on the aggregate desired quantity of electricity for all vehicles and network constraints. Then, determine how to distribute that electricity to individual vehicles while satisfying those vehicles' needs.
2. Vehicle-Specific Optimization – Determine optimal purchase setpoints for each vehicle first, independently of system constraints. Then, apply system-level constraints and readjust the purchasing scheme until all system-level constraints are satisfied.

The second scheme is employed here. A major reason for this decision is that the vast majority of timesteps encountered throughout the year are not expected to result in constraint violations. Therefore, it was considered to be a better use of resources to treat all vehicles independently, and then only apply additional complexity for the few cases where adjustments are needed to ensure system constraints are met. Otherwise, the problem of redistributing a system-optimal purchase solution would have to be undertaken for each timestep. Additionally, centering the overall optimization strategy on individual vehicles can allow some decentralization of the computational resources needed to execute the algorithm.

Based on a vehicle-centric approach, the PEV charging algorithm is comprised of the following steps (A more detailed description is provided in Appendix B):

1. Construct statistical price distributions, $\Phi(p)$, for every vehicle and timestep based on $\left[p_{stat}^{i}\right]$ (specifies mean and standard deviation of prices at each timestep).

2. Scale each statistical price distribution into a price-energy distribution so that the integral over the entire distribution is equal to the maximum quantity of electricity which can be purchased by the applicable vehicle at the timestep of interest. The scaling factors of all price distributions is compactly saved in a scaling matrix, $\left[S^{iv}\right]$.

3. Calculate a vector of vehicle-specific purchase setpoints ($[SP^{v}]$) which satisfies, for each vehicle:

$$\sum_{i=1}^{ni}\left(c_{v}^{v}(\mathrm{v}) * S^{iv}(\mathrm{i},\mathrm{v}) * \int_{-\infty}^{SP}\Phi(p)dp\right) = q_{des}^{v}(v) \qquad \textbf{(5-5)}$$

4. Resolve potential network constraint violations by iteratively rescaling price-energy distributions (via $\left[S^{iv}\right]$) and recalculating purchase setpoint prices ($[SP^{v}]$).

$[SP^{v}]$ and $\left[S^{iv}\right]$ are independent. For a given price, higher price setpoints increase demand and therefore increase the potential for a network constraint violation. The constraint violation, in turn, implies that electricity supply at a given price is limited, so setpoints must be further increased to ensure the electricity shortfall is made up at a higher price. $[SP^{v}]$ and $\left[S^{iv}\right]$ both converge when calculated setpoint price becomes high enough to ensure that the desired quantity of electricity can be purchased.

Since only a statistical forecast of future prices exists, the method of resolving the impact of network constraints must also be performed stochastically. A desired maximum likelihood of a constraint violation ($lf_{des}$) is selected. For a given value of $lf_{des}$, there is an electricity price which statistically bounds an equivalent percentage of expected prices (e.g., $23 may bound the lowest 20% of potential electricity prices for a given timestep). This price is compared to projected purchase setpoints to anticipate timesteps where excess capacity (e(i), below) is inadequate (i.e., negative) for aggregate demand.

$$e(i) = c_x - ld_x^i(i) - [a_x^v]^T\left([c_v^v].*\left[Purchase_{proj}^v\right]\right) \quad \textbf{(5-6)}$$

where $\left[Purchase_{proj}^v\right]$ indicates which vehicles are likely to be charging, given $lf_{des}$ and the given price distribution. If e(i) is negative, the price-energy distributions must be rescaled via:

$$S^{iv}(i) = \left[Purchase_{proj}^v\right].*\left(1 + \frac{e(i)}{\left[Purchase_{proj}^v\right]^T * [c_v^v]}\right) + \left([n_1^v] - \left[Purchase_{proj}^v\right]\right).*\left[A_{proj}^{iv}\right] \quad \textbf{(5-7)}$$

Here, capacity is reduced to within the network constraint for only vehicles likely to be charging.

5. Make purchase decision based on $[SP^v]$ for given timestep price

6. Update uncertain parameters to reflect latest information on vehicle arrivals, departures, and charge levels.

7. Repeat above steps for each timestep to dynamically adapt setpoints given forecast errors.

### 5.1.3 Simulation Framework Summary

The simulation studies performed in this chapter are characterized in the following table. Rationale follows.

**Table 5.1 – Simulation Framework for Heterogeneous Vehicle Charging With Network Constraints.**

| | |
|---|---|
| Network Characteristics | Prices: Real Time Market, New York City<br>Network Charging Rate Constraint: PEV Load ≤ 1.44 – 1.2*(Ld/Ldmax) [MW]<br>Ld is the actual load of New York City (from NYISO)<br>$Ld_{max}$ is yearly maximum load in New York City (11,500 MW) |
| Vehicle Characteristics | Number: 40, 60, 80, 100, 120, 140, 160, 180, 200 (Independent Vehicles)<br>Charger Maximum Rate: 9.6kW (0.80 kWhr per five-minute timestep)<br>Daily Energy Usage (per Vehicle):<br>4 hour periods – Not Simulated<br>8 hour periods – Not Simulated<br>15 hour periods – 42.50 of 85 kWhr (50%), equivalent to 125 miles/day usage |
| Charging Period Characteristics | AM - Not Simulated<br>DAY - Not Simulated<br>PM – Not Simulated<br>NIGHT – Max Connection Time: 6pm to 9am (15 hours, or 180 five minutes steps)<br>Avg Connection Time: 8pm to 7am (11 hours, or 132 five minutes steps)<br>Charging Time: 4.43 hours (40.3%, or 53.125 of 132 timesteps) |
| Years Simulated | 2012 – Not Simulated<br>2013 – Not Simulated<br>2014 – all days except Jan 1, Mar 8, Nov 2, and Dec 31<br><br>Days within a year are simulated interdependently (initial charge is the maximum of either zero or 42.5kWhr less than the final charge of the previous night). |

With a generalized PEV charging algorithm developed above, the remaining goal of this chapter is to assess the performance of that algorithm. A rigorous assessment of the algorithm would require simulating PEV charging for a fleet of vehicles with unique charging characteristics, driving characteristics, and driving schedules. However, a less rigorous assessment is pursued here for simplicity.

Simulations are performed for a fleet of vehicles with random arrival and departure times, since that dimension of variability is at the heart of the enhancement made in this chapter. Also, simulations cover a range of PEV penetration levels to evaluate the behavior of the generalized network constraint resolution algorithm. However, other dimensions of variability are eliminated. All vehicles are assumed to have identical charging characteristics (i.e., $[c_v^v] = c_v * [n_1^v]$) and daily electricity use (i.e., ½ $L_{max}$) as assumed in earlier chapters. Given the variability in vehicle charging schedules, vehicles are considered to be sufficiently diverse so that additional diversity in charging and driving characteristics is not necessary. Also, the scope of simulation is reduced to only the NIGHT charging period in 2014.

Consideration of random PEV arrival and departure times requires reframing the charging period. The evaluated daily charging period spans from 6pm to 9am, with all arrivals and departures occurring somewhere within that period. Fleet-average arrival and departure times are set to 8pm and 7am respectively. This increase in average vehicle charging period (from 8 hours to 11 hours) represents a significant increase in deferability with additional cost savings, and is balanced by consideration of deeper PEV penetration levels to reduce cost savings. The fleet-average arrival/departure times form anchor points for randomized selection of vehicle-specific arrival and departure times. Each individual vehicle is randomly assigned its own average arrival and departure time which is retained for a given simulation month. These monthly vehicle-specific base times are selected from a Gaussian distribution centered at the fleet average arrival/departure times, with a standard deviation of 1.5 hours (arrivals) and 1.1 hours (departures). Daily arrival/departure times are also randomized about each vehicle's monthly average times. Specifically, daily arrival/departure times are selected from a Gaussian distribution centered about the monthly vehicle-specific times with a standard deviation of 0.35 hours.

The randomly-generated arrival and departure times used for each vehicle are contained in vehicle availability matrices, $[A_{act}^{iv}]$. To facilitate fair comparisons between simulations, the exact same availability matrices are used for all simulations of a given day. That is, simulation of charging 80 vehicles on January 20th via the LATE charging method and simulation of charging 120 vehicles on January 20th via the STAT method use the same availability matrix, with the exception that data for vehicles 81 through 120 is removed from the 80 vehicle simulation. Equivalent forecast availability matrices are also used. These forecast matrices are derived by aggregating the actual availability matrices for a given month, which yields forecast matrices that represent each vehicles average arrival and departure times for that month.

A single network constraint, identical to that in the previous chapter, is assumed. That is, overall network capacity is set to 1.44MW, and PEV-independent actual and forecast load is based on normalizing New York City load to a neighborhood of 300 homes (each with a maximum household load of 4kW). As described above, in the generalized algorithm used in this chapter, network constraints are resolved by reducing expected charging capacity for a given time period until the likelihood of a constraint violation falls below a user-selected threshold. In this study, the supplied threshold is a 20% likelihood of constraint violation. Consideration of additional network constraints was not pursued because any additional insight was not considered worth the substantial increase in algorithm complexity and its attendant increase in required computational resources.

Given the network constraint and general parameter uncertainty, PEVs may fail to completely charge by the end of the charging period. This is especially true for cases with high PEV penetration, and for vehicles which arrive relatively late and depart relatively early. To understand the impact this potential has on drivers, simulations base initial charge level on the previous day's final charge level. Simulation results are evaluated to detect cases where multi-day charging behavior results in a failure to support daily driving requirements.

### 5.1.4 Comparison Algorithm Development

As in previous chapters, comparison algorithms are needed to establish a basis for evaluating performance of the main, statistical algorithm. Reframing the main algorithm and the accompanying simulation studies to consider random PEV arrivals and departures requires reworking the baseline algorithms used in previous chapters. The revised basline algorithms are discussed as follows:

#### 5.1.4.1 Average Price Purchasing (Simple - AVG)

This algorithm considers the simple case of purchasing electricity at the average price throughout the charging period, and represents the extreme case of being completely unselective. Each vehicle is charged at a unique rate determined by dividing the desired charge by the duration of its expected availability. The final aggregate price of electricity for each vehicle roughly represents the average price in effect throughout the vehicles availability. However, purchasing shortfalls occur if the vehicle departs early or network constraints force temporary reductions in the vehicle-specific charge rate. These shortcomings highlight the practical limitations of this practically realizable baseline charging scheme.

#### 5.1.4.2 Linear Programming (Ideal - LP)

This ideal case represents the case of optimal selectivity, and is used to benchmark how well an algorithm performs when compared to the best possible, but not practically realizable, performance. This algorithm represents the case where electricity is purchased only during the lowest priced 5-minute periods. Charging is simulated by using Matlab's linear programming routine with the formulation described in the "General Problem Definition" section above, except that actual prices, loads, availabilities, and initial levels are used exclusively. There is no use of forecast parameters. It is noted that if a vehicle has insufficient availability to result in a complete charge by the end of the charging period, the algorithm maximizes charging to the extent possible.

#### 5.1.4.3 Late Night Purchasing (Simple - LATE)

This algorithm considers purchasing electricity as late in the charging period as possible to make use of the fact that prices are typically lowest late at night. The latest possible time to initiate charging and still conclude the charging period with a full charge is calculated for each vehicle. This is done by subtracting the required time for fully charging at the maximum charge rate from the projected departure time. As this is intended to be a simple and practically realizable baseline case, charging shortfalls will occur if the vehicle departs before its expected departure time or if network constraints force temporary reductions in each vehicles charging rate.

#### 5.1.4.4 Late Night Linear Programming (Simple - LATELP)

This algorithm represents a comparison point similar to the LATE case, except that it represents a not-practically-realizable version. Matlab's linear programming routine is used to simulate the idealized case of charging all vehicles as late as possible. The main difference from the LATE case is the this case anticipates periods where charging rate is limited by network constraints, and shifts charging to earlier periods.

#### 5.1.4.5 Midnight Centered Purchasing (Simple - MIDNIGHT)

This algorithm represents a comparison point similar to the LATE and LATELP cases in that a simple rule attempts to reduce costs by leveraging the fact that prices are generally cheaper late at night. However, the MIDNIGHT case recognizes that prices begin to rise as morning approaches. Therefore, rather than initiate charging as late as possible, the MIDNIGHT algorithm initiates charging near midnight. For many cases, this leads to charging completion prior to the morning price rise. For cases where post-midnight charging is insufficient, the balance of charging is performed just prior to midnight. This algorithm is implemented via Matlab's linear programming routine where the objective function is weighted to prefer charging at midnight and later (reduced weighting as time progresses after midnight), and then prior to midnight (reduced weighting for times earlier than midnight). As with other linear programming based algorithms, the MIDNIGHT case represents an ideal, but not-physically realizable, case.

## 5.2 Performance Results Summary and Conclusions

Performance results are presented in Appendix A and discussed in this section.

In general, the cost performance exhibited in previous chapters is unaltered by adding the complexity of heterogeneous PEV charging. Figure A.1 shows the effective electricity price achieved for various charging scenarios (i.e., homogeneous with and without constraints and heterogeneous with constraints), charging methods (e.g., LATE, STAT2, AVG, etc.), and number of vehicles, all simulated on a consistent framework. The results show very close performance across charging scenarios for all charging methods. Note that effective prices for the homogeneous and heterogeneous constrained cases match well for all vehicle penetration levels while those results align well with the unconstrained case at the 40 vehicle penetration level only. This is expected because the effect of the constraint is to increase effective price with vehicle penetration, while penetration has no effect on price for the unconstrained case. Numerical confirmation of the general alignment of results across charging scenarios is shown in Table A.2 for 40 vehicle penetration. The table shows that STAT2 method can save approximately 24% out of a maximum possible 32% for an AVG baseline, regardless of scenario. Similarly, the STAT2 method can save approximately 16% out of a maximum possible 24% for a MIDNIGHT baseline, regardless of scenario. These results show promising savings potential.

It is also worth describing the cost performance of the various algorithms relative to each other. Again, this can be visualized in Figure A.1. As expected, the AVG method yields the highest effective prices (due to its inability to capitalize on deferability) while the LP method yields the lowest effective prices (since it uses deferability optimally). The STAT2 method tracks the LP method in trend, though its performance is understandably not as good as the optimal case. The three late-night methods as a group track between the AVG and STAT2 methods although, like the AVG method, results tend to be invariant of penetration level due to not capitalizing on deferability. Performance between the three late-night methods shows the MIDNIGHT method as the best performing algorithm since it considers the effect that latest charging is not always cheapest. This is because, although late night costs are generally cheapest, prices rise again as consumers wake up. The MIDNIGHT algorithm favors purchasing in the midnight to 4am period rather than just the latest times in the charging period. Since the MIDNIGHT algorithm is the most competitive realistic baseline method, it is the preferred point for realistic cost savings comparison.

While cost performance is the primary metric for assessing algorithm viability, trends in final charging level must also be considered. If an algorithm is unable to ensure minimum charging levels are met, it will not be accepted by consumers regardless of cost performance.

Figure 5.1 shows the number of calendar days within 2014 where the minimum charging level of 50% is not achieved by the STAT2 and LP charging methods.

Numerical values for this heterogeneous scenario are tabulated in Table A.6. These results indicate that, for penetration levels between 80 and 180, there are 12 calendar days in 2014 where about 30 vehicles are incapable of charging above 50%. These LP results prioritize fully charging PEVs above cost savings, so the reason for the level shortfall must be that the statistical arrival/departure characteristics of those vehicles precludes them from being available long enough to obtain the necessary charge. This behavior can then be "blamed" on the poor charging habits of the drivers; they arrive late and leave early. However, level performance with the STAT2 algorithm for equivalent simulations is much worse. Figure 5.1 shows a steep increase in affected calendar days for penetration levels above 140. Such performance calls the viability of the STAT method into question for high penetrations levels.

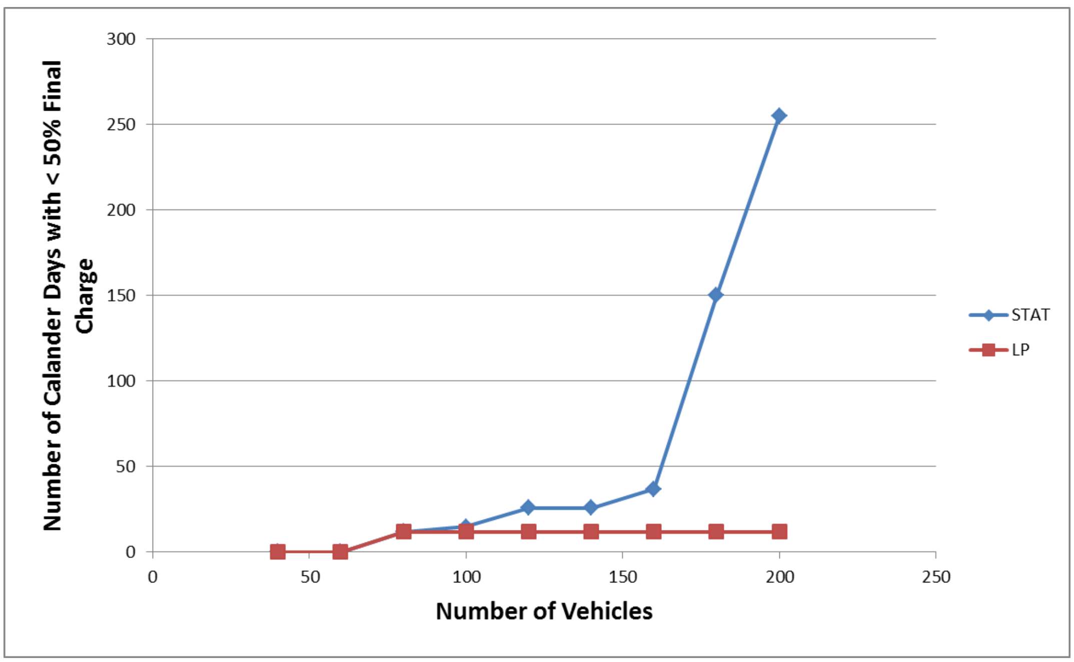


**Figure 5.1 – Charging Level Performance as a Function of Number of Vehicles Charging for 2014 Heterogeneous with Network Constraints Scenario with Statistical Charging Algorithm.**

Level performance results for the STAT2 and LP methods was further investigated by creating color matrices for heterogenous charging with 160 vehicles. These matrices are shown in Figure 5.2. Each matrix element has final charge level indicated by color, with ‘red’ indicating full charge and ‘green’ indicating low charge (near or below 50%). The horizontal matrix dimension is vehicle number, and the vertical matrix dimension is the day of the year. The rightmost plot represents results for the LP method, and shows that a full charge is available for all days except several associated with Vehicle Number 65 in September (note that arrival/departure trends are consistent within a given month). The left plot represents results for the STAT2 method, and shows that most days results in full (red) or near full (yellow) charge except for three vehicle-month groups. Overall, the results confirm that level shortfalls are tied to month-specific poor vehicle charging behavior. Although some vehicles are incapable of fully charging throughout a month (based on LP), there is also a trend for some vehicles which have unfavorable availability behavior to fail to fully charge even though it is physically possible. Further investigation has shown this trend within the STAT2 algorithm to be a result of its constraint violation resolution mechanism.

