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LAKE GEORGE MODELING PROJECT
1971 FINAL REPORT

A final technical report for Union Carbide Subcontract Nos. 3567 and 3581 for the Eastern Deciduous Forest Biome, IBP, Lake George Site

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PROJECT OVERVIEW

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Many people have contributed to the Lake George modeling effort. Faculty participants have included Frank DiCesare, Assistant Professor of Systems Engineering, Samuel Katz, Professor of Geology, R. Bruce Simpson, Assistant Professor of Mathematics, and William A. Wallace, Associate Professor of Public Management. Graduate assistants have included Robert J. Ashdown, Jay A. Bloomfield, James M. Hill, Irving K. Hwang, Robert C. Kohberger, Edgardo P. Rodriquez, Mitchell T. Silver, and John P. Weyant. Programmers have included John S. Fisher and Richard Roth.

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Introduction

In general, the progress in developing an integrated set of ecosystem models has been most encouraging. It appears that the coordinated efforts by the multidisciplinary Lake George team are on the verge of yielding the comprehensive synthesis that was originally envisaged.

Much of the modeling research has been discussed previously (Park and Wilkinson, 1971); therefore, this progress report is intended as a supplement which updates the earlier report and more adequately describes certain important models.

Because the modeling project is dependent on a large number of other studies, there has to be an almost constant re-ordering of priorities as data and information become available from both field and laboratory research. For this reason the modeling effort is ahead of schedule on some tasks and is somewhat behind schedule on other tasks.

Process-oriented functional simulation models for the most important biological components of the ecosystem are well advanced;

and empirical models for the physical and chemical components are in varying stages of completion, depending on the availability of adequately processed data from ongoing studies, including cognate projects that are not funded by IBP.

Modeling Status

At this time it is possible to evaluate realistically the present status and to ascertain with some degree of confidence the future direction of each of the models which represent the Lake George ecosystem. Figure 1 portrays the progression and interrelationships of these models.

Phytoplankton Model. This model is the most advanced of the models at this time (see "PHYTOPLANKTON-HERBIVOROUS ZOOPLANKTON MODEL"). It is presently calibrated for diatom concentrations. Validation and sensitivity analysis are in progress now, and it will soon be calibrated for overall phytoplankton productivity. Disaggregation of the model into bionomic submodels will be attempted this year using preliminary compositional data. The time-step for the model will be decreased to one day in order to evaluate hypotheses concerning daily phytoplankton dynamics.

Herbivorous Zooplankton Model. A functional herbivorous zooplankton model is operational (see "PHYTOPLANKTON-HERBIVOROUS ZOOPLANKTON MOD and Park and Wilkinson, 1971). However, it needs to be better calibrated -- a procedure which is in progress at the present time. The model will be converted to a population-dynamic model in the near future, using data recently made available by D. C. McNaught.