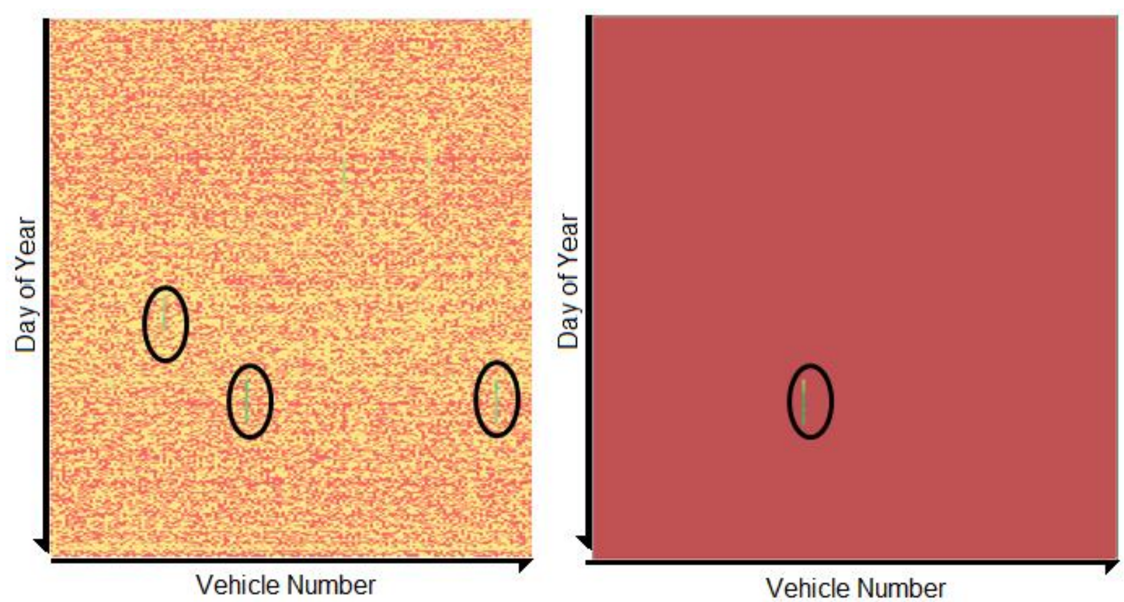


**Figure 5.2 – Color Matrices of Final Charging Level for Heterogeneous Charging with 160 Vehicles: The Left Block Shows Results from the STAT Method, and the Right Block Shows Results from the LP Method.**

The STAT method’s constraint violation resolution method is based on redistributing charge capacity equally among all vehicles whose price setpoints are above the Real Time Market (RTM) price at a given timestep. This means that vehicles whose availability is barely long enough to obtain a good charge level when charging at the maximum rate may fail to charge adequately if the charge rate must be reduced by network constraints. This contrasts with the LP method which, when a timestep becomes constraint limited, will preferentially allocate charge to vehicles which would otherwise have trouble achieving a full charge level. Thus, it is possible to revise the STAT2 method's charge capacity redistribution method to preferentially allocate charging capacity to vehicles with poor availability behavior and, in general, reduce the number of vehicles impacted by failure to charge.

Such an algorithm revision was performed and simulated for the heterogeneous case with 200 vehicles. When network constraint limited, charge capacity was redistributed proportionally to the number of time periods projected to be remaining for a given vehicle. The revised algorithm reduced the number of vehicle days with less than 50% charge levels from 768 to 2 while increasing the effective purchase price of electricity from \$43.19 to \$43.70.

# 6. IMPACT OF REGULATION COSTS

The goal of this chapter is to evaluate the impact that increased system regulation and its associated costs can have on the overall cost of using the statistical Plugin Electric Vehicle (PEV) charging method described in previous chapters.

This chapter is broken into several parts:

1. Background
2. Method
3. Performance Results Summary and Conclusions

## 6.1 Background

While the PEV charging methodology described earlier appears to offer considerable cost savings to the aggregator, one important factor reduces the feasibility of practically implementing such a method on a large scale; the need for fast-acting generation to support the significant load swings inherent to the method.

The optimality of the STAT2 algorithm is due, in part, to its “bang-bang” behavior. That is, as soon as price becomes high, the PEV charging system is to discontinue charging as quickly as possible. Similarly, as soon as price switches to a low value, the PEV charging system is to resume charging as quickly as possible. This quick, price-following behavior maximizes the energy purchased at low cost and minimizes the energy purchased at high cost. Although this behavior is optimal from a naïve charging cost point of view, when implemented on a significant scale it can generate severe external costs to the system which will impact all users.

In the extreme case, large scale bang-bang charging behavior can lead to load ramping that is too extreme (in rate and/or magnitude) for grid reliability to be maintained. As grid loads change, an equivalent quantity of generation must also be changed in unison to keep supply balanced with demand. If the variation in generation cannot keep up with the variation in load or vice versa, grid voltage and/or frequency will not be able to be maintained within the required range. Parts of the system may then trip off line and result in outages. Such behavior has been observed in Tokyo, Japan, when an unprecedented high rate of load increase caused parts of the network to trip off line [29]. Also, deliberately tripping off wind power plants in response to periods of negative Real-Time Market (RTM) prices has “created operational challenges” in New York State [30]. The costs of such behavior are clearly unacceptable regardless of the potential savings estimated in previous chapters.

In a less severe case, generation will be able to keep up with variations in load. However, additional regulation resources (i.e., generators with the ability to quickly change output) will likely need to be deployed to dynamically balance energy supply and demand. These resources are deployed at a cost premium.

In a strict sense, the additional costs described above do not affect PEV savings potential for the aggregator. Current NYISO pricing rules treat regulation as overhead and the costs are spread evenly across all users. However, any significant use of the PEV charging algorithm may change market and grid dynamics enough to warrant an out-of-market intervention which prohibits its use. Such an intervention would likely prevent any aggregator from extracting any benefit from the STAT2 algorithm.

At this point, it is worth commenting on the appropriateness of the NYISO RTM pricing scheme for facilitating appropriate demand response in general. In this thesis, PEV charging demand is controlled to respond to RTM electricity prices. In the ideal case, responding to such price signals would result in better load balancing. A momentary dip in demand should result in a dip in prices, and the PEV charging algorithm should act to “fill” the dip in demand and keep overall demand fairly consistent. Similarly, a momentary jump in demand should result in a jump in prices and a corresponding fall in PEV charging. However, actual behavior does not match this ideal. NYISO RTM prices reflect market conditions at least one 5-minute cycle earlier. Furthermore, prices generally remain in effect for 5 minutes, regardless of the demand response, so there is no mechanism to moderate the magnitude of the response to the magnitude of the initial imbalance. Thus, RTM prices do not accurately reflect instantaneous supply/demand conditions. In fact, the conditions-to-price time lag may drive instabilities rather eliminate them. This fundamental shortcoming in RTM pricing explains why a demand response to RTM prices may have the deleterious effects on grid stability described above.

Use of PEVs' charging flexibility for demand response is better suited for interaction with the regulation marketplace rather than the RTM. Regulation is controlled every 6 seconds (vice every 5 minutes), and is directly dispatched to balance grid conditions. This timely and direct (vice using prices as an intermediary signal) control method ensures that grid conditions remain balanced. The PEV charging method considered in this paper, by contrast, exacerbates imbalances and drives the need for additional regulation resources to restore balance.

In this chapter, an effort is made to model the added regulation costs which are necessitated by the large demand swings inherent to the PEV charging algorithm, and understand the extent to which they counteract the potential overall cost savings offered by the statistical charging algorithm.

## 6.2 Method

This sub-study recalculates PEV charging costs calculated for the scenarios considered in previous chapters by including regulation cost penalties. The simulation results included in Appendix A for various charging scenarios under a common simulation framework are post-processed by adding load-swing costs based on NYISO regulation pricing. Four movement penalty price structures are considered:

### 6.2.1 Baseline

Charging costs are identical to those determined in earlier studies; no regulation-based costs are added.

### 6.2.2 Strictly Movement

Baseline charging costs are revised to reflect "movement" costs every time PEV load changes. That is, each time PEV load changes, the absolute value of the change in demand is multiplied by the NYISO regulation movement price at that time. Although the actual cost of regulation in NYS is covered by the combination of a movement cost and a capacity cost, this price structure deliberately neglects the capacity component for a conservatively low cost estimate.

### 6.2.3 Augmented Movement

Baseline charging costs are revised to reflect movement costs every time PEV load changes. Each time PEV load changes, the absolute value of the change in demand is multiplied by the NYISO regulation movement price at that time, as well as $1/12^{th}$ the capacity price at that time. Regulation capacity is generally dispatched for longer periods than five minutes, with the price paid per hour in service, regardless of movement. However, this price structure assumes that capacity is only dispatched for one five-minute period at a time. This price structure attempts to represent a more realistic view of regulation costs, but without considering the complexity of dispatching regulation capacity for longer periods.

### 6.2.4 Movement + Capacity

Baseline charging costs are revised to reflect both movement and capacity prices. Each time PEV load changes, the absolute value of the change in demand is multiplied by the NYISO regulation movement price at that time (as described in [31] and [32]). Additionally, the maximum absolute change in movement within a given hour is used to determine the capacity required for the entire hour. The average capacity price for that hour is then applied to the capacity required for that hour. This pricing scheme attempts to most realistically represent the true cost of added regulation. Applying capacity requirements on an hourly basis reflects the fact that regulation capacity must be on line even if there is only the potential, but not the certainty, of movement. This would be the case for the algorithms of interest because the extent of movement is not known until the RTM price is discovered.

It should be noted that it is possible that the Movement + Capacity pricing structure may not be represent the highest possible costs. In the extreme, use of the smart PEV charging algorithm would require a considerable expansion in the number of regulation-capable generators tied to the system. Building new generator capacity would likely drive up regulation costs beyond the historical costs assumed in this study.

## 6.3 Performance Results Summary and Conclusions

Simulation results are presented in Appendix A. Figure A.1 through Figure A.4 show cost results for baseline regulation costs, strictly movement regulation costs, augmented movement regulation costs, and movement + capacity regulation costs, respectively. Within each figure, results are broken down into subplots according to charging scenario (i.e., homogeneous charging – no constraints, homogeneous charging – with constraints, and heterogeneous charging – with constraints). Each subplot, in turn shows effective charging price as a function of vehicle penetration for several charging methods. The numerical values which support the plot results are tabulated in Table A.4 through Table A.15.

As explained in earlier chapters, the STAT method shows considerable cost savings over the AVG and MIDNIGHT baseline cases, although the magnitude of those savings decreases with increased PEV penetration level. These results are seen in the baseline regulation cost model of Figure A.1. Adding "strictly movement" costs has a near negligible effect on overall costs, as seen in Figure A.2 and supporting tables, since the NYSIO movement costs are very small compared with capacity costs (e.g., movement costs are less than one dollar per MW of movement, whereas capacity costs can be tens of dollars per MWhr of capacity).

Including "augmented movement" costs significantly changes cost performance (e.g., potential savings for the STAT2 method in the heterogeneous charging scenario drop from 25% of a maximum potential 32% to 16% of a maximum possible 24% by adding augmented movement costs). However, the general cost performance trends do not change (see Figure A.3). That is, the relative performance (ranking) of the various PEV charging algorithms do not change with respect to each other, and the same cost behavior exists relative to increased PEV penetration levels. Therefore, the statistical charging algorithm can be seen as having a net-positive societal benefit, even though it necessitates that additional regulation resources must be committed to support grid operations.

Including "movement + capacity" costs results in a complete reversal of the conclusion of earlier work (see Figure A.4). The statistical algorithm drives the highest overall costs of all methods, followed by the LP method. The MIDNIGHT method develops the lowest costs. These results are understandable. If regulation costs must be applied regardless of whether they are needed at a given moment, cost for the STAT2 and LP methods will both become significant, but the LP method will better utilize the now-available regulation capacity. The MIDNIGHT algorithm, by contrast selects the cheapest overall times for charging and exercises movement only at the beginning and end of its preferred charging period.

Modified versions of the STAT and LP methods were developed and simulated for all scenarios and regulation cost models, but only for the case of 40 vehicle penetration. The modified algorithms reduced the periodicity of purchase decision making from every 5 minutes to every 10 minutes. This modification was performed to see whether the reduced periodicity could reduce net costs since the frequency of movement could be reduced while potential savings through selective purchasing might be minimally reduced.

For the "no regulation" and "strictly movement" cost models, overall cost increased slightly as regulation costs are already minimal, but selectivity based savings decreased.

For the "augmented movement" cost model, overall cost for the LP algorithm decreased slightly, but overall cost for the STAT algorithm increased slightly. Again, this can be explained by the fact that both algorithms will incur similar new regulations costs, but the LP algorithm is better able to utilize movement for cost savings.

For the "movement + capacity" cost model, reducing charging decision periodicity made no significant impact for the STAT algorithm, but resulted in a net cost decrease in the LP algorithm. For both algorithms with this cost model, movement costs are minimal, so reduced periodicity provides little benefit. However, as noted earlier, the LP algorithm is better able to utilize regulation capacity, once committed, for cost savings.

Generally, the results of this chapter are discouraging from the standpoint of demonstrating the viability of cost savings through a statistical PEV charging algorithm. The most realistic model of regulation costs (movement + capacity) needed to compensate for the bang-bang nature of the statistical algorithm show that use of that algorithm is a net loss.

Even though any regulation costs are likely to be external to the balance sheet of an aggregator capitalizing on use of a statistical PEV charging algorithm, the external costs are likely to be significant enough to require an intervention to prohibit the algorithms use. Furthermore, the actual cost of regulation is likely to be more severe as scaling up historical costs (as was done in this study) underrepresents the cost of building the new regulation capacity required for substantial use of the STAT algorithm. Also, such fundamental shifts away from current market behavior call into question a fundamental assumption of this thesis; that historic price and load data can be used to understand the behavior of deferability-based algorithms.

In the best-case scenario, the fundamental premise of this thesis, that selectively charging PEVs in the RTM can practically result in cost savings, needs to be evaluated via alternate means that inherently take regulation costs into account. Best-case LP simulation would need to be performed while taking the regulation capacity cost structure into account, and any practical algorithm will need to balance regulation capacity costs against the potential gains which that added capacity can provide. The optimal algorithm, in that case, will focus on determining a “sweet spot” of additional regulation capacity. Such studies go beyond the scope of this thesis.

# 7. CONCLUSION AND FUTURE WORK

This thesis explores the cost savings potential of charging PEVs in a RTM price environment by selectively charging at select times when prices are low. Simulation studies of various developmental and baseline algorithms lead to the following conclusions.

Significant cost savings potential exists for selectively charging PEVs during low price periods while still meeting PEV charging goals. Simulation studies of nightly PEV charging in New York City throughout 2014 indicate that, for the case where a PEV must charge for 40% of a nightly charging period, selective charging methods can reduce charging costs by as much as 32% (when compared to the cost of charging at the nightly average price) and 23% (when compared to charging immediately after midnight). However, as the level of PEVs in a neighborhood increase and the aggregate charging rate of all PEVs exceeds network capacity limits, potential savings decrease. This is because the constraint forces the fraction of time when charging must occur to increase, with a corresponding decrease in opportunities for selectively charging.

A practical general purpose algorithm for extracting a significant portion (over 50%) of the potential cost savings was successfully developed. The algorithm facilitates charging of multiple independent PEVs with diverse charging requirements in a network with multiple power flow constraints. Also, the algorithm is designed to operate with only forecasts of future system parameters (such as prices, non-PEV loads, and vehicle availabilities) until those parameters values are discovered in real time.

The algorithm is sufficiently general so that it can be invoked similarly to a generic linear programming algorithm, except that 1) the objective function is described statistically, 2) values for prices and constraints are populated with forecast data until the actual values are discovered in real-time, and 3) algorithm output is a set of price setpoint values which continually update in real time (i.e., a PEV will charge for a timestep if RTM price is below the current setpoint). A correspondingly general solution method was developed which is centered around constructing and scaling statistical price-energy distributions to represent the potential cost of all available units of energy.  These distributions are iteratively scaled to accurately represent network constraints and facilitate calculation of setpoints which will adaptively work to satisfy the desired charge demand at minimal cost.

Simulation studies consistent with those used to quantify cost savings potential (above) show that the algorithm can reduce charging costs by 25% when compared to the cost of charging at the nightly average price (vice 32% savings in the ideal case) and 16% when compared to charging immediately after midnight (vice 23% savings in the ideal case).  As expected, these savings rate decrease as the number of PEVs charging in the network increase.

A fundamental assumption in motivating price-selectivity based energy purchasing was that using PEVs to quickly respond to RTM prices would benefit both consumers (through PEV charging cost reduction) and grid operation (by balancing out demand volatility with time).  Cost savings associated with selective PEV charging would be compensated by the grid benefits of better balanced demand.  However, this "win-win" assumption is invalid, and <u>potential cost savings realized by selective PEV charging come at the expense of added costs to other consumers</u> (i.e., a win-lose scenario).

In the ideal case, RTM prices would reflect current load perturbations, and responding to the RTM price signal would always act to steady temporal load variations. However, current RTM behavior does not match this ideal.  Prices reflect grid conditions at least five minutes earlier which, in terms of temporal load balancing, are independent from current conditions.  The price signal can actually drive a destabilizing response. Additionally, since RTM prices generally remain in effect for five minutes regardless of demand response, the price signal has no way of moderating the magnitude of a demand response to the magnitude of the imbalance.

Simulation studies were performed to determine the overall cost savings potential of the selective charging algorithm, and included both the energy cost savings considered previously, as well as the cost of additional regulation resources needed to counterbalance the load swings associated with the algorithm.  These simulation studies indicate that, for the most realistic regulation cost model, price selective PEV charging drives overall costs higher.

## 7.1 Future Work

As with the development of any complex system, opportunities for developing and understanding enhancements of increased sophistication abound. The following list includes such opportunities for the PEV charging method of this thesis:

1. Application to Non-PEV Systems – This thesis explored the general topic of using systems with energy storage capacity to selectively purchase electricity by exploring one specific target application; PEV charging. PEV charging was chosen because of its simplicity relative to other systems with energy storage. Future work could enhance the methods developed herein for use in more complex applications. This could include systems where stored energy can be lost with time, and where energy accumulation and consumption occur concurrently. Such characteristics are found in building heating and cooling systems. Stored heat can be lost with time, so any optimization algorithm would need to include modeling of system dynamics, such as heat loss.
2. Expanded Statistical Modeling – The PEV charging algorithm developed herein makes use of a statistical price model to deal with the fact that future price information is both uncertain and necessary for effective optimization. However, future values of several other parameters (e.g., vehicle availability, non-PEV system load, initial charge level) used in the optimization are also uncertain, but are not treated with a statistical model. Future work could expand the use of statistical modeling for parameters other than price.
3. Price Environment Loading – This thesis made use of historic price data for simulating the performance of a system which has not yet been fielded. If the PEV charging methodologies developed herein were to be fielded on a large scale, the price behavior imposed in these simulations would no longer be representative of actual price behavior, and the cost performance indicated in these studies would be invalidated. Future work could model the interrelationship between selective PEV charging and price development

to determine how cost savings would change with widespread use of the charging method.

4. Network Constraint Resolution – Although the general PEV charging algorithm developed herein could be applied to a distribution network with multiple constraints, only a single network constraint was included for simulations. Future work could explore the behavior of PEV charging in a neighborhood with multiple network constraints, and assess the performance of the proposed multiple constraint resolution methodology. Additionally, for the heterogeneous charging scenario, resolution of network constraints relied on a user-supplied constant, $lf_{des}$. That constant represents the desired maximum likelihood of constraint violation, and was set to a value of 0.20 for all simulations. Future work could explore the effect that various values of the constant have on cost and level performance.
5. Co-optimization of PEV Charging and Regulation – All algorithms simulated in this thesis made charging decisions without considering impact on the need for regulation. Future studies could explore algorithms which minimize overall charging costs by including a regulation cost model. Such algorithms would likely search for a "sweet spot" in committed regulation resources instead of rewarding maximum use of regulation. Alternatively, grid performance could become unacceptable and result in outages if inadequate grid resources (including, but not limited to regulation) are available for the types of load swings brought on by widespread use of the PEV charging algorithm. Future work could explore the potential for explicitly modeling grid stability limits, or other grid performance parameters, as an optimization constraint.