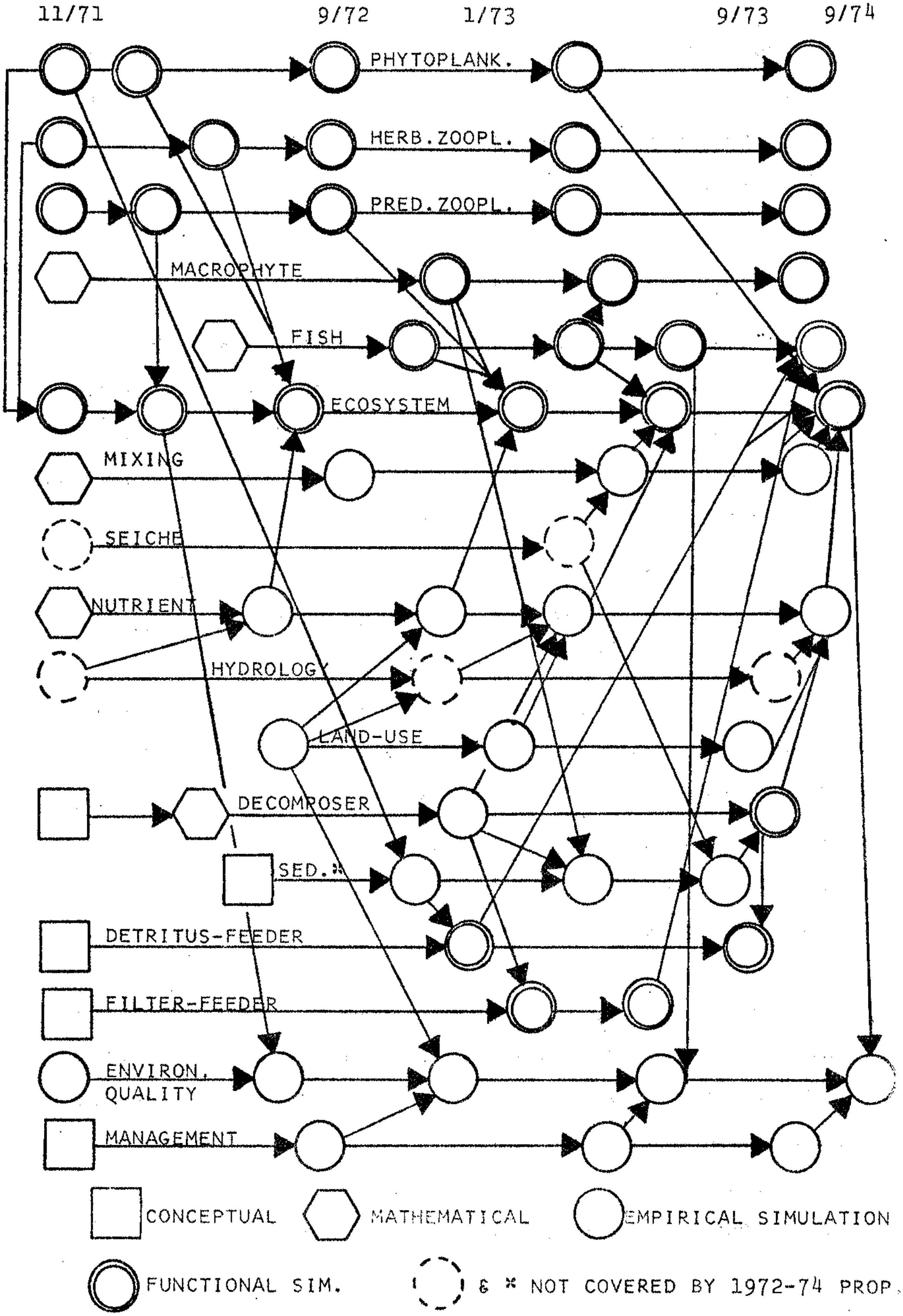

Figure 1 - Lake George Modeling Flowchart.

Predatory Zooplankton Model. This model (Park and Wilkinson, 1971) is operational, but has yet to be calibrated with Lake George data; however, these data (collected by E. J. LaRow) are in the Lake George data management system, and the calibration will be performed shortly. The model will then be converted to a population-dynamic model.

Macrophyte Model. The mathematical formulation for this model is available (see "MACROPHYTE MODEL"). Further development is dependent on data for determination of the model constructs.

Fish Model. The Lake Wingra model will be adapted as soon as it is available in functional form. A proposal to collect fish data at Lake George is pending; the data will be used to modify and validate the model so that it will be applicable to an oligotrophic lake.

Aquatic Productivity Model. The various biologic and physical-chemical models are being linked so as to obtain a comprehensive predictive model of the ecosystem. At this time only the phytoplankton and herbivorous zooplankton models have been linked (see "PHYTOPLANKTON - HERBIVOROUS ZOOPLANKTON MODEL"). Additional models will be added as they are validated; with the addition of the vertical mixing model, the overall model will integrate over depth.

Circulation Model. The mathematical formulation for this model is discussed elsewhere (see "VERTICAL MIXING MODEL"). The model incorporates the phytoplankton model and generalizes it for depth. Presently the model is awaiting the development of a construct to represent the combined mixing processes. Horizontal transport eventually will be incorporated into the comprehensive ecosystem model, probably as an empirical model with stochastic elements. A seiche model has been developed as a part of the physical limnologic study (Stewart, 1971).