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# APPENDICES

The following appendices supplement the technical discussions provided in earlier chapters:

# APPENDIX A. SIMULATION RESULTS

This appendix presents PEV charging simulation results from the various charging scenarios explored earlier in this thesis, but with a consistent simulation framework which readily supports results comparison between scenarios. For the "Homogenous Charging – No Network Constraints" (Chapter 3) and "Homogeneous Charging – with Network Constraints" (Chapter 4) scenarios, these results are entirely new from the results presented within the respective chapters since a different charging period is assumed. For the "Heterogeneous Charging – with Network Constraints" (Chapter 5) and "Regulation Impact" (Chapter 6) scenarios, this appendix presents the detailed results in lieu of those chapters. In all cases, the individual chapters should be read to understand the general behavior exhibited.

Table A.1 summarizes the characteristics of the common set of simulations.

**Table A.1 – Framework for Consistent Simulation of Thesis Scenarios.**

| | |
|---|---|
| Network Characteristics | Prices: Real Time Market, New York City<br>Network Charging Rate Constraint: PEV Load ≤ 1.44 – 1.2*(Ld/$Ld_{max}$) [MW]<br>Ld is the actual load of New York City (from NYISO)<br>$Ld_{max}$ is yearly maximum load in New York City (11,500 MW) |
| Vehicle Characteristics | Number: 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150<br>Charger Maximum Rate: 9.6kW (0.80 kWhr per five-minute timestep)<br>Daily Energy Usage (per Vehicle):<br>4 hour periods – Not Simulated<br>8 hour periods – Not Simulated<br>15 hour periods – 42.50 of 85 kWhr (50%), equivalent to 125 miles/day usage |
| Charging Period Characteristics | Homogeneous Scenarios - 8pm to 7am (11 hours, or 132 steps of five minutes each)<br>Heterogeneous Scenario - 6pm to 9am (15 hours, or 180 steps of five minutes each) |
| Year Simulated | 2012 – Not Simulated<br>2013 – Not Simulated<br>2014 – all days except Jan 1, Mar 8, Nov 2, and Dec 31<br><br>Days within a year are simulated interdependently (initial charge is the maximum of either zero or 42.5kWhr less than the final charge of the previous night). |
| Regulation Schemes | Baseline<br>Strictly Movement<br>Augmented Movement<br>Movement + Capacity |

The need for a consistent simulation framework is driven by the Heterogeneous charging scenario of Chapter 5. That scenario is based on a 15 hour nightly simulation window, with an average vehicle charging period of 11 hours. As a result, the simulations of Chapter 5 cannot be readily compared with those of Chapters 3 and 4. To remedy this, the simulation scenarios of Chapters 3 and 4 were repeated with an 11 hour charging window, and with the comparative algorithms used in Chapter 5. Since the regulation cost studies of Chapter 6 are merely the result of post-processing already-performed simulations, all three charging scenarios (i.e., Homogeneous – No Constraints, Homogeneous – With Constraints, and Heterogeneous – With Constraints) are processed for all four regulation cost models (i.e., No Regulation, Strictly Movement, Augmented Movement, and Movement + Capacity). The numerical results for each of these 12 simulation sets are included in Table A.4 through Table A.15 (at the end of this Appendix). As a large number of numerical results cannot easily be absorbed by the reader, some graphical presentations are included and select numerical results are tabulated to highlight key trends (below).

Cost savings results are illustrated in Figure A.1 through Figure A.4. These plots show the yearly average price of electricity resulting from charging for the various scenarios and charging methods, as a function of number of vehicles charging. Each plot represents a separate regulation cost model, and each subplot represents a different charging scenario. The overall cost trends with respect to number of vehicles are explained in Chapters 5 and 6.

Throughout all scenarios, greater cost savings occur when there are fewer vehicles charging because, when network constraints are assumed, maximum deferability exists. Table A.2 and Table A.3 therefore present cost results for charging 40 vehicles only. The results are expressed in terms of savings potential of test cases when compared to base cases. For example, the first row of Table A.2 indicates that the statistical charging algorithm saves 24.7% when compared to the average charging algorithm for the homogenous scenario without regulation costs. LP results are included as a test case to allow comparison with both an “unintelligent” base case as well as the best possible base case. As an example, the first two rows of Table A.2 indicate that the statistical algorithm can save 24.7% over the maximum possible savings of 31.6% (LP). These results are provided for all four regulation scenarios. Table A.3 provides similar results to Table A.2, except the test case algorithms only update purchasing decisions every 10 minutes vice every 5 minutes (hence, they are identified as “skip” algorithms). The skip algorithms reflect a sensitivity study for regulation cost models in Chapter 6.

**Table A.2 – Cost Comparison for Select PEV Charging Simulations – Standard Charging Methods.**

| Cases Considered | | | | Regulation Cost Model | | | |
|---|---|---|---|---|---|---|---|
| Scenario | Number of Vehicles | Test Case | Base Case | No Regulation | Strictly Movement | Augmented Movement | Movement + Capacity |
| Homogeneous - No Network Constraints | 40 | STAT | AVG | 0.2473 | 0.2473 | 0.1383 | -0.1272 |
| Homogeneous - No Network Constraints | 40 | LP | AVG | 0.3158 | 0.3158 | 0.2289 | -0.0347 |
| Homogeneous - No Network Constraints | 40 | STAT | MIDNIGHT | 0.1640 | 0.1640 | 0.0469 | -0.2169 |
| Homogeneous - No Network Constraints | 40 | LP | MIDNIGHT | 0.2400 | 0.2400 | 0.1472 | -0.1170 |
| Homogenous - with Network Constraints | 40 | STAT | AVG | 0.2473 | 0.2473 | 0.1383 | -0.1272 |
| Homogenous - with Network Constraints | 40 | LP | AVG | 0.3158 | 0.3158 | 0.2290 | -0.0346 |
| Homogenous - with Network Constraints | 40 | STAT | MIDNIGHT | 0.1640 | 0.1640 | 0.0469 | -0.2169 |
| Homogenous - with Network Constraints | 40 | LP | MIDNIGHT | 0.2400 | 0.2400 | 0.1472 | -0.1169 |
| Heterogeneous - with Network Constraints | 40 | STAT | AVG | 0.2530 | 0.2530 | 0.1562 | -0.0860 |
| Heterogeneous - with Network Constraints | 40 | LP | AVG | 0.3193 | 0.3193 | 0.2373 | -0.0222 |
| Heterogeneous - with Network Constraints | 40 | STAT | MIDNIGHT | 0.1597 | 0.1597 | 0.0550 | -0.1381 |
| Heterogeneous - with Network Constraints | 40 | LP | MIDNIGHT | 0.2342 | 0.2342 | 0.1458 | -0.0712 |

**Table A.3 – Cost Comparison for Select PEV Charging Simulations – Skip Charging Methods.**

| Cases Considered | | | | Regulation Cost Model | | | |
|---|---|---|---|---|---|---|---|
| Scenario | Number of Vehicles | Test Case | Base Case | No Regulation | Strictly Movement | Augmented Movement | Movement + Capacity |
| Homogeneous - No Network Constraints | 40 | STAT (skip) | AVG | 0.2014 | 0.2014 | 0.1252 | -0.1365 |
| Homogeneous - No Network Constraints | 40 | LP (skip) | AVG | 0.2970 | 0.2970 | 0.2472 | 0.0405 |
| Homogeneous - No Network Constraints | 40 | STAT (skip) | MIDNIGHT | 0.1130 | 0.1130 | 0.0324 | -0.2269 |
| Homogeneous - No Network Constraints | 40 | LP (skip) | MIDNIGHT | 0.2192 | 0.2192 | 0.1674 | -0.0358 |
| Homogenous - with Network Constraints | 40 | STAT (skip) | AVG | 0.2014 | 0.2014 | 0.1252 | -0.1365 |
| Homogenous - with Network Constraints | 40 | LP (skip) | AVG | 0.2970 | 0.2970 | 0.2472 | 0.0405 |
| Homogenous - with Network Constraints | 40 | STAT (skip) | MIDNIGHT | 0.1130 | 0.1130 | 0.0324 | -0.2269 |
| Homogenous - with Network Constraints | 40 | LP (skip) | MIDNIGHT | 0.2192 | 0.2192 | 0.1674 | -0.0358 |
| Heterogeneous - with Network Constraints | 40 | STAT (skip) | AVG | 0.2086 | 0.2086 | 0.1390 | -0.1142 |
| Heterogeneous - with Network Constraints | 40 | LP (skip) | AVG | 0.3016 | 0.3016 | 0.2543 | 0.0337 |
| Heterogeneous - with Network Constraints | 40 | STAT (skip) | MIDNIGHT | 0.1097 | 0.1097 | 0.0357 | -0.1677 |
| Heterogeneous - with Network Constraints | 40 | LP (skip) | MIDNIGHT | 0.2144 | 0.2144 | 0.1648 | -0.0127 |

The numerical results of Table A.4 through Table A.15 also contain relevant data with respect to achieving daily charging level goals. This includes information on the number of calendar days for each year-long simulation where there were any vehicles which failed to charge up to either a 99% or 50% level. As it is also important to get an idea of the number of vehicles which failed to fully charge on an affected calendar day, equivalent metrics are included for the number of vehicle-days where final charge level is less than 99% or 50%. A steep rise in the number of affected days exists and is explained further in Chapter 5.

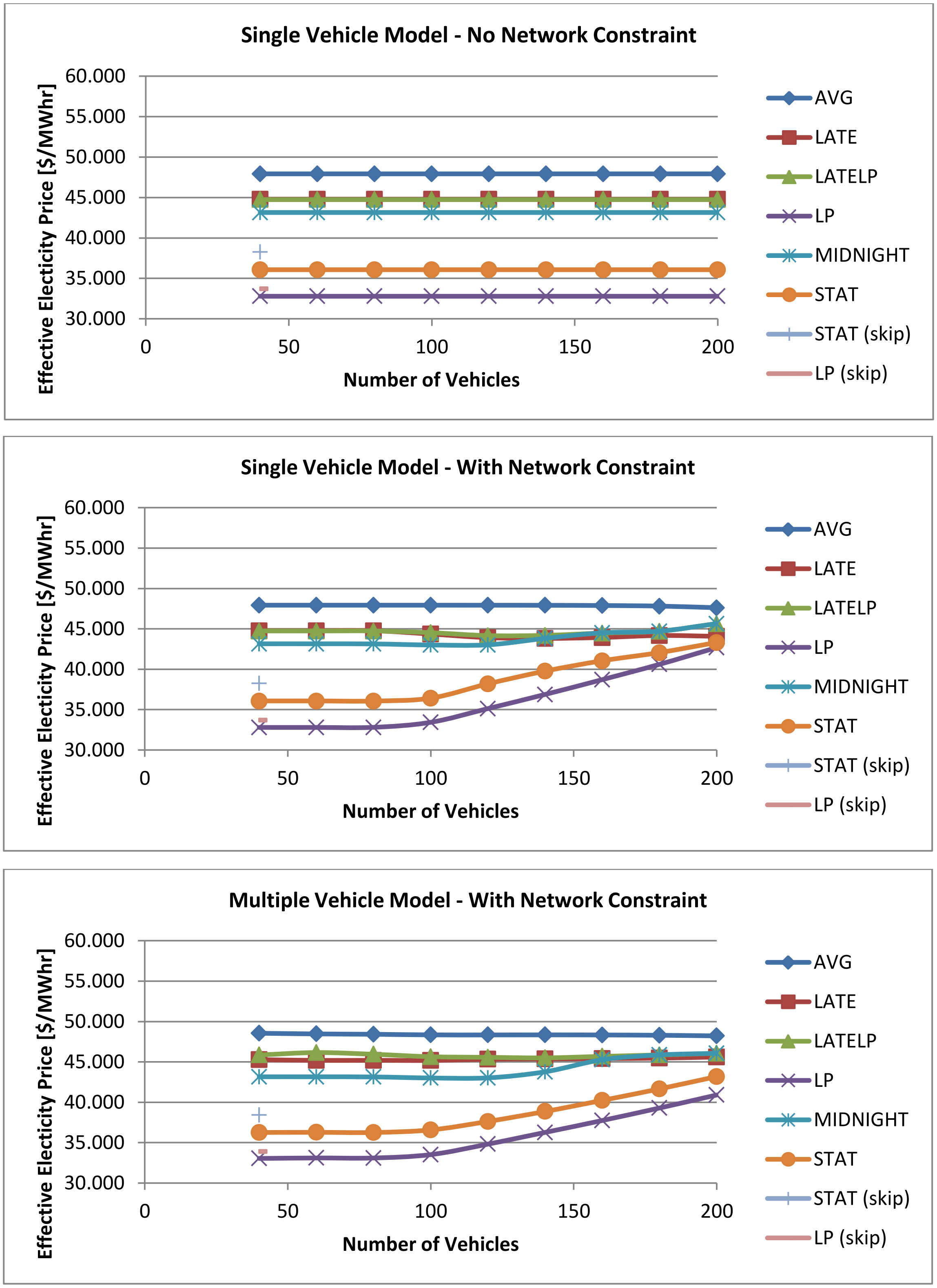



**Figure A.1 – Cost Performance of Various PEV Charging Algorithms: No Regulation Costs.**

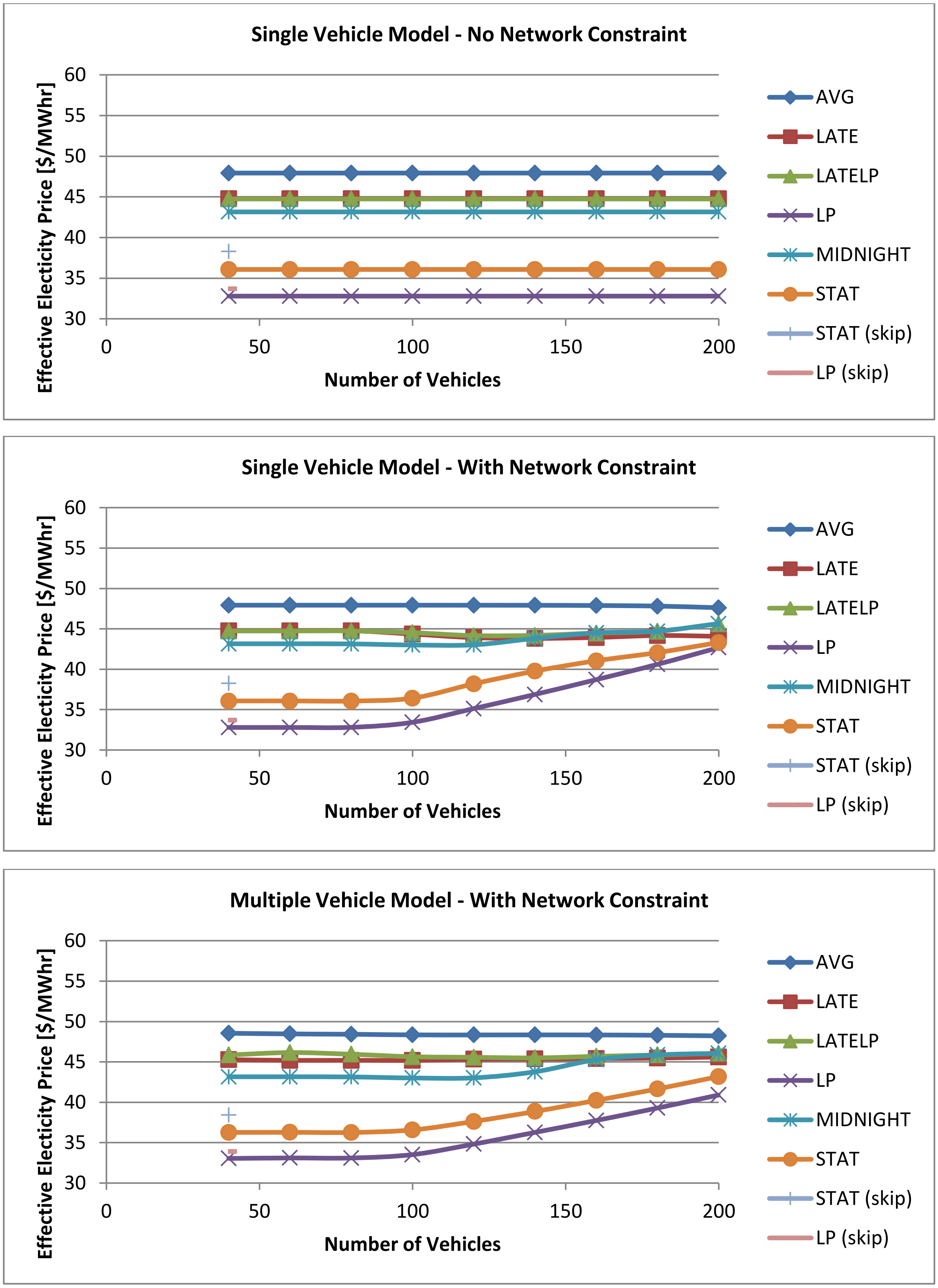



**Figure A.2 – Cost Performance of Various PEV Charging Algorithms: Strictly Movement Costs.**

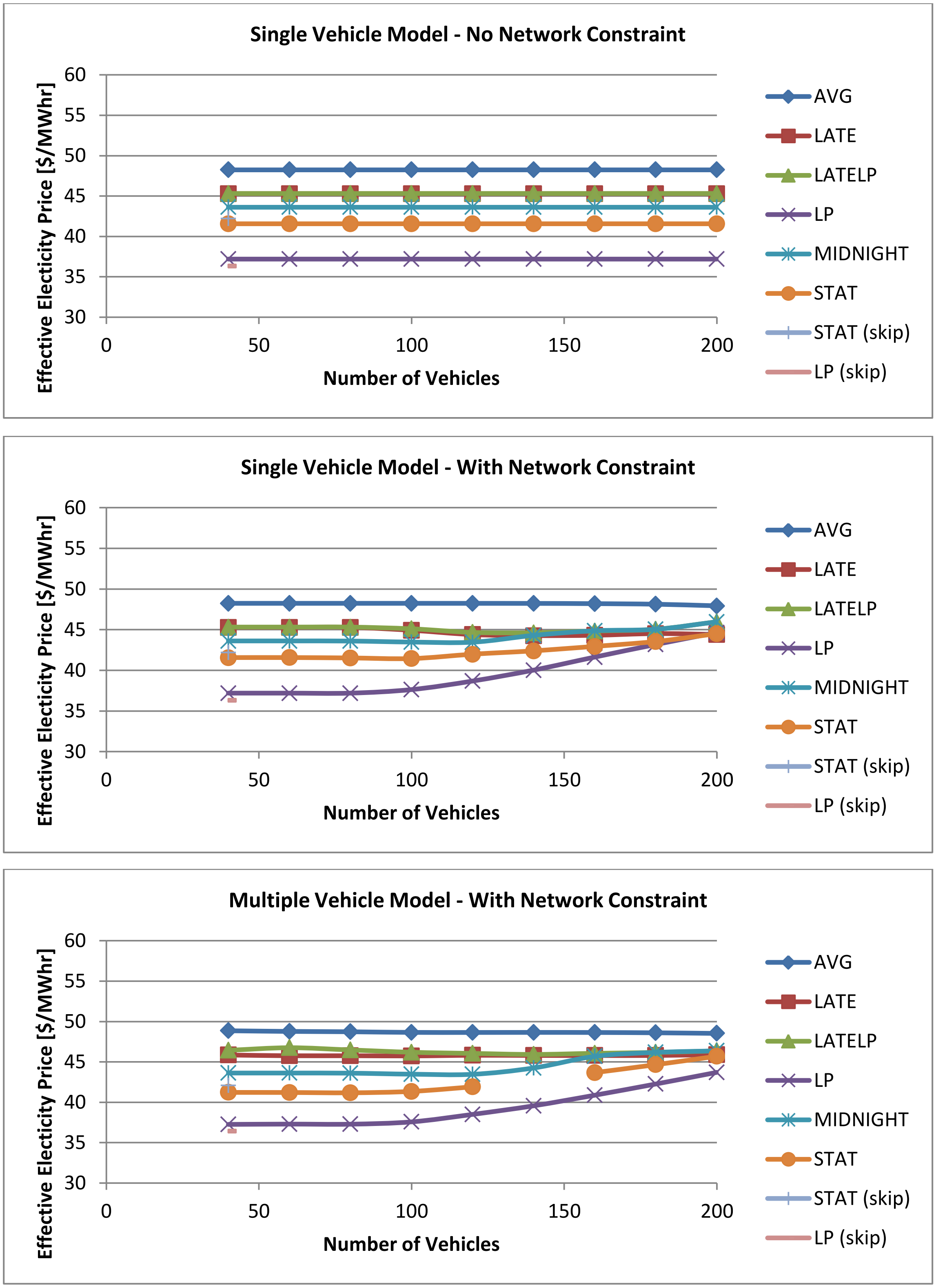
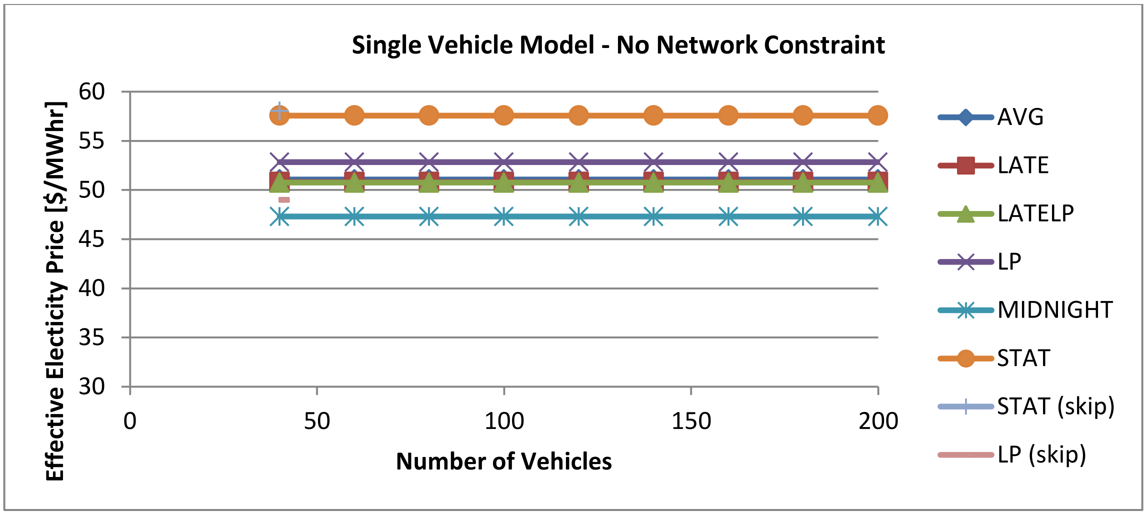
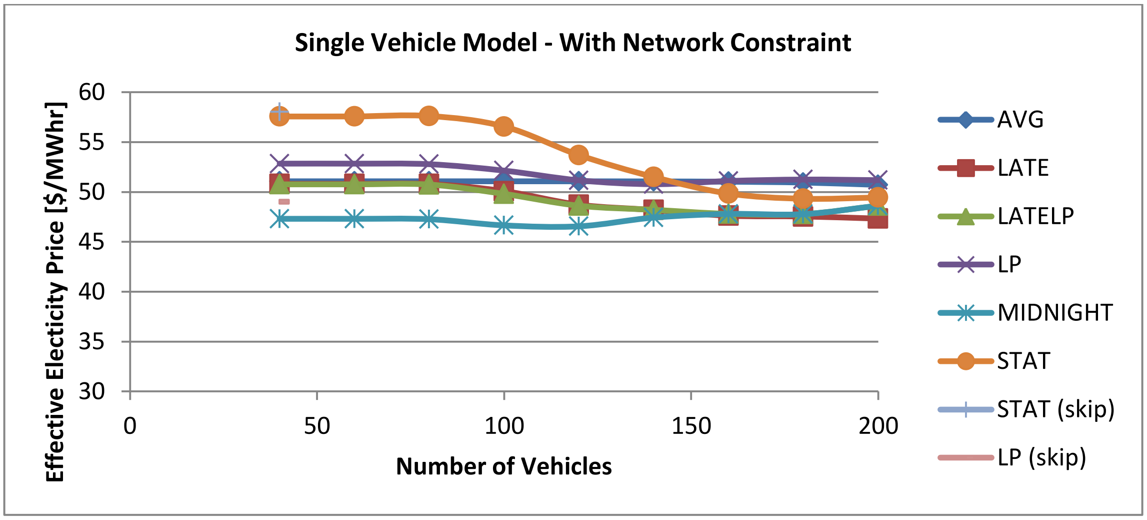



**Figure A.3 – Cost Performance of Various PEV Charging Algorithms: Augmented Movement Costs.**



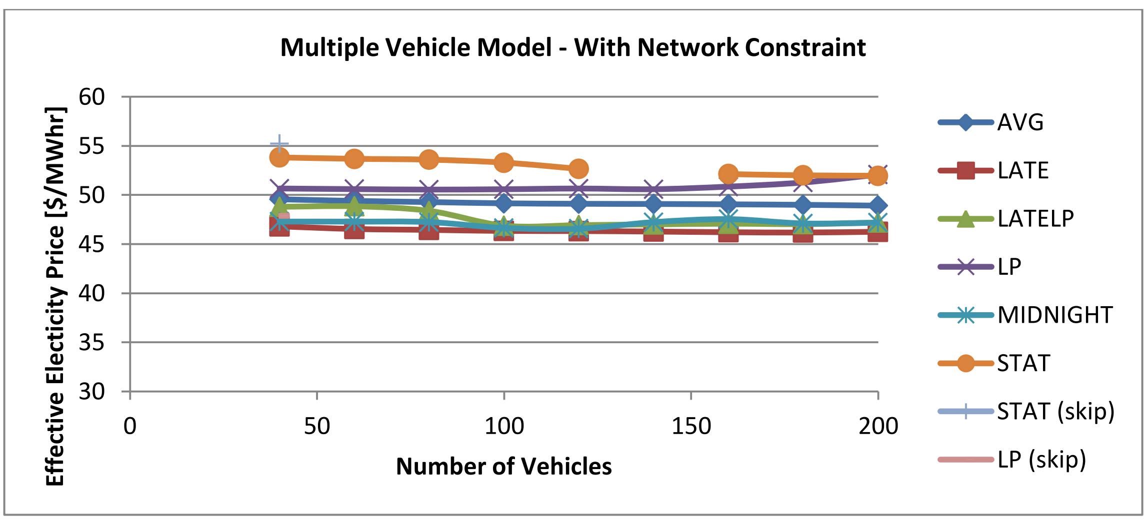




**Figure A.4 – Cost Performance of Various PEV Charging Algorithms: Movement + Capacity Costs.**

## Table A.4 – Simulation Results - Homogeneous without Network Constraints – No Regulation Costs.

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 47.93 | 29251.31 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 47.93 | 43876.97 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 47.93 | 58502.62 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 47.93 | 73128.28 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 47.93 | 87753.93 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 47.93 | 102379.59 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 160 | 47.93 | 117005.24 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 180 | 47.93 | 131630.90 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 200 | 47.93 | 146256.55 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 40 | 44.78 | 27327.20 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 44.78 | 40990.80 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 44.78 | 54654.40 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 100 | 44.78 | 68318.00 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 120 | 44.78 | 81981.60 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 140 | 44.78 | 95645.20 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 160 | 44.78 | 109308.80 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 180 | 44.78 | 122972.40 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 200 | 44.78 | 136636.00 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 40 | 44.74 | 27306.82 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 44.74 | 40960.23 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 44.74 | 54613.64 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 44.74 | 68267.05 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 44.74 | 81920.46 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 44.74 | 95573.87 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 44.74 | 109227.28 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 44.74 | 122880.69 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 200 | 44.74 | 136534.10 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LP (skip) | 40 | 33.69 | 20562.23 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 32.79 | 20014.52 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 32.79 | 30021.78 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 32.79 | 40029.04 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 32.79 | 50036.30 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 32.79 | 60043.56 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 32.79 | 70050.82 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 32.79 | 80058.08 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 32.79 | 90065.34 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 200 | 32.79 | 100072.60 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 40 | 43.15 | 26335.45 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.15 | 39503.18 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.15 | 52670.90 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.15 | 65838.63 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.15 | 79006.35 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 43.15 | 92174.08 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 43.15 | 105341.80 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 43.15 | 118509.53 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 200 | 43.15 | 131677.25 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| STAT (skip) | 40 | 38.27 | 23348.82 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 36.08 | 22014.58 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 36.08 | 33021.87 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 36.08 | 44029.16 | 1220.45 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 100 | 36.08 | 55036.45 | 1525.56 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 120 | 36.08 | 66043.74 | 1830.67 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 140 | 36.08 | 77051.03 | 2135.78 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 160 | 36.08 | 88058.32 | 2440.90 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 180 | 36.08 | 99065.61 | 2746.01 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 200 | 36.08 | 110072.90 | 3051.12 | 0.995 | 0 | 0 | 0 | 0 |

## Table A.5 – Simulation Results - Homogeneous with Network Constraints - No Regulation Costs.

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 47.93 | 29251.30 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 47.93 | 43877.01 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 47.93 | 58502.85 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 47.93 | 73128.48 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 47.93 | 87749.37 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 47.92 | 102356.76 | 2136.05 | 1.000 | 1 | 1 | 0 | 0 |
| AVG | 160 | 47.89 | 116910.57 | 2441.20 | 0.998 | 12 | 12 | 0 | 0 |
| AVG | 180 | 47.82 | 131309.63 | 2745.97 | 0.993 | 34 | 34 | 0 | 0 |
| AVG | 200 | 47.62 | 145191.17 | 3049.03 | 0.964 | 84 | 84 | 0 | 0 |
| LATE | 40 | 44.78 | 27327.20 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 44.78 | 40990.84 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 44.77 | 54649.09 | 1220.60 | 1.000 | 2 | 2 | 0 | 0 |
| LATE | 100 | 44.36 | 67585.13 | 1523.55 | 0.966 | 140 | 140 | 0 | 0 |
| LATE | 120 | 43.91 | 79772.16 | 1816.67 | 0.818 | 359 | 359 | 0 | 0 |
| LATE | 140 | 43.82 | 92215.29 | 2104.52 | 0.698 | 359 | 359 | 0 | 0 |
| LATE | 160 | 43.94 | 104847.90 | 2386.38 | 0.606 | 359 | 359 | 3 | 3 |
| LATE | 180 | 44.18 | 116548.31 | 2638.11 | 0.534 | 359 | 359 | 56 | 56 |
| LATE | 200 | 44.08 | 123843.82 | 2809.63 | 0.478 | 359 | 359 | 210 | 210 |
| LATELP | 40 | 44.74 | 27306.82 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 44.74 | 40960.29 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 44.74 | 54610.93 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 44.53 | 67941.93 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 44.16 | 80858.27 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 44.20 | 94404.19 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 44.48 | 108587.39 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 44.71 | 122789.88 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LATELP | 200 | 45.65 | 139221.25 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| LP (skip) | 40 | 33.69 | 20562.23 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 32.79 | 20014.52 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 32.79 | 30021.95 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 32.81 | 40046.57 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 33.43 | 51008.78 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 35.13 | 64324.76 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 36.88 | 78778.37 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 38.72 | 94517.15 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 40.61 | 111527.21 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LP | 200 | 42.68 | 130177.08 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| MIDNIGHT | 40 | 43.15 | 26335.45 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.15 | 39503.13 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.14 | 52651.82 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.00 | 65613.71 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.02 | 78772.69 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 43.83 | 93613.94 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 44.51 | 108667.52 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 44.71 | 122789.88 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| MIDNIGHT | 200 | 45.65 | 139221.25 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| STAT (skip) | 40 | 38.27 | 23348.82 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 36.08 | 22014.58 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 36.08 | 33022.24 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 36.07 | 44013.34 | 1220.37 | 0.994 | 6 | 6 | 0 | 0 |
| STAT | 100 | 36.42 | 55522.84 | 1524.48 | 0.979 | 89 | 89 | 0 | 0 |
| STAT | 120 | 38.20 | 69727.18 | 1825.41 | 0.920 | 265 | 265 | 0 | 0 |
| STAT | 140 | 39.77 | 84350.50 | 2121.01 | 0.834 | 336 | 336 | 0 | 0 |
| STAT | 160 | 41.05 | 98903.21 | 2409.18 | 0.732 | 357 | 357 | 0 | 0 |
| STAT | 180 | 42.06 | 112836.81 | 2682.56 | 0.633 | 359 | 359 | 23 | 23 |
| STAT | 200 | 43.31 | 126061.71 | 2910.36 | 0.549 | 359 | 359 | 72 | 72 |

**Table A.6 – Simulation Results - Heterogeneous with Network Constraints - No Regulation Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 48.55 | 29619.18 | 610.06 | 0.988 | 359 | 3658 | 0 | 0 |
| AVG | 60 | 48.47 | 44351.86 | 915.08 | 0.988 | 359 | 5428 | 0 | 0 |
| AVG | 80 | 48.42 | 59076.25 | 1220.02 | 0.987 | 359 | 7230 | 12 | 12 |
| AVG | 100 | 48.35 | 73730.30 | 1525.04 | 0.987 | 359 | 8995 | 12 | 12 |
| AVG | 120 | 48.34 | 88461.85 | 1830.04 | 0.987 | 359 | 10808 | 12 | 12 |
| AVG | 140 | 48.35 | 103225.80 | 2135.05 | 0.987 | 359 | 12689 | 12 | 12 |
| AVG | 160 | 48.34 | 117945.50 | 2440.10 | 0.987 | 359 | 14648 | 12 | 12 |
| AVG | 180 | 48.30 | 132581.97 | 2745.08 | 0.986 | 359 | 17607 | 12 | 12 |
| AVG | 200 | 48.22 | 147058.93 | 3049.57 | 0.979 | 359 | 23844 | 17 | 17 |
| LATE | 40 | 45.28 | 27613.42 | 609.84 | 0.977 | 359 | 4860 | 0 | 0 |
| LATE | 60 | 45.19 | 41338.49 | 914.76 | 0.977 | 359 | 7245 | 0 | 0 |
| LATE | 80 | 45.20 | 55119.55 | 1219.57 | 0.977 | 359 | 9679 | 12 | 12 |
| LATE | 100 | 45.18 | 68875.48 | 1524.31 | 0.974 | 359 | 12724 | 13 | 13 |
| LATE | 120 | 45.33 | 82807.85 | 1826.67 | 0.941 | 359 | 26457 | 24 | 24 |
| LATE | 140 | 45.38 | 96391.10 | 2124.17 | 0.865 | 359 | 46735 | 27 | 27 |
| LATE | 160 | 45.42 | 109738.05 | 2416.31 | 0.774 | 359 | 57323 | 36 | 39 |
| LATE | 180 | 45.47 | 122859.49 | 2701.96 | 0.690 | 359 | 64615 | 125 | 209 |
| LATE | 200 | 45.61 | 135685.82 | 2974.92 | 0.615 | 359 | 71798 | 269 | 2305 |
| LATELP | 40 | 45.86 | 27986.37 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 46.16 | 42256.10 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 45.95 | 56078.59 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 100 | 45.64 | 69635.95 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 120 | 45.57 | 83429.99 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 140 | 45.50 | 97186.12 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 160 | 45.69 | 111530.72 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LATELP | 180 | 45.86 | 125950.25 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LATELP | 200 | 46.07 | 140573.08 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| LP (skip) | 40 | 33.91 | 20692.73 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 33.05 | 20170.73 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 33.11 | 30309.20 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 33.10 | 40404.69 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 100 | 33.52 | 51145.69 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 120 | 34.82 | 63754.34 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 140 | 36.27 | 77478.33 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 160 | 37.75 | 92150.24 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LP | 180 | 39.29 | 107908.82 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LP | 200 | 40.91 | 124820.06 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 40 | 43.16 | 26340.05 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.16 | 39510.01 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.14 | 52655.26 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 100 | 43.01 | 65622.45 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 120 | 43.03 | 78777.33 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 140 | 43.78 | 93505.74 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 160 | 45.27 | 110520.54 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 180 | 45.87 | 125963.39 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 200 | 46.07 | 140573.08 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| STAT (skip) | 40 | 38.43 | 23434.34 | 609.87 | 0.981 | 358 | 4992 | 0 | 0 |
| STAT | 40 | 36.27 | 22118.31 | 609.87 | 0.981 | 357 | 4390 | 0 | 0 |
| STAT | 60 | 36.28 | 33192.78 | 914.81 | 0.980 | 357 | 6581 | 0 | 0 |
| STAT | 80 | 36.26 | 44229.46 | 1219.63 | 0.980 | 358 | 8747 | 12 | 12 |
| STAT | 100 | 36.58 | 55767.30 | 1524.56 | 0.979 | 358 | 10431 | 15 | 15 |
| STAT | 120 | 37.63 | 68835.20 | 1829.31 | 0.978 | 359 | 12735 | 26 | 26 |
| STAT | 140 | 38.86 | 82933.04 | 2134.01 | 0.977 | 359 | 15622 | 26 | 26 |
| STAT | 160 | 40.24 | 98106.66 | 2438.15 | 0.974 | 359 | 19566 | 37 | 49 |
| STAT | 180 | 41.66 | 114177.15 | 2740.73 | 0.965 | 359 | 24743 | 150 | 237 |
| STAT | 200 | 43.19 | 131247.02 | 3038.93 | 0.945 | 359 | 31561 | 255 | 768 |

**Table A.7 – Simulation Results - Homogeneous without Network Constraints - Strictly Movement Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 47.93 | 29251.31 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 47.93 | 43876.97 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 47.93 | 58502.62 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 47.93 | 73128.28 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 47.93 | 87753.93 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 47.93 | 102379.59 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 160 | 47.93 | 117005.24 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 180 | 47.93 | 131630.90 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 200 | 47.93 | 146256.55 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 40 | 44.78 | 27327.20 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 44.78 | 40990.80 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 44.78 | 54654.40 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 100 | 44.78 | 68318.00 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 120 | 44.78 | 81981.60 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 140 | 44.78 | 95645.20 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 160 | 44.78 | 109308.80 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 180 | 44.78 | 122972.40 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 200 | 44.78 | 136636.00 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 40 | 44.74 | 27306.82 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 44.74 | 40960.23 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 44.74 | 54613.64 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 44.74 | 68267.05 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 44.74 | 81920.46 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 44.74 | 95573.87 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 44.74 | 109227.28 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 44.74 | 122880.69 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 200 | 44.74 | 136534.10 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LP (skip) | 40 | 33.69 | 20562.23 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 32.79 | 20014.52 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 32.79 | 30021.78 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 32.79 | 40029.04 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 32.79 | 50036.30 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 32.79 | 60043.56 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 32.79 | 70050.82 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 32.79 | 80058.08 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 32.79 | 90065.34 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 200 | 32.79 | 100072.60 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 40 | 43.15 | 26335.45 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.15 | 39503.18 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.15 | 52670.90 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.15 | 65838.63 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.15 | 79006.35 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 43.15 | 92174.08 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 43.15 | 105341.80 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 43.15 | 118509.53 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 200 | 43.15 | 131677.25 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| STAT (skip) | 40 | 38.27 | 23348.82 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 36.08 | 22014.58 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 36.08 | 33021.87 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 36.08 | 44029.16 | 1220.45 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 100 | 36.08 | 55036.45 | 1525.56 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 120 | 36.08 | 66043.74 | 1830.67 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 140 | 36.08 | 77051.03 | 2135.78 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 160 | 36.08 | 88058.32 | 2440.90 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 180 | 36.08 | 99065.61 | 2746.01 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 200 | 36.08 | 110072.90 | 3051.12 | 0.995 | 0 | 0 | 0 | 0 |