Nutrient Model. A nutrient budget model was developed as a part of the lead-time study (Park and Wilkinson, 1970); however, it contained several nude procedures due to lack of information. Since then the model has been reformulated in more elegant mathematical form (Park and Wilkinson, 1971).

Further development of a generalized model, capable of simulating perturbations within the drainage basin, is dependent on data presently being collected by a study of the Lake George Village sewage treatment facility and a study of the Hulett's Landing community leach field. Representation of the dynamics of sewage treatment is very important because sewage effluents account for about 25% of the nitrogen and about 75% of the phosphorus added to the lake each year (D. B. Aulenbach, personal communication, 1971).

Hydrology Model. Development of a water budget model and incorporation of simulations using the Lake Wingra group's hydrology transport model are well advanced (Colon, 1971). The modeling group gave programming assistance to this research.

Decomposer Model. A conceptual model was developed in close cooperation with L. S. Clesceri (see "DECOMPOSER MODELING"). Formulation of a mathematical model will begin shortly; however, due to the lack of prior published research, the determination of model constructs will be a slow process. Model development will have to proceed at the same rate as the basic research. Therefore, it is anticipated that the coluble organic terms of the model will be determined first, followed by determination of the sediment terms. This latter stage of the model development will be aided by the availability of empirical data on nutrient regeneration from from a study proposed by S. Kobayashi.

Sedimentation Model. Although it has not been formally proposed, development of an organic sedimentation model would be helpful in the modeling of both decomposition and benthic organisms. Such a model could be driven primarily by symbolic water turbulence as a function of depth, perhaps with a component representative of seiche nodal currents. Field data on organic carbon sediments are available from a completed IBP study (Friedman and Schoettle, 1971).

Detritus-feeder and Filter-feeder Models. A proposal to study benthic organisms at Lake George is pending. In anticipation of this study, conceptual models have already been developed. A reasonable approach seems to be to divide the benthos into detritus feeders and filter feeders. Both groups are quite amenable to modeling, so that the main requirement will be for field data to validate the models.

Management Models. One of the most important contributions of the Lake George study is a quantified documentation of the impact of Man on an aquatic ecosystem. Lake George is well suited to this type of study because it is just beginning to feel the effects of the impact. It is still possible to find examples of both "before" and "after" and to observe the progression of cultural eutrophication.

However, the IBP objectives go beyond mere description. Ecosystem simulation models, such as those being developed at Lake George, permit symbolic experimentation or perturbation analysis. In this way one can produce generalized scenarios for given management decisions. Furthermore, given an objective function, optimization techniques can be used to find an optimal solution.

The population-pollution model (Stern, 1971; Park and Wilkinson, 1971), developed in close cooperation with the Lake George modeling group, is an example of the incorporation of pertinent models into a macromodel capable of forecasting the impact of Man on Lake George. Additional macromodels will be developed and refined as the modeling proceeds.

Preliminary management modeling at Lake George has been concerned with the formulation of optimization models for placement of sewage treatment facilities (see "ENVIRONMENTAL MANAGEMENT MODELING FOR LAKE GEORGE BASIN"). Although the formulations are of general application, the research is quite timely because of the concern of the Lake George citizenry. Example sets were prepared as a part of the study, but were not included in the attached report because of space limitations. Mathematical programming is a rapidly developing field with great potential application to environmental problem solving. One of the Lake George modeling objectives is to demonstrate the usefulness of this approach in management models that are closely tied to specific environmental macromodels.

Data Management and Analysis

As described by Park and Wilkinson (1971), a system for storage, retrieval, updating and analysis of Lake George data has been developed on the Rensselaer Polytechnic Institute IBM 360 model 50 computer. Many transformation routines commonly desried for preparing the raw data for analysis have been written and loaded into the system. These routines, as well as the full system library, are callable for assistance in the analysis and manipulation of the data. Although considerable refinement is needed, and is in process, the system is operational. With completion of the procedures manual, the individual investigators will have the capability of accessing the system from remote terminals by means of time-sharing and remote-job-entry, using the ALPHA system.