**Table A.8 – Simulation Results - Homogeneous with Network Constraints - Strictly Movement Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 47.93 | 29251.30 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 47.93 | 43877.01 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 47.93 | 58502.85 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 47.93 | 73128.48 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 47.93 | 87749.37 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 47.92 | 102356.76 | 2136.05 | 1.000 | 1 | 1 | 0 | 0 |
| AVG | 160 | 47.89 | 116910.57 | 2441.20 | 0.998 | 12 | 12 | 0 | 0 |
| AVG | 180 | 47.82 | 131309.63 | 2745.97 | 0.993 | 34 | 34 | 0 | 0 |
| AVG | 200 | 47.62 | 145191.17 | 3049.03 | 0.964 | 84 | 84 | 0 | 0 |
| LATE | 40 | 44.78 | 27327.20 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 44.78 | 40990.84 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 44.77 | 54649.09 | 1220.60 | 1.000 | 2 | 2 | 0 | 0 |
| LATE | 100 | 44.36 | 67585.13 | 1523.55 | 0.966 | 140 | 140 | 0 | 0 |
| LATE | 120 | 43.91 | 79772.16 | 1816.67 | 0.818 | 359 | 359 | 0 | 0 |
| LATE | 140 | 43.82 | 92215.29 | 2104.52 | 0.698 | 359 | 359 | 0 | 0 |
| LATE | 160 | 43.94 | 104847.90 | 2386.38 | 0.606 | 359 | 359 | 3 | 3 |
| LATE | 180 | 44.18 | 116548.31 | 2638.11 | 0.534 | 359 | 359 | 56 | 56 |
| LATE | 200 | 44.08 | 123843.82 | 2809.63 | 0.478 | 359 | 359 | 210 | 210 |
| LATELP | 40 | 44.74 | 27306.82 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 44.74 | 40960.29 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 44.74 | 54610.93 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 44.53 | 67941.93 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 44.16 | 80858.27 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 44.20 | 94404.19 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 44.48 | 108587.39 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 44.71 | 122789.88 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LATELP | 200 | 45.65 | 139221.25 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| LP (skip) | 40 | 33.69 | 20562.23 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 32.79 | 20014.52 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 32.79 | 30021.95 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 32.81 | 40046.57 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 33.43 | 51008.78 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 35.13 | 64324.76 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 36.88 | 78778.37 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 38.72 | 94517.15 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 40.61 | 111527.21 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LP | 200 | 42.68 | 130177.08 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| MIDNIGHT | 40 | 43.15 | 26335.45 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.15 | 39503.13 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.14 | 52651.82 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.00 | 65613.71 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.02 | 78772.69 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 43.83 | 93613.94 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 44.51 | 108667.52 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 44.71 | 122789.88 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| MIDNIGHT | 200 | 45.65 | 139221.25 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| STAT (skip) | 40 | 38.27 | 23348.82 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 36.08 | 22014.58 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 36.08 | 33022.24 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 36.07 | 44013.34 | 1220.37 | 0.994 | 6 | 6 | 0 | 0 |
| STAT | 100 | 36.42 | 55522.84 | 1524.48 | 0.979 | 89 | 89 | 0 | 0 |
| STAT | 120 | 38.20 | 69727.18 | 1825.41 | 0.920 | 265 | 265 | 0 | 0 |
| STAT | 140 | 39.77 | 84350.50 | 2121.01 | 0.834 | 336 | 336 | 0 | 0 |
| STAT | 160 | 41.05 | 98903.21 | 2409.18 | 0.732 | 357 | 357 | 0 | 0 |
| STAT | 180 | 42.06 | 112836.81 | 2682.56 | 0.633 | 359 | 359 | 23 | 23 |
| STAT | 200 | 43.31 | 126061.71 | 2910.36 | 0.549 | 359 | 359 | 72 | 72 |

**Table A.9 – Simulation Results - Heterogeneous with Network Constraints - Strictly Movement Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 48.55 | 29619.18 | 610.06 | 0.988 | 359 | 3658 | 0 | 0 |
| AVG | 60 | 48.47 | 44351.86 | 915.08 | 0.988 | 359 | 5428 | 0 | 0 |
| AVG | 80 | 48.42 | 59076.25 | 1220.02 | 0.987 | 359 | 7230 | 12 | 12 |
| AVG | 100 | 48.35 | 73730.30 | 1525.04 | 0.987 | 359 | 8995 | 12 | 12 |
| AVG | 120 | 48.34 | 88461.85 | 1830.04 | 0.987 | 359 | 10808 | 12 | 12 |
| AVG | 140 | 48.35 | 103225.80 | 2135.05 | 0.987 | 359 | 12689 | 12 | 12 |
| AVG | 160 | 48.34 | 117945.50 | 2440.10 | 0.987 | 359 | 14648 | 12 | 12 |
| AVG | 180 | 48.30 | 132581.97 | 2745.08 | 0.986 | 359 | 17607 | 12 | 12 |
| AVG | 200 | 48.22 | 147058.93 | 3049.57 | 0.979 | 359 | 23844 | 17 | 17 |
| LATE | 40 | 45.28 | 27613.42 | 609.84 | 0.977 | 359 | 4860 | 0 | 0 |
| LATE | 60 | 45.19 | 41338.49 | 914.76 | 0.977 | 359 | 7245 | 0 | 0 |
| LATE | 80 | 45.20 | 55119.55 | 1219.57 | 0.977 | 359 | 9679 | 12 | 12 |
| LATE | 100 | 45.18 | 68875.48 | 1524.31 | 0.974 | 359 | 12724 | 13 | 13 |
| LATE | 120 | 45.33 | 82807.85 | 1826.67 | 0.941 | 359 | 26457 | 24 | 24 |
| LATE | 140 | 45.38 | 96391.10 | 2124.17 | 0.865 | 359 | 46735 | 27 | 27 |
| LATE | 160 | 45.42 | 109738.05 | 2416.31 | 0.774 | 359 | 57323 | 36 | 39 |
| LATE | 180 | 45.47 | 122859.49 | 2701.96 | 0.690 | 359 | 64615 | 125 | 209 |
| LATE | 200 | 45.61 | 135685.82 | 2974.92 | 0.615 | 359 | 71798 | 269 | 2305 |
| LATELP | 40 | 45.86 | 27986.37 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 46.16 | 42256.10 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 45.95 | 56078.59 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 100 | 45.64 | 69635.95 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 120 | 45.57 | 83429.99 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 140 | 45.50 | 97186.12 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 160 | 45.69 | 111530.72 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LATELP | 180 | 45.86 | 125950.25 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LATELP | 200 | 46.07 | 140573.08 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| LP (skip) | 40 | 33.91 | 20692.73 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 33.05 | 20170.73 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 33.11 | 30309.20 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 33.10 | 40404.69 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 100 | 33.52 | 51145.69 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 120 | 34.82 | 63754.34 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 140 | 36.27 | 77478.33 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 160 | 37.75 | 92150.24 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LP | 180 | 39.29 | 107908.82 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LP | 200 | 40.91 | 124820.06 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 40 | 43.16 | 26340.05 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.16 | 39510.01 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.14 | 52655.26 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 100 | 43.01 | 65622.45 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 120 | 43.03 | 78777.33 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 140 | 43.78 | 93505.74 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 160 | 45.27 | 110520.54 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 180 | 45.87 | 125963.39 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 200 | 46.07 | 140573.08 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| STAT (skip) | 40 | 38.43 | 23434.34 | 609.87 | 0.981 | 358 | 4992 | 0 | 0 |
| STAT | 40 | 36.27 | 22118.31 | 609.87 | 0.981 | 357 | 4390 | 0 | 0 |
| STAT | 60 | 36.28 | 33192.78 | 914.81 | 0.980 | 357 | 6581 | 0 | 0 |
| STAT | 80 | 36.26 | 44229.46 | 1219.63 | 0.980 | 358 | 8747 | 12 | 12 |
| STAT | 100 | 36.58 | 55767.30 | 1524.56 | 0.979 | 358 | 10431 | 15 | 15 |
| STAT | 120 | 37.63 | 68835.20 | 1829.31 | 0.978 | 359 | 12735 | 26 | 26 |
| STAT | 140 | 38.86 | 82933.04 | 2134.01 | 0.977 | 359 | 15622 | 26 | 26 |
| STAT | 160 | 40.24 | 98106.66 | 2438.15 | 0.974 | 359 | 19566 | 37 | 49 |
| STAT | 180 | 41.66 | 114177.15 | 2740.73 | 0.965 | 359 | 24743 | 150 | 237 |
| STAT | 200 | 43.19 | 131247.02 | 3038.93 | 0.945 | 359 | 31561 | 255 | 768 |

**Table A.10 – Simulation Results - Homogeneous without Network Constraints - Augmented Movement Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 48.24 | 29441.80 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 48.24 | 44162.70 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 48.24 | 58883.60 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 48.24 | 73604.50 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 48.24 | 88325.40 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 48.24 | 103046.30 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 160 | 48.24 | 117767.20 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 180 | 48.24 | 132488.10 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 200 | 48.24 | 147209.00 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 40 | 45.30 | 27644.71 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 45.30 | 41467.07 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 45.30 | 55289.42 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 100 | 45.30 | 69111.78 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 120 | 45.30 | 82934.13 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 140 | 45.30 | 96756.49 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 160 | 45.30 | 110578.84 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 180 | 45.30 | 124401.20 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 200 | 45.30 | 138223.55 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 40 | 45.32 | 27658.75 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 45.32 | 41488.13 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 45.32 | 55317.50 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 45.32 | 69146.88 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 45.32 | 82976.25 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 45.32 | 96805.63 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 45.32 | 110635.00 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 45.32 | 124464.38 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 200 | 45.32 | 138293.75 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LP (skip) | 40 | 36.32 | 22163.17 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 37.20 | 22701.10 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 37.20 | 34051.65 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 37.20 | 45402.20 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 37.20 | 56752.75 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 37.20 | 68103.30 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 37.20 | 79453.85 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 37.20 | 90804.40 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 37.20 | 102154.95 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 200 | 37.20 | 113505.50 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 40 | 43.62 | 26619.25 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.62 | 39928.88 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.62 | 53238.50 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.62 | 66548.13 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.62 | 79857.75 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 43.62 | 93167.38 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 43.62 | 106477.00 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 43.62 | 119786.63 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 200 | 43.62 | 133096.25 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| STAT (skip) | 40 | 42.20 | 25745.13 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 41.57 | 25367.21 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 41.57 | 38050.82 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 41.57 | 50734.42 | 1220.45 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 100 | 41.57 | 63418.03 | 1525.56 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 120 | 41.57 | 76101.63 | 1830.67 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 140 | 41.57 | 88785.24 | 2135.78 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 160 | 41.57 | 101468.84 | 2440.90 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 180 | 41.57 | 114152.45 | 2746.01 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 200 | 41.57 | 126836.05 | 3051.12 | 0.995 | 0 | 0 | 0 | 0 |

**Table A.11 – Simulation Results - Homogeneous with Network Constraints - Augmented Movement Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 48.24 | 29441.80 | 610.30 | 1.000 | 30 | 30 | 29 | 29 |
| AVG | 60 | 48.24 | 44161.79 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 48.24 | 58883.88 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 48.24 | 73603.68 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 48.24 | 88318.65 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 48.23 | 103022.65 | 2136.05 | 1.000 | 1 | 1 | 0 | 0 |
| AVG | 160 | 48.20 | 117670.84 | 2441.20 | 0.998 | 12 | 12 | 0 | 0 |
| AVG | 180 | 48.13 | 132167.24 | 2745.97 | 0.993 | 34 | 34 | 0 | 0 |
| AVG | 200 | 47.93 | 146149.06 | 3049.03 | 0.964 | 84 | 84 | 0 | 0 |
| LATE | 40 | 45.30 | 27644.71 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 45.30 | 41466.89 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 45.29 | 55283.72 | 1220.60 | 1.000 | 2 | 2 | 0 | 0 |
| LATE | 100 | 44.93 | 68447.88 | 1523.55 | 0.966 | 140 | 140 | 0 | 0 |
| LATE | 120 | 44.40 | 80660.13 | 1816.67 | 0.818 | 359 | 359 | 0 | 0 |
| LATE | 140 | 44.27 | 93163.24 | 2104.52 | 0.698 | 359 | 359 | 0 | 0 |
| LATE | 160 | 44.32 | 105754.17 | 2386.38 | 0.606 | 359 | 359 | 3 | 3 |
| LATE | 180 | 44.52 | 117450.31 | 2638.11 | 0.534 | 359 | 359 | 56 | 56 |
| LATE | 200 | 44.41 | 124788.13 | 2809.63 | 0.478 | 359 | 359 | 210 | 210 |
| LATELP | 40 | 45.32 | 27658.75 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 45.32 | 41488.28 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 45.32 | 55314.79 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 45.11 | 68822.88 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 44.66 | 81771.26 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 44.62 | 95317.83 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 44.84 | 109469.05 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 45.05 | 123721.76 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LATELP | 200 | 45.97 | 140207.86 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| LP (skip) | 40 | 36.32 | 22163.31 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 37.20 | 22701.08 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 37.20 | 34051.91 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 37.20 | 45403.02 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 37.63 | 57419.46 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 38.68 | 70826.09 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 40.03 | 85503.82 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 41.63 | 101627.62 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 43.18 | 118586.35 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LP | 200 | 44.65 | 136164.27 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| MIDNIGHT | 40 | 43.62 | 26619.25 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.62 | 39928.96 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.60 | 53219.16 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 43.48 | 66334.66 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 43.46 | 79576.42 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 44.30 | 94633.90 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 44.87 | 109535.22 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 45.05 | 123721.76 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| MIDNIGHT | 200 | 45.97 | 140207.86 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| STAT (skip) | 40 | 42.20 | 25745.13 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 41.57 | 25367.21 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 41.57 | 38053.59 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 41.52 | 50674.27 | 1220.37 | 0.994 | 6 | 6 | 0 | 0 |
| STAT | 100 | 41.45 | 63187.36 | 1524.48 | 0.979 | 89 | 89 | 0 | 0 |
| STAT | 120 | 41.98 | 76638.21 | 1825.41 | 0.920 | 265 | 265 | 0 | 0 |
| STAT | 140 | 42.40 | 89932.49 | 2121.01 | 0.834 | 336 | 336 | 0 | 0 |
| STAT | 160 | 42.95 | 103468.39 | 2409.18 | 0.732 | 357 | 357 | 0 | 0 |
| STAT | 180 | 43.55 | 116814.51 | 2682.56 | 0.633 | 359 | 359 | 23 | 23 |
| STAT | 200 | 44.51 | 129542.47 | 2910.36 | 0.549 | 359 | 359 | 72 | 72 |

## Table A.12 – Simulation Results - Heterogeneous with Network Constraints - Augmented Movement Costs.

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 48.86 | 29803.73 | 610.01 | 0.988 | 359 | 3895 | 0 | 0 |
| AVG | 60 | 48.78 | 44631.24 | 915.00 | 0.988 | 359 | 5769 | 0 | 0 |
| AVG | 80 | 48.73 | 59448.87 | 1219.91 | 0.987 | 359 | 7709 | 12 | 12 |
| AVG | 100 | 48.65 | 74194.15 | 1524.93 | 0.987 | 359 | 9518 | 12 | 12 |
| AVG | 120 | 48.65 | 89018.04 | 1829.92 | 0.987 | 359 | 11431 | 12 | 12 |
| AVG | 140 | 48.66 | 103875.45 | 2134.92 | 0.987 | 359 | 13391 | 12 | 12 |
| AVG | 160 | 48.64 | 118687.58 | 2439.93 | 0.987 | 359 | 15419 | 12 | 12 |
| AVG | 180 | 48.61 | 133419.68 | 2744.91 | 0.986 | 359 | 18406 | 12 | 12 |
| AVG | 200 | 48.53 | 147993.27 | 3049.41 | 0.979 | 359 | 24592 | 17 | 17 |
| LATE | 40 | 45.86 | 27964.17 | 609.83 | 0.977 | 359 | 4860 | 0 | 0 |
| LATE | 60 | 45.76 | 41863.41 | 914.75 | 0.977 | 359 | 7245 | 0 | 0 |
| LATE | 80 | 45.77 | 55815.14 | 1219.56 | 0.977 | 359 | 9679 | 12 | 12 |
| LATE | 100 | 45.74 | 69723.95 | 1524.30 | 0.974 | 359 | 12725 | 13 | 13 |
| LATE | 120 | 45.82 | 83701.13 | 1826.67 | 0.941 | 359 | 26461 | 24 | 24 |
| LATE | 140 | 45.80 | 97292.11 | 2124.19 | 0.865 | 359 | 46732 | 27 | 27 |
| LATE | 160 | 45.79 | 110645.72 | 2416.30 | 0.774 | 359 | 57325 | 36 | 39 |
| LATE | 180 | 45.80 | 123751.52 | 2701.96 | 0.690 | 359 | 64615 | 125 | 210 |
| LATE | 200 | 45.91 | 136583.29 | 2974.92 | 0.615 | 359 | 71798 | 269 | 2308 |
| LATELP | 40 | 46.44 | 28342.86 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 46.76 | 42803.66 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 46.48 | 56730.59 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 100 | 46.19 | 70475.50 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 120 | 46.05 | 84315.29 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 140 | 45.93 | 98095.15 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 160 | 46.07 | 112450.42 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LATELP | 180 | 46.17 | 126807.41 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LATELP | 200 | 46.37 | 141496.26 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| LP (skip) | 40 | 36.43 | 22236.12 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 37.27 | 22742.92 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 37.30 | 34147.12 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 37.29 | 45513.96 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 100 | 37.58 | 57328.36 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 120 | 38.50 | 70494.46 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 140 | 39.57 | 84515.69 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 160 | 40.88 | 99785.82 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LP | 180 | 42.26 | 116068.71 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LP | 200 | 43.71 | 133368.39 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 40 | 43.62 | 26624.06 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 43.62 | 39935.99 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 43.61 | 53222.30 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 100 | 43.48 | 66339.97 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 120 | 43.47 | 79579.16 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 140 | 44.25 | 94520.59 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 160 | 45.69 | 111532.75 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 180 | 46.18 | 126818.98 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 200 | 46.37 | 141496.42 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| STAT (skip) | 40 | 42.07 | 25655.26 | 609.86 | 0.981 | 358 | 4992 | 0 | 0 |
| STAT | 40 | 41.23 | 25142.41 | 609.87 | 0.981 | 357 | 4390 | 0 | 0 |
| STAT | 60 | 41.21 | 37703.71 | 914.81 | 0.980 | 357 | 6581 | 0 | 0 |
| STAT | 80 | 41.18 | 50221.83 | 1219.63 | 0.980 | 358 | 8747 | 12 | 12 |
| STAT | 100 | 41.35 | 63041.18 | 1524.56 | 0.979 | 358 | 10435 | 15 | 15 |
| STAT | 120 | 41.92 | 76677.61 | 1829.31 | 0.978 | 359 | 12760 | 26 | 26 |
| STAT | 140 | | | | | | | | |
| STAT | 160 | 43.68 | 106489.85 | 2438.15 | 0.974 | 359 | 19573 | 37 | 49 |
| STAT | 180 | 44.66 | 122392.24 | 2740.73 | 0.965 | 359 | 24743 | 150 | 237 |
| STAT | 200 | 45.71 | 138894.45 | 3038.93 | 0.945 | 359 | 31560 | 255 | 768 |

**Table A.13 – Simulation Results - Homogeneous without Network Constraints - Movement + Capacity Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 51.07 | 31170.18 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 60 | 51.07 | 46755.27 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 51.07 | 62340.36 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 51.07 | 77925.45 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 51.07 | 93510.54 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 51.07 | 109095.63 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 160 | 51.07 | 124680.72 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 180 | 51.07 | 140265.81 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 200 | 51.07 | 155850.90 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 40 | 50.82 | 31012.83 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 50.82 | 46519.25 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 50.82 | 62025.66 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 100 | 50.82 | 77532.08 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 120 | 50.82 | 93038.49 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 140 | 50.82 | 108544.91 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 160 | 50.82 | 124051.32 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 180 | 50.82 | 139557.74 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 200 | 50.82 | 155064.15 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 40 | 50.76 | 30981.68 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 50.76 | 46472.52 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 50.76 | 61963.36 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 50.76 | 77454.20 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 50.76 | 92945.04 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 50.76 | 108435.88 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 50.76 | 123926.72 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 50.76 | 139417.56 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 200 | 50.76 | 154908.40 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| LP (skip) | 40 | 49.00 | 29906.80 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 52.84 | 32250.52 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 52.84 | 48375.78 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 52.84 | 64501.04 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 52.84 | 80626.30 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 52.84 | 96751.56 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 52.84 | 112876.82 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 52.84 | 129002.08 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 52.84 | 145127.34 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 200 | 52.84 | 161252.60 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 40 | 47.31 | 28873.47 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 47.31 | 43310.21 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 47.31 | 57746.94 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 47.31 | 72183.68 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 47.31 | 86620.41 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 47.31 | 101057.15 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 47.31 | 115493.88 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 47.31 | 129930.62 | 2746.35 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 200 | 47.31 | 144367.35 | 3051.50 | 1.000 | 0 | 0 | 0 | 0 |
| STAT (skip) | 40 | 58.04 | 35408.97 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 57.57 | 35131.49 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 57.57 | 52697.24 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 57.57 | 70262.98 | 1220.45 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 100 | 57.57 | 87828.73 | 1525.56 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 120 | 57.57 | 105394.47 | 1830.67 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 140 | 57.57 | 122960.22 | 2135.78 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 160 | 57.57 | 140525.96 | 2440.90 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 180 | 57.57 | 158091.71 | 2746.01 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 200 | 57.57 | 175657.45 | 3051.12 | 0.995 | 0 | 0 | 0 | 0 |

**Table A.14 – Simulation Results - Homogeneous with Network Constraints - Movement + Capacity Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 51.07 | 31170.18 | 610.30 | 1.000 | 30 | 30 | 29 | 29 |
| AVG | 60 | 51.07 | 46753.88 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 80 | 51.07 | 62340.06 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 100 | 51.07 | 77924.02 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 120 | 51.07 | 93502.89 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| AVG | 140 | 51.06 | 109069.04 | 2136.05 | 1.000 | 1 | 1 | 0 | 0 |
| AVG | 160 | 51.03 | 124573.56 | 2441.20 | 0.998 | 12 | 12 | 0 | 0 |
| AVG | 180 | 50.95 | 139896.98 | 2745.97 | 0.993 | 34 | 34 | 0 | 0 |
| AVG | 200 | 50.71 | 154628.41 | 3049.03 | 0.964 | 84 | 84 | 0 | 0 |
| LATE | 40 | 50.82 | 31012.83 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 60 | 50.82 | 46519.25 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATE | 80 | 50.80 | 62011.47 | 1220.60 | 1.000 | 2 | 2 | 0 | 0 |
| LATE | 100 | 50.08 | 76306.75 | 1523.55 | 0.966 | 140 | 140 | 0 | 0 |
| LATE | 120 | 48.72 | 88515.34 | 1816.67 | 0.818 | 359 | 359 | 0 | 0 |
| LATE | 140 | 48.19 | 101423.41 | 2104.52 | 0.698 | 359 | 359 | 0 | 0 |
| LATE | 160 | 47.62 | 113642.40 | 2386.38 | 0.606 | 359 | 359 | 3 | 3 |
| LATE | 180 | 47.55 | 125432.29 | 2638.11 | 0.534 | 359 | 359 | 56 | 56 |
| LATE | 200 | 47.33 | 132984.27 | 2809.63 | 0.478 | 359 | 359 | 210 | 210 |
| LATELP | 40 | 50.76 | 30981.68 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 50.76 | 46472.51 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 50.74 | 61938.42 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 100 | 49.81 | 75991.23 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 120 | 48.62 | 89022.34 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 140 | 48.21 | 102980.52 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 160 | 47.81 | 116702.70 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 180 | 47.78 | 131211.95 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LATELP | 200 | 48.63 | 148306.32 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| LP (skip) | 40 | 49.00 | 29906.86 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 52.84 | 32250.18 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 52.84 | 48375.75 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 52.79 | 64430.53 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 100 | 52.14 | 79556.11 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 120 | 51.17 | 93686.58 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 140 | 50.78 | 108469.07 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 160 | 51.10 | 124754.32 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 180 | 51.25 | 140746.66 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| LP | 200 | 51.19 | 156113.59 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| MIDNIGHT | 40 | 47.31 | 28873.47 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 47.31 | 43310.15 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 47.27 | 57698.52 | 1220.60 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 100 | 46.65 | 71176.96 | 1525.75 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 120 | 46.56 | 85244.07 | 1830.90 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 140 | 47.44 | 101325.82 | 2136.05 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 160 | 47.80 | 116685.75 | 2441.20 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 180 | 47.78 | 131211.95 | 2746.35 | 0.998 | 10 | 10 | 0 | 0 |
| MIDNIGHT | 200 | 48.63 | 148306.32 | 3049.92 | 0.971 | 55 | 55 | 0 | 0 |
| STAT (skip) | 40 | 58.04 | 35408.97 | 610.03 | 0.986 | 150 | 150 | 0 | 0 |
| STAT | 40 | 57.57 | 35131.49 | 610.22 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 60 | 57.58 | 52704.26 | 915.34 | 0.995 | 0 | 0 | 0 | 0 |
| STAT | 80 | 57.62 | 70318.56 | 1220.37 | 0.994 | 6 | 6 | 0 | 0 |
| STAT | 100 | 56.57 | 86236.08 | 1524.48 | 0.979 | 89 | 89 | 0 | 0 |
| STAT | 120 | 53.70 | 98028.38 | 1825.41 | 0.920 | 265 | 265 | 0 | 0 |
| STAT | 140 | 51.50 | 109232.71 | 2121.01 | 0.834 | 336 | 336 | 0 | 0 |
| STAT | 160 | 49.84 | 120085.40 | 2409.18 | 0.732 | 357 | 357 | 0 | 0 |
| STAT | 180 | 49.32 | 132313.92 | 2682.56 | 0.633 | 359 | 359 | 23 | 23 |
| STAT | 200 | 49.45 | 143906.07 | 2910.36 | 0.549 | 359 | 359 | 72 | 72 |

**Table A.15 – Simulation Results - Heterogeneous with Network Constraints - Movement + Capacity Costs.**

| | | Charge Cost Results | | | Charge Level Results | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Charge Method | Number of Vehicles | Price [$/MWhr] | Total Cost [$] | Total Energy Purchased [MWhr] | Capacity Factor | Calander Days < 99% | Vehicle Days < 99% | Calander Days < 50% | Vehicle Days < 50% |
| AVG | 40 | 49.57 | 30236.31 | 610.01 | 0.988 | 359 | 3895 | 0 | 0 |
| AVG | 60 | 49.40 | 45199.71 | 915.00 | 0.988 | 359 | 5769 | 0 | 0 |
| AVG | 80 | 49.28 | 60122.85 | 1219.91 | 0.987 | 359 | 7709 | 12 | 12 |
| AVG | 100 | 49.16 | 74958.01 | 1524.93 | 0.987 | 359 | 9518 | 12 | 12 |
| AVG | 120 | 49.11 | 89864.11 | 1829.92 | 0.987 | 359 | 11431 | 12 | 12 |
| AVG | 140 | 49.09 | 104797.35 | 2134.92 | 0.987 | 359 | 13391 | 12 | 12 |
| AVG | 160 | 49.05 | 119687.25 | 2439.93 | 0.987 | 359 | 15419 | 12 | 12 |
| AVG | 180 | 49.00 | 134505.03 | 2744.91 | 0.986 | 359 | 18406 | 12 | 12 |
| AVG | 200 | 48.92 | 149172.04 | 3049.41 | 0.979 | 359 | 24592 | 17 | 17 |
| LATE | 40 | 46.81 | 28545.66 | 609.83 | 0.977 | 359 | 4860 | 0 | 0 |
| LATE | 60 | 46.54 | 42573.96 | 914.75 | 0.977 | 359 | 7245 | 0 | 0 |
| LATE | 80 | 46.45 | 56649.00 | 1219.56 | 0.977 | 359 | 9679 | 12 | 12 |
| LATE | 100 | 46.34 | 70638.87 | 1524.30 | 0.974 | 359 | 12725 | 13 | 13 |
| LATE | 120 | 46.33 | 84632.99 | 1826.67 | 0.941 | 359 | 26461 | 24 | 24 |
| LATE | 140 | 46.28 | 98304.84 | 2124.19 | 0.865 | 359 | 46732 | 27 | 27 |
| LATE | 160 | 46.21 | 111664.13 | 2416.30 | 0.774 | 359 | 57325 | 36 | 39 |
| LATE | 180 | 46.18 | 124778.66 | 2701.96 | 0.690 | 359 | 64615 | 125 | 210 |
| LATE | 200 | 46.26 | 137615.76 | 2974.92 | 0.615 | 359 | 71798 | 269 | 2308 |
| LATELP | 40 | 48.80 | 29783.62 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 60 | 48.85 | 44723.85 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LATELP | 80 | 48.39 | 59055.96 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 100 | 46.88 | 71521.42 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 120 | 46.93 | 85921.62 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 140 | 47.01 | 100415.07 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LATELP | 160 | 47.08 | 114923.72 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LATELP | 180 | 47.04 | 129175.67 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LATELP | 200 | 47.19 | 143987.92 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| LP (skip) | 40 | 47.90 | 29232.07 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 40 | 50.66 | 30920.69 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 60 | 50.60 | 46324.09 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| LP | 80 | 50.55 | 61700.73 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 100 | 50.59 | 77184.03 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 120 | 50.66 | 92752.78 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 140 | 50.59 | 108065.78 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| LP | 160 | 50.86 | 124145.76 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| LP | 180 | 51.28 | 140828.34 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| LP | 200 | 52.07 | 158900.15 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 40 | 47.30 | 28864.44 | 610.30 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 60 | 47.29 | 43293.94 | 915.45 | 1.000 | 0 | 0 | 0 | 0 |
| MIDNIGHT | 80 | 47.24 | 57662.42 | 1220.51 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 100 | 46.64 | 71160.23 | 1525.66 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 120 | 46.57 | 85263.89 | 1830.81 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 140 | 47.24 | 100904.41 | 2135.96 | 0.999 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 160 | 47.54 | 116058.05 | 2441.11 | 1.000 | 30 | 30 | 12 | 12 |
| MIDNIGHT | 180 | 47.11 | 129366.81 | 2746.26 | 1.000 | 45 | 45 | 12 | 12 |
| MIDNIGHT | 200 | 47.19 | 143987.88 | 3051.41 | 1.000 | 45 | 45 | 12 | 12 |
| STAT (skip) | 40 | 55.23 | 33680.41 | 609.86 | 0.981 | 358 | 4992 | 0 | 0 |
| STAT | 40 | 53.83 | 32828.51 | 609.87 | 0.981 | 357 | 4390 | 0 | 0 |
| STAT | 60 | 53.69 | 49116.87 | 914.81 | 0.980 | 357 | 6581 | 0 | 0 |
| STAT | 80 | 53.60 | 65373.68 | 1219.63 | 0.980 | 358 | 8747 | 12 | 12 |
| STAT | 100 | 53.30 | 81263.66 | 1524.56 | 0.979 | 358 | 10435 | 15 | 15 |
| STAT | 120 | 52.66 | 96334.92 | 1829.31 | 0.978 | 359 | 12760 | 26 | 26 |
| STAT | 140 | | | | | | | | |
| STAT | 160 | 52.13 | 127106.84 | 2438.15 | 0.974 | 359 | 19573 | 37 | 49 |
| STAT | 180 | 52.01 | 142534.69 | 2740.73 | 0.965 | 359 | 24743 | 150 | 237 |
| STAT | 200 | 51.96 | 157915.74 | 3038.93 | 0.945 | 359 | 31560 | 255 | 768 |

# APPENDIX B. HETEROGENEOUS CHARGING ALGORITHM DESCRIPTION

This appendix presents a detailed description of the statistical algorithm used for heterogeneous Plugin Electric Vehicle (PEV) charging initially described in Chapter 5.

This appendix is broken into several parts:

1. General Problem Description
2. General Solution Algorithm Description
3. Example of Network Constraint Resolution

## B.1. General Problem Description

The algorithm can be used to solve general problems of the form:

Minimize:

Objective function

$$Cost = \left[p^i_{act}\right]^T * \left[Q^{iv}_{act}\right]^T * \left[n^i_1\right] \tag{B-1}$$

$$(\boldsymbol{where}\ [\boldsymbol{p^i_{stat}}] \rightarrow [\boldsymbol{p^i_{act}}]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

Subject to:

Consumer Accumulation (Goal) Constraint:

$$\left[Q^{iv}_{act}\right]^T * \left[n^i_1\right] = \left[q^v_{des,act}\right] = \left[L^v_{des,final}\right] - \left[L^v_{act,init}\right] \tag{B-2}$$

$$(\boldsymbol{where}\ [\boldsymbol{L^v_{fc,init}}] \rightarrow [\boldsymbol{L^v_{act,init}}]\ \boldsymbol{on}\ '\boldsymbol{i}'\ \boldsymbol{at\ which\ vehicle}\ '\boldsymbol{v}'\ \boldsymbol{arrives})$$

Consumer Rate Constraint:

$$0 * \left[n^i_1\right][n^v_1]^T \leq \left[Q^{iv}_{act}\right] \leq \left[A^{iv}_{act}\right].* \left(\left[n^i_1\right][c^v_v]^T\right) \tag{B-3}$$

$$(\boldsymbol{where}\ [\boldsymbol{A^{iv}_{fc}}] \rightarrow [\boldsymbol{A^{iv}_{act}}]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

System Rate Constraints:

$$\left[Q^{iv}_{act}\right] * [a^v_a] \leq c_a * \left[n^i_1\right] - \left[ld^i_{a,act}\right] \tag{B-4a}$$

$$\left[Q^{iv}_{act}\right] * [a^v_b] \leq c_b * \left[n^i_1\right] - \left[ld^i_{b,act}\right] \tag{B-4b}$$

...

$$\left[Q_{act}^{iv}\right] * \left[a_x^v\right] \leq c_x * \left[n_1^i\right] - \left[ld_{x,act}^i\right] \quad \textbf{(B-4c)}$$

...

$$\left[Q_{act}^{iv}\right] * \left[a_{nc}^v\right] \leq c_{nc} * \left[n_1^i\right] - \left[ld_{nc,act}^i\right] \quad \textbf{(B-4d)}$$

$$(\boldsymbol{where}\ \left[\boldsymbol{ld_{x,fc}^i}\right] \rightarrow \left[\boldsymbol{ld_{x,act}^i}\right]\ \boldsymbol{on}\ '\boldsymbol{i}')$$

This formulation is identical to that used for a Linear Programming (LP) problem, except that:

1. The dominant dimension of the solution, ‘i’, represents time, and the solution results $\left[Q_{act}^{iv}\right]$, are determined step-by-step in real time.
2. Values for many parameters (indicated in bold text) are initially only guessed for future timesteps until the actual values become discovered in real time.
3. The solution mechanism has the form of a time-evolving setpoint price rule, $[SP^v]$, which is compared to actual real-time price at each timestep to determine the quantity of electricity to be charged at each timestep, $\left[Q_{act}^{iv}\right]$

<u>Variables are defined as follows</u>:

| **Name** | **Dimension** | **Description** |
|---|---|---|
| A→B on ‘i’ | Operator | Represents transformation from one vector or matrix (A) to another (B), updated as timesteps ‘i’ increment. |
| Cost | Scalar | Aggregate Cost for all Consumers |
| i | Scalar | Index for timesteps (maximum is ‘ni’) |
| v | Scalar | Index for consumer (maximum is ‘nv’) |
| c | Scalar | Index for network constraints (maximum is ‘nc’) |
| $c_x$ | Scalar | Magnitude of network accumulation constraint ‘x’ |
| $[c_v^v]$ | Vector (v x 1) | Consumer-specific accumulation constraint, for each consumer ‘v’ |
| $\left[n_1^i\right]$ | Vector (i x 1) | Vector of ones for each timestep ‘i’ |
| $[n_1^v]$ | Vector (v x 1) | Vector of ones for each timestep ‘v’ |
| $\left[p_{act}^i\right]$ | Vector (i x 1) | Actual resource price at each timestep ‘i’ |
| $\left[p_{stat}^i\right]$ | Vector (i x 1) | Forecast resource price (statistically described) at each timestep ‘i’.<br>Statistical description at each timestep includes: |

| | | |
|---|---|---|
| | | mean expected price (μ)<br>price standard deviation (σ) |
| $[Q_{act}^{iv}]$ | Matrix (i x v) | Actual resource quantity purchased at each timestep 'i', and for each consumer 'v' |
| $[q_{des,act}^{v}]$ | Vector (v x 1) | Actual desired resource quantity to be purchased, for each consumer 'v' |
| $[L_{des,final}^{v}]$ | Vector (v x 1) | Actual desired final resource level, for each consumer 'v' |
| $[L_{act,init}^{v}]$ | Vector (v x 1) | Actual initial resource level, for each consumer 'v' |
| $[L_{fc,init}^{v}]$ | Vector (v x 1) | Forecast initial resource level, for each consumer 'v' |
| $[A_{act}^{iv}]$ | Matrix (i x v) | Actual availability of each consumer for resource accumulation at each timestep 'i', for each consumer 'v' |
| $[A_{fc}^{iv}]$ | Matrix (i x v) | Forecast availability of each consumer for resource accumulation at each timestep 'i', for each consumer 'v' |
| $[a_{x}^{v}]$ | Vector (v x 1) | Applicability of network accumulation constraint 'x' to consumers |
| $[ld_{x,act}^{i}]$ | Vector (i x 1) | Actual consumer-independent load at network point 'x', for each timestep 'i' |
| $[ld_{x,fc}^{i}]$ | Vector (i x 1) | Forecast consumer-independent load at network point 'x', for each timestep 'i' |

## B.2. General Solution Algorithm Description

The general solution method is comprised of the following steps, executed in sequence for each timestep of the charging period.

### B.2.1. Construct Price Distributions

A statistical price distribution, $\Phi(p)$, is constructed for every vehicle and timestep based on $[p_{stat}^{i}]$ (specifies mean and standard deviation of prices at each timestep). This is similar to the method used in previous chapters. The statistical parameters of $[p_{stat}^{i}]$ are selected using the STAT2 method described in Chapter 3. However, two significant modifications are made from previously used statistical algorithms.

First, a unique distribution is generated on the smallest level of granularity (i.e., unique distribution for each 5-minute timestep vice for each hour). Generating distributions for the smallest level of granularity allows the effects of individual vehicle arrivals and departures to be modeled more precisely. Also, the complexity of aggregating information from a 5-minute timescale to a larger timescale is removed. However, working on a smaller level of granularity comes at the expense of requiring additional computational resources.

Second, simple triangular price distributions are used in lieu of Gaussian distributions to reduce the computational resources required for algorithm execution. Many price distributions must be constructed for setpoint calculation (36,000 for simulating charging of 200 vehicles over 15 hours), and Cumulative Density Functions (CDFs) for these distributions must be continually recalculated within setpoint calculation loops, constraint violation resolution loops, and while stepping through each nightly timestep. Using a triangular distribution of roughly equivalent variance results in an approximately 100 fold increase in execution speed without significantly impacting results.

### B.2.2. Scale Price Distributions

Each statistical price distribution is scaled so that the integral over the entire distribution is equal to the maximum quantity of electricity which can be purchased by the applicable vehicle at the timeperiod of interest. If there are no network constraint limitations, the scaling is a function of $[A^{iv}_{proj}]$ (which scales distributions to zero if the vehicle is not available), and $[c^v_v]$ (which scales the distribution's integral to the available electricity based on a vehicles charger size). In the general case, where network constraint violations are to be resolved, a scaling matrix $[S^{iv}]$ is used which combines the effects of availability $[A^{iv}_{proj}]$ as well as capacity limits due to network constraints. This is discussed further later. A key simplification over the methods used in previous chapters is that the distributions are scaled to natural units of resource (i.e., units of electrical energy) rather than scaling to an indirect surrogate parameter (such as fraction of an hour for which charging is required). Hence, the distributions are referred to as price-energy distributions.

### B.2.3. Calculate Purchase Setpoints

A vector of vehicle-specific purchase setpoints ($SP^v$) is calculated. Like in earlier chapters, maximum and minimum bounding SPs are guessed, and a binomial search is performed until the SP for each vehicle stabilizes and the expected amount of charging matches the desired amount of charging. This criterion can be stated for each vehicle, v, as follows:

$$\sum_{i=1}^{ni}\left(c_v^v(\mathrm{v}) * S^{iv}(\mathrm{i},\mathrm{v}) * \int_{-\infty}^{SP}\Phi(p)dp\right) = q_{des}^v(v) \qquad \textbf{(B-5)}$$

### B.2.4. Iterate to Resolve Network Constraints

Once setpoints are calculated for all vehicles at a given timestep, the entire system needs to be evaluated to ensure that no network constraints will be violated throughout the charging period. If setpoints will result in a network constraint violation, the effective charge rate for all vehicles being charged at that timestep will need to be reduced to prevent the violation. Such "throttling" of charge rate in real time is fairly straight forward. However, the impact of potential future constraint violations on the overall determination of optimal setpoints is more complex. Potential limitations need to be modeled via price-energy distribution scaling, and the setpoint calculation then needs to be re-performed. Simply stated, if there is a potential limitation in the availability of cheap energy in the future, the setpoint will have to be raised so that the potential shortfall can be made up from a more expensive source (i.e., other future timestep). The methodology used to resolve network constraints is described as follows, with an example provided at the end of this appendix.

Network constraint violation can be tested by determining whether demand exceeds capacity. The excess capacity (e(i)) is calculated as follows. If e(i) is negative, the constraint is violated for the timestep of interest.

$$e(i) = c_x - ld_x^i(i) - [a_x^v]^T([c_v^v].*[purchase^v]) \qquad \textbf{(B-6)}$$

where $[purchase^v]$ is a vector which identifies the vehicles that are to charge at the timestep of interest. The purchase vector is known deterministically for only the current timestep. For future timesteps, the purchase vector depends on price. Since only a statistical description of probable price exists, future constraint violations are also described stochastically.

Constraint violation and its dependence on price is demonstrated in the following figure. The top three subplots represent price likelihood for three different vehicles which are all subject to an identical price environment. Although the price distribution is identical for each vehicle, each vehicle has a unique purchase setpoint (green vertical line which bounds the quantity of electricity to be purchased) based on the desired extent of charging. The bottom subplot represents the aggregate charging rate as a function of price (note that each step corresponds to one of the vehicle setpoints), with a network constraint limit illustrated as a horizontal red line. The constraint will only be violated for low prices where two or more vehicles will charge concurrently. The likelihood of a constraint violation can be determined by considering the price at which the constraint will be violated ($22.71; the price below which vehicle 2 will charge). Since the probability of price being below $22.71 is approximately 55%, the likelihood of a constraint violation is also 55% for this timestep.

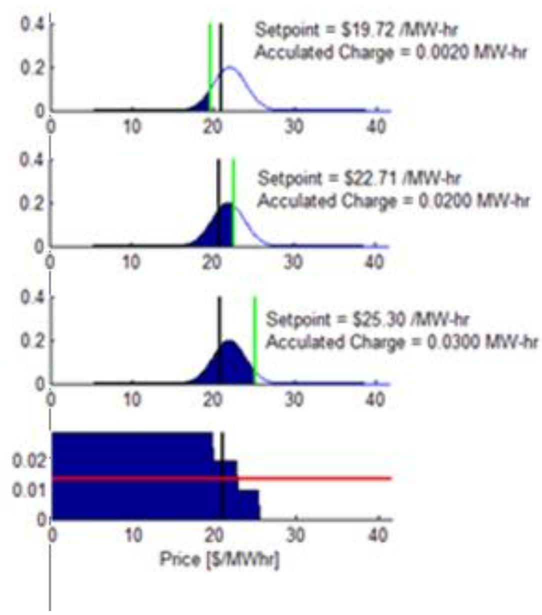



**Figure B.1 – Example of Resolving of a Network Constraint Violation while Calculating a Price Setpoint for a System with Three Vehicles.**

The insight gained from the example is used for algorithm development. The user must specify a desired likelihood of constraint violation ($lf_{des}$, vertical black line in figure example), and the algorithm will iterate SP calculation and distribution rescaling until the likelihood of constraint violation is below that threshold. Thus, constraints are only resolved by adjusting charging for vehicles likely to be charging, identified in $\left[purchase^{v}_{proj}\right]$. Selecting a useful value for $lf_{des}$ is important. If too low a value is selected, too many vehicles will be assumed to charge at too many timesteps. This will result in over-anticipating demand. The setpoint will then be driven up artificially, and vehicles risk being prematurely charged at a higher-than-necessary cost. By contrast, if too high a value of $lf_{des}$ is selected, demand will be under-anticipated, the setpoint will be driven lower, and the vehicles risk not being fully charged at the end of the charging period.

If a constraint violation is anticipated for a given $lf_{des}$, the available charging rate in the statistical model is reduced (scaled) by adjusting the scaling matrix, $S^{iv}$, as follows.

$$S^{iv}(i) = \left[Purchase^{v}_{proj}\right] .* \left(1 + \frac{e(i)}{\left[Purchase^{v}_{proj}\right]^{T} * [c^{v}_{v}]}\right) + \left([n^{v}_{1}] - \left[Purchase^{v}_{proj}\right]\right) .* \left[A^{iv}_{proj}\right] \quad \textbf{(B-7)}$$

The first half of the equation rescales only vehicles expected to be charging (i.e., those vehicles whose setpoint is above the price associated with $lf_{des}$, as identified in $\left[purchase^{v}_{proj}\right]$). The rescaling is a ratio of capacity shortfall to capacity of vehicles expected to be charging. The second half of the equation leaves scaling unaffected for vehicles not likely to be charging, but retains scaling dependence on the availability matrix. In the example, the charging rate associated with vehicles 2 and 3 will be reduced (from 0.01 to 0.075) so that their combined charging rate (0.015) is equal to the charging rate constraint. The assumed charging rate for vehicle 1 is unchanged as its setpoint is sufficiently low (below the \$21 associated with $lf_{des}$ = 20%, illustrated with vertical black line) that it is unlikely to charge in the timestep of interest.

The general methodology for resolving potential network constraint violations described above is iterated until setpoints stabilize and no constraint violations are anticipated. For the case of a single constraint violation (as in the previous paragraph), a reduction in charging capacity is distributed proportionally to vehicle charging capacity of vehicles likely to be charging. In the general case, multiple independent constraints throughout a distribution network may be violated and charging rate reductions are not necessarily distributed proportionally to charging capacity of offending vehicles. To maximize the total quantity of electricity purchased at an advantageous price, the following LP formulation can be used to calculate $S^{iv}$ for the affected timestep.

Given: vehicles likely to be charging, $[purchase^{v}_{proj}]$, at timestep 'i'

Find: $S^{iv}(i)$

Which Maximizes:

$$[purchase^{v}_{proj}]^{T} * \left([c^{v}_{v}] .* \left[S^{iv}(i)\right]\right) \qquad \textbf{(B-8)}$$

Subject to:

$$0 * [n^{v}_{1}] \leq \left[S^{iv}(i)\right] \leq \left[A^{iv}_{proj}(i)\right] \qquad \textbf{(B-9)}$$

$$[a^{v}_{x}]^{T} * \left([c^{v}_{v}] .* \left[S^{iv}(i)\right]\right) \leq c_{x} - ld^{i}_{x}(\mathrm{i}) \quad \forall\, network\ constraints \qquad \textbf{(B-10)}$$

### B.2.5. Make Purchase Decisions

Once a set of SPs compatible with network constraints is calculated, a purchase decision can be made for the current timestep. The decision rule is simple; charge the PEV if the price is lower than the setpoint. This decision sets the purchase vector ($[purchase^{v}_{act}]$) for the current timestep.

The next part of the decision is to determine the quantity of electricity to be purchased by the contending vehicles. Since the determination has already been made that purchasing at the given price is advantageous for the vehicles identified in the purchase, every attempt should be made to maximize the amount of electricity purchased for them. If no network constraints will be violated, each vehicle charges at the maximum charging rate. If network constraints will be violated by doing so, the charging rates must be reduced. For a single constraint violation, the charging reduction necessary to prevent constraint violation is split evenly among all charging vehicles. If multiple constraint violations will occur, the linear programming based allocation scheme described above is performed, except that the list of vehicles expected to be charging $[purchase^{v}_{proj}]$ is replaced with the list of vehicles identified in the purchase vector $[purchase_{act}^{v}]$. Also, the result of the reallocation of charging is $\left[Q_{act}^{iv}(i)\right]$ rather than $\left[S^{iv}(i)\right]$.

It should be noted that $[purchase_{act}^{v}]$ differs from $\left[purchase_{proj}^{v}\right]$ beyond the fact that the former represents an actual decision and the later represents a hypothetical decision. The former includes vehicles which were not anticipated to be charging but actually are. The inevitability that vehicles which were not expected to be charging at a given set of timesteps will charge at some of them is a source of prediction error.

### B.2.6. Update Uncertain Parameters

Once a charging decision has been effected, parameter values must be updated to reflect the changed conditions for the next timestep. Doing so is necessary to continually keep the ongoing optimization properly framed. Certain parameters are updated to reflect the discovery of new data associated with the new timestep.

For example, $[q_{des}^{v}]$ is updated to reflect the increase in vehicle charge level for vehicles which had just undergone charging. Also, discovery of new price data allows for updating statistics which are used for better price forecasting (i.e., execution on the STAT2 methodology). Similarly, discovery of new vehicle arrivals and departures allows forecast availability to be updated, and $[L_{init}^{v}]$ can be updated to reflect the actual initial charge level of a just-arrived vehicle. Details for the vehicle availability matrix, $\left[A^{iv}\right]$, is updated follow.

The vehicle availability matrix, $\left[A^{iv}\right]$, lists the availability of each vehicle for charging at each timestep. The dimensions of are dictated by the number of timesteps and the number of vehicles. Each element is either a '1', if the vehicle is available for charging, or a '0' if the vehicle is not available for charging. The forecast availability of vehicles is expressed as $\left[A_{fc}^{iv}\right]$ and, as time progresses and actual availability is discovered, the forecast values are updated to the actual values. Thus, $\left[A_{fc}^{iv}\right]$ evolves into $\left[A_{act}^{iv}\right]$. However, an additional level of complexity exists. If a vehicle arrives earlier than forecast, the elemenst for current and future "pre-arrival" timesteps are updated from '0' to a '1'. Similarly, if a vehicle is discovered to depart early, current and future "post-departure" timesteps can be updated without waiting for discovery in later timesteps. Since, as just discussed, availability is inferred for some timesteps, neither the forecast nor the actual availability matricies are appropriate for price model simulation. Rather, a continuously evolving projected availability matrix, $\left[A_{proj}^{iv}\right]$, is used. It is further noted that as time progresses through a simulation, past timestep elements of $\left[A_{proj}^{iv}\right]$ are set to zero to reflect the fact that charging can no longer occur at those timesteps.

### B.2.7. Repeat for Each Timestep

An iterative strategy is used to compensate for inaccurate forecast data. Hypothetically, it may be sufficient to calculate vehicle specific setpoints only at the beginning of the charging period and execute that setpoint based charging rule for all remaining timesteps. However, some degree of iteration of setpoint values is desirable since many of the parameters on which the initial optimization is based are uncertain. The above steps are therefore repeated throughout the charging window, potentially as frequently as at every timestep.

Iteration reduces the impact of forecasting error. For example, price prediction may improve with time with the introduction of additional data for trending. The main benefit of iteration, however, deals with the progress of achieving the charging goal. As the algorithm progresses through the charging window, the setpoint is continually adjusted based on the ratio of quantity of electricity which remains to be charged to the remaining quantity of electricity available for charging. If the remaining desired quantity of electricity is small relative to available quantity, significant purchase deferability exists and the setpoint can be lowered to lower overall cost. If the remaining desired quantity of electricity is large relative to the available quantity, there is little ability to defer purchasing electricity and meet the charging goal. As a result, the setpoint is increased; cost savings are to be sacrificed to achieve the charging goal. Thus, the setpoint is dynamically recalculated throughout the charging window to ensure the charging goal is met at minimal cost. This is consistent with the control system methodology of using feedback to reduce steady-state error.

It should be noted that iteration can be employed to reframe the scope of the problem from optimizing a finite charging period to continually optimizing for the immediate future. The algorithm can then be run for an infinite period of time. The optimization problem is continuously reframed for the specified optimization horizon (e.g., the next 14 hours). A new setpoint is calculated at each timestep based on forecasting parameters through that time horizon. Iteration and adaptation throughout a given vehicle's charging would drive the effects of any forecasting errors to be small by the time charging is completed.

## B.3. Example of Network Constraint Resolution

This section provides a simplified example of the calculational method used to resolve network constraint violations. The example consists of price setpoint calculation for a single timestep in a system with three vehicles, five timesteps, and a single network constraint. The example graphically steps through each iteration of constraint resolution on the following figures.

In all graphics, price-energy distributions for each timestep are constructed from $[p^i_{stat}]$, and data from $[A^{iv}_{proj}]$ is used to "zero-out" distributions where the vehicle is unavailable (i.e., timestep 1 (vehicles 1 and 2) and timestep 5 (vehicle 5)). A price setpoint (green vertical line) is calculated for each vehicle so that the anticipated charging over the entire charging period is equal to the desired amount of charging. The rightmost column represents the expected aggregate charging rate due to all vehicles as a function of price for each timestep. Note that each step in expected aggregate charging rate reflects the setpoint associated with one of the three vehicles. The network constraint limit is shown as a horizontal red line. Vertical black lines represent the price where the likelihood of a network constraint violation, $lf_{des}$, is 20%

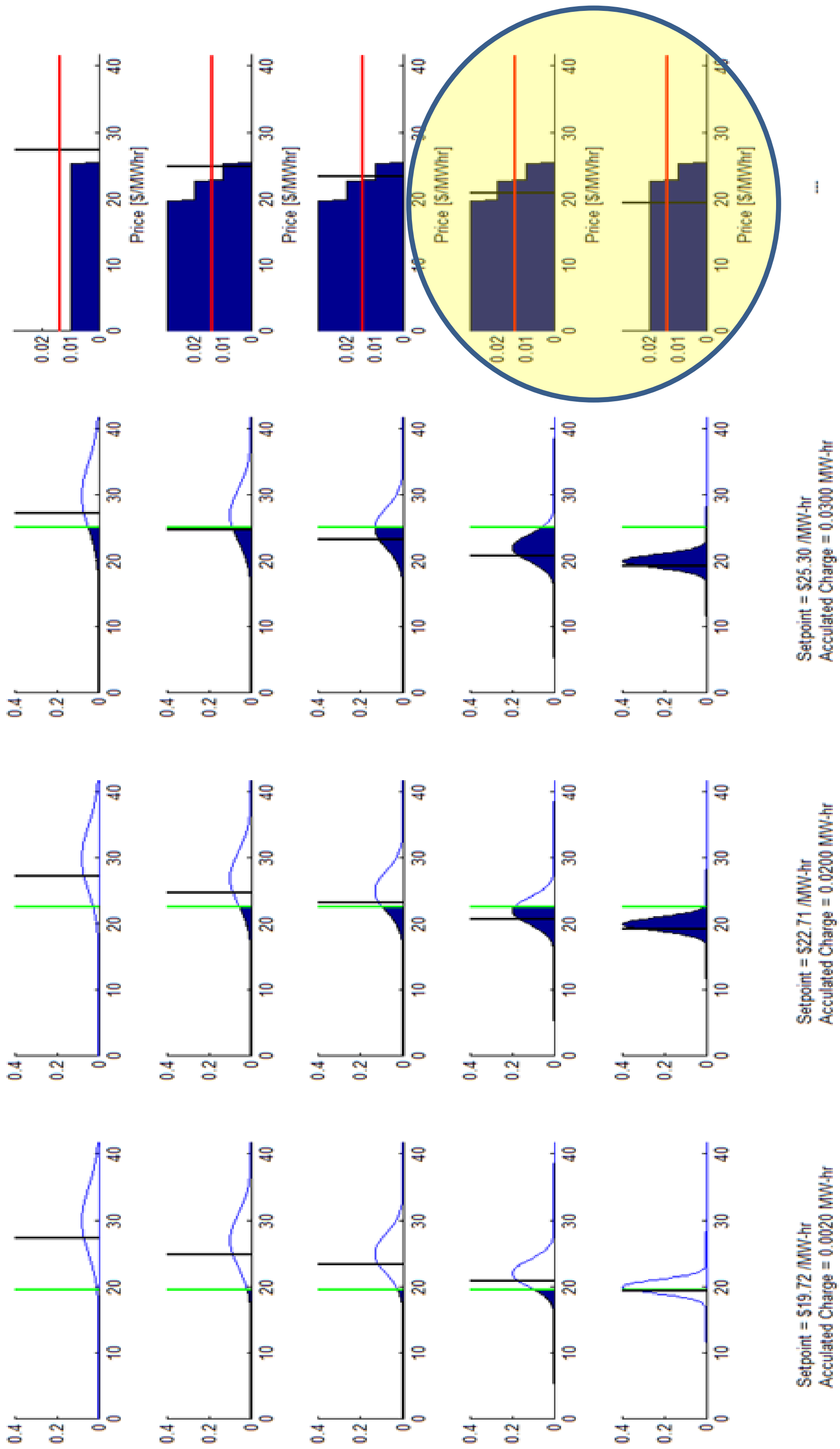



**Figure B.2 – Step 1 of Network Constraint Resolution: The network constraint is likely to be violated to timesteps 4 and 5 (i.e., the intersection of the red and black lines overlaps an area of charging). This necessitates rescaling the price-energy distributions for vehicles 2 and 3 for the last two timesteps.**

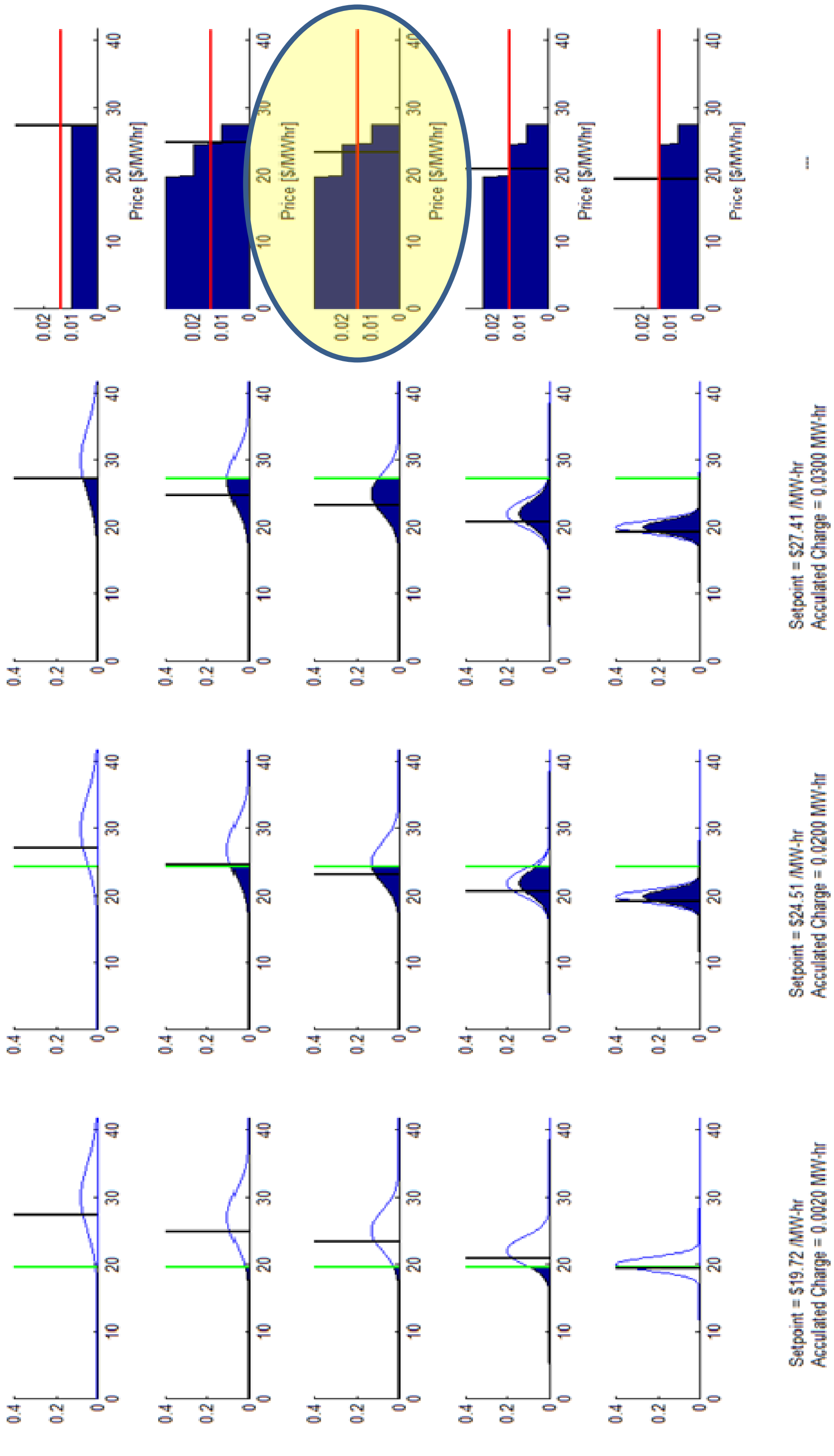



**Figure B.3 – Step 2 of Network Constraint Resolution: In this step, the price-energy distributions for vehicles 2 and 3 are reduced for timesteps 4 and 5, which eliminates the anticipated network constraint violation. However, setpoints are then re-adjusted higher to support vehicle charging goals, which results in a new network constraint violation at timestep 3.**

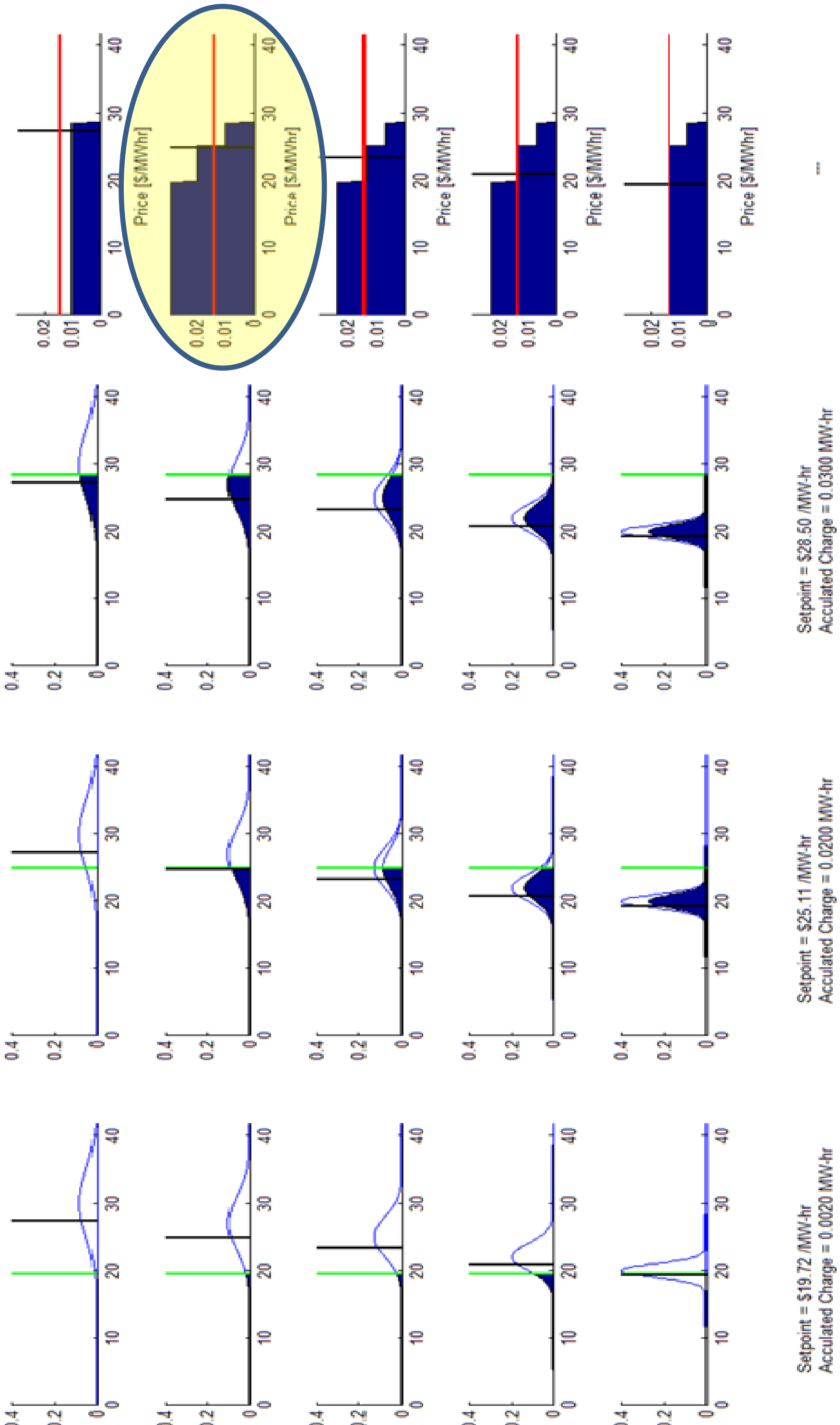



**Figure B.4 – Step 3 of Network Constraint Resolution: In this step, the price-energy distributions for vehicles 2 and 3 are reduced for timesteps 3, which eliminates the anticipated network constraint violation. However, setpoints are then re-adjusted higher to support vehicle charging goals, which results in a new network constraint violation at timestep 2.**

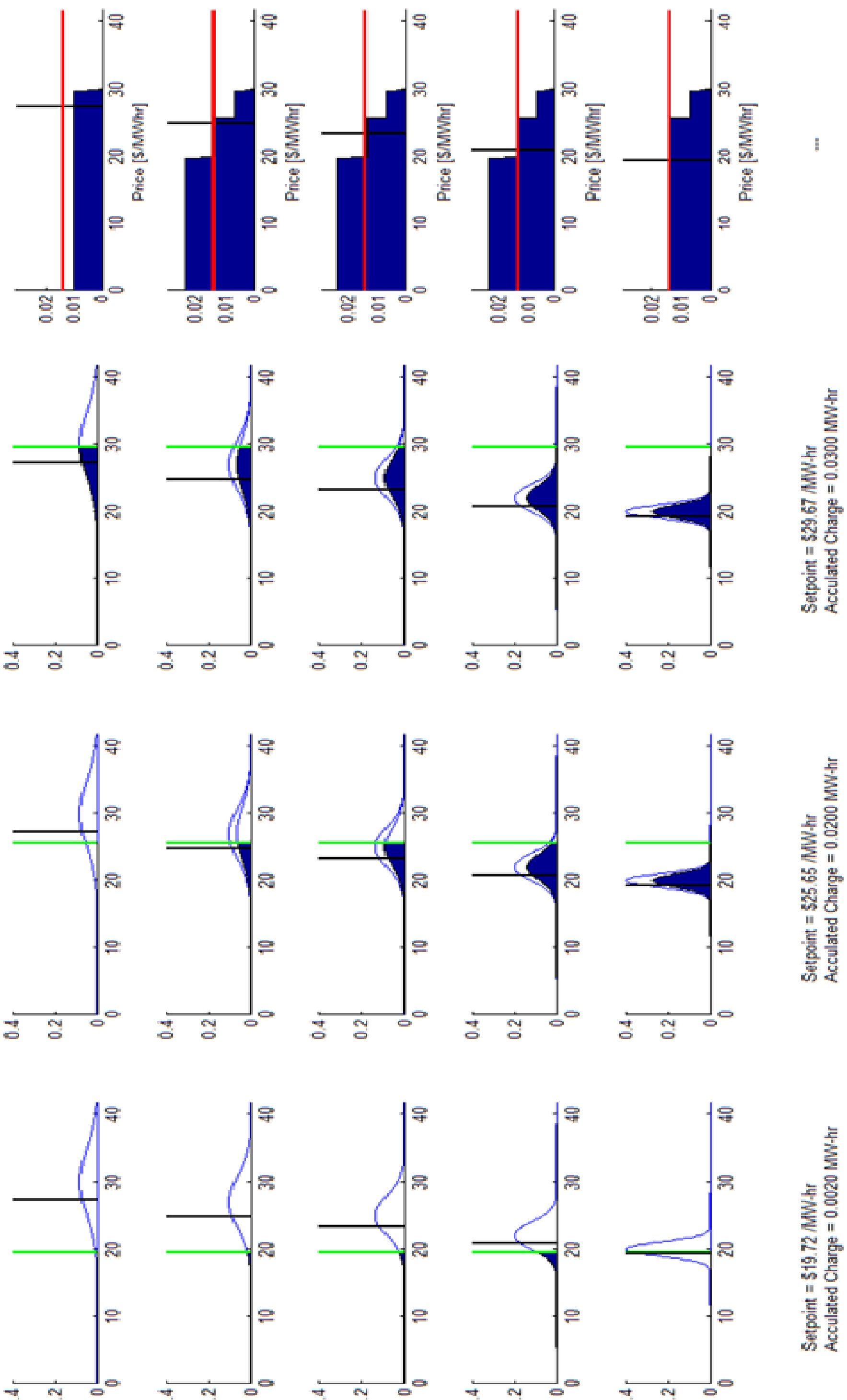



**Figure B.5 – Step 4 of Network Constraint Resolution: In this final step, the price-energy distributions for vehicles 2 and 3 are reduced for timesteps 2, which eliminates the anticipated network constraint violation. The resulting setpoint recalculation does not result in any further anticipated constraint violations, so setpoint calculation is complete.**

# APPENDIX C. REPRESENTATIVE TRANSIENT RESULTS

This appendix presents representative transient PEV charging behavior for various charging methods for the heterogeneous charging scenario. The presented results are taken for a single vehicle on a single night of charging (January 20, 2014) for the case where 120 vehicles charge in the neighborhood every night.

Figure C.1 presents aggregate electricity demand for a 300 home neighborhood with 120 vehicles for various PEV charging methods. Also included (top sub-plot) are Real Time Market (RTM) prices which drives charging behavior for some of the charging methods. For reference, a horizontal black line indicates the average price over the entire charging period.

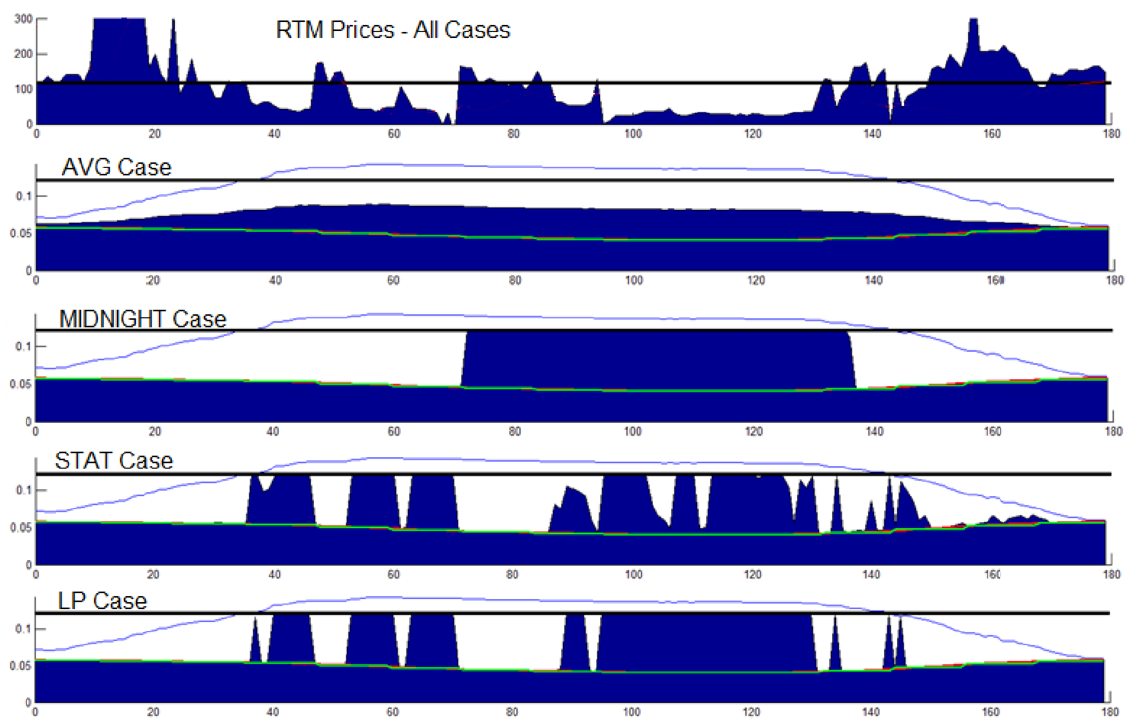



**Figure C.1 – RTM Prices and Neighborhood-Aggregate Electricity Demand (Various Cases) for the Night of January 20th, 2014.**

For each of the four charging cases, several parameters are displayed. A black horizontal line represents the maximum overall neighborhood load permitted by the network constraint. A green, convex up, line represents aggregate electricity demand from only the 300 homes. A blue curve represents the potential electricity demand given the total number of vehicles available for charging; the transient behavior indicates the gradual arrival of vehicles early in the charging period, and the gradual departure of vehicles late in the charging period.

The behavior of the four charging methods is described as follows.

1. The shaded area in the AVG case shows aggregate electricity demand if each vehicle's charging rate is "throttled down" to charge at a constant rate throughout the entire duration of a vehicle's forecast availability. The plot shows that when 120 independent vehicles use this charging methodology, aggregate demand gradually builds and then dies throughout the charging period; more than filling up the nightly lull in household demand.
2. The shaded area in the MIDNIGHT case reflects a charging method where each vehicle begins charging at its maximum rate at midnight and continues until charging is fully completed. The aggregate effect is a sharp increase in aggregate load (sufficient to be limited by the network constraint) at midnight, with a sharp load decrease several hours later. The sharp transitions at the beginning and end of the charging period reflect that all vehicles are executing almost identical charging behavior.
3. The shaded area in the STAT and LP cases are very similar as the STAT case was developed to emulate the behavior of the LP method. In both cases, aggregate load swings drastically depending on price; demand rises to the network constraint when price is low and decreases to only the household load when price is high. The main difference between the two cases is the method for determining which prices are low enough to warrant charging. Only minor differences in the times selected for charging can be observed.

Figure C.2 presents vehicle availability and charge level for a single vehicle over the course of the January 20$^{th}$ 2014 nightly charging period. The top sub-plot shows forecast vehicle availability (red line) and actual availability (blue shaded area); the vehicle arrives and departs earlier than forecast.

The remaining sub-plots show charge levels for the same vehicle, but with different charging methodologies. In all cases, blue shaded areas represents actual charge level, horizontal black lines represents a full charge level, green lines represent charge level if the vehicle was charged at the maximum rate immediately after arrival, and red lines represent charge level if the vehicle was charged at the maximum level as late as possible to ensure full charge at the forecast departure time.

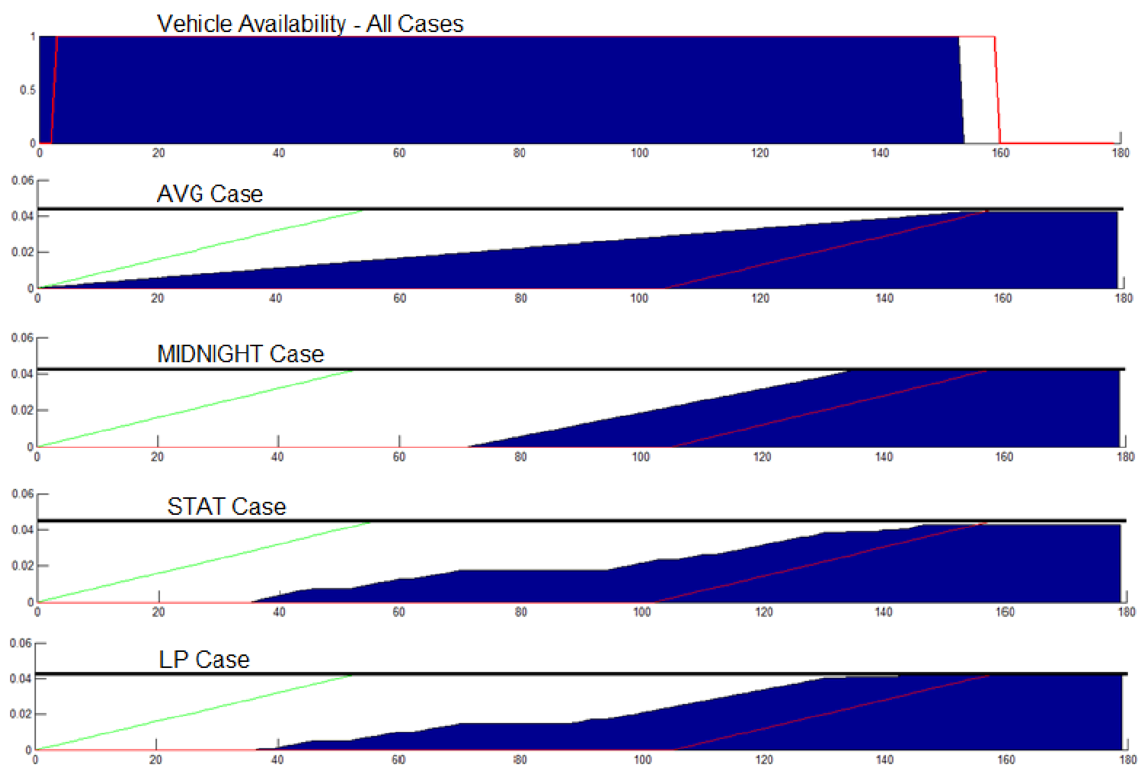



**Figure C.2 – Availability and Charge Level for a Single Vehicle for the Night of January 20th, 2014.**

For the AVG case, level increases at a constant rate throughout the charging period. Since the vehicle departs early, the final charge level is slightly lower than complete. The MIDNIGHT case shows charging at the maximum rate, but with charging beginning at midnight. The STAT and LP case show similar behavior; charging rate intermittently fluctuates between the maximum rate and zero throughout the charging period. In the STAT case, the earlier-than-forecast departure results in failure to fully charge. However, the shortfall in charge level is minor.