The library on the Rensselaer computer has been expanded considerably during the past year, primarily through the efforts of the modeling group. The library now includes procedures for cluster, ordination, and trend-surface analysis, as well as computing packages BMD, MPS, SSP, SPSS, OMNITAB, and SYMAP. This capability will enhance the role of consultation with the investigators with respect to data collection, organization, and analysis.

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PHYTOPLANKTON-HERBIVOROUS ZOOPLANKTON MODEL

by

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and

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Introduction

A combined phytoplankton-herbivorous zooplankton model has been calibrated for diatom and zooplankton concentrations at 9 m depth. The model appears to have a sound biologic basis and yields a very close fit to the observed data used in the calibration. Preliminary validation and sensitivity analysis suggest that the model constitutes a powerful tool for both understanding and predicting ecosystem dynamics.

Development of the model began with experimentation with the classical Riley model (Park and Wilkinson, 1970) and gained considerable impetus with the adaptation of the Manhattan College group's model (DiToro, O'Connor, and Thomann, 1970). Experimentation with a slightly modified Manhattan model, using Lake George data from the Fall, 1970 period, was inconclusive in that only a downward trend for phytoplankton and zooplankton was obtained (Park and Wilkinson, 1971).

Since then the model has been examined in detail, using as input values data from Lake George for all of 1970. The original model assumed linear temperature relationships - a simplifying assumption that is invalid for Lake George where very low winter temperatures are attained. Therefore, the temperature terms were modified in accordance with the Van't Hoff equation (Belehradek, 1935) in order to correct the underestimates of biological reaction rates at low temperatures.

In calibrating the model, so as to obtain a good fit between the predicted and observed values for diatom and zooplankton concentrations, the phytoplankton growth and respiration and zooplankton respiration coefficients were changed within the ranges reported in the literature. The validity of these constructs will be determined using data that are now becoming available from the ongoing primary and secondary productivity studies at Lake George.

Mathematical Model

The mathematical models of phytoplankton and herbivorous zooplankton have been discussed previously (Park and Wilkinson, 1971). However, since modifications have been made, updated model equations and notations are included in the Appendix of this report.

The underestimation of reaction rates at low temperature was detected when experimenting with the Manhattan model in which

$$R = C_1 * T \qquad (1$$

where R is the reaction rate, C_1 is the rate constant, and T is temperature in $^{\circ}$C. The linear temperature relationship worked well in the original study because the water temperature in the San Joaquin River ranges approximately from 9°C to 28°C; however, values for Lake George range from just greater than 0°C to 24°C. Therefore, the simplifying assumption of linear temperature relationships appeared to be unjustified for Lake George.

In general, the effect of temperature on chemical reactions can be described by the Van't Hoff equation

$$Q_{10} = \left(\frac{K_2}{K_1} \right)^{\frac{10}{T_2 - T_1}} ; \qquad (2$$

the instantaneous rate of a chemical reaction at any temperature can be approximated by

$$R = C_2 * Q_{10}^{\frac{T}{10}} \qquad (3$$

where R is the reaction rate, Q_{10} is the Van't Hoff temperature coefficient, T is temperature in °C, K_1 and K_2 are constants, and C_2 is the reaction constant.

Exogenous Variables

To validate the model it is necessary to have values for exogenous variables. For purposes of validation, processed field data are used as input values for solar radiation and nutrients; however, values for temperature are calculated from an equation (see Eq. A-1.6) derived from field data of 1969-70.

Incident Solar Radiation

The solar radiation data show great variations, precluding any deterministic prediction based on day of year. As an alternative, weekly average values are being used at the present time. These weekly averages seem to provide a fairly stable seasonal pattern (Figure 1). An empirical predictive model has been developed in conjunction with the hydrology study; this model currently gives monthly averages, but will be modified to give weekly and daily values based on a stochastic procedure.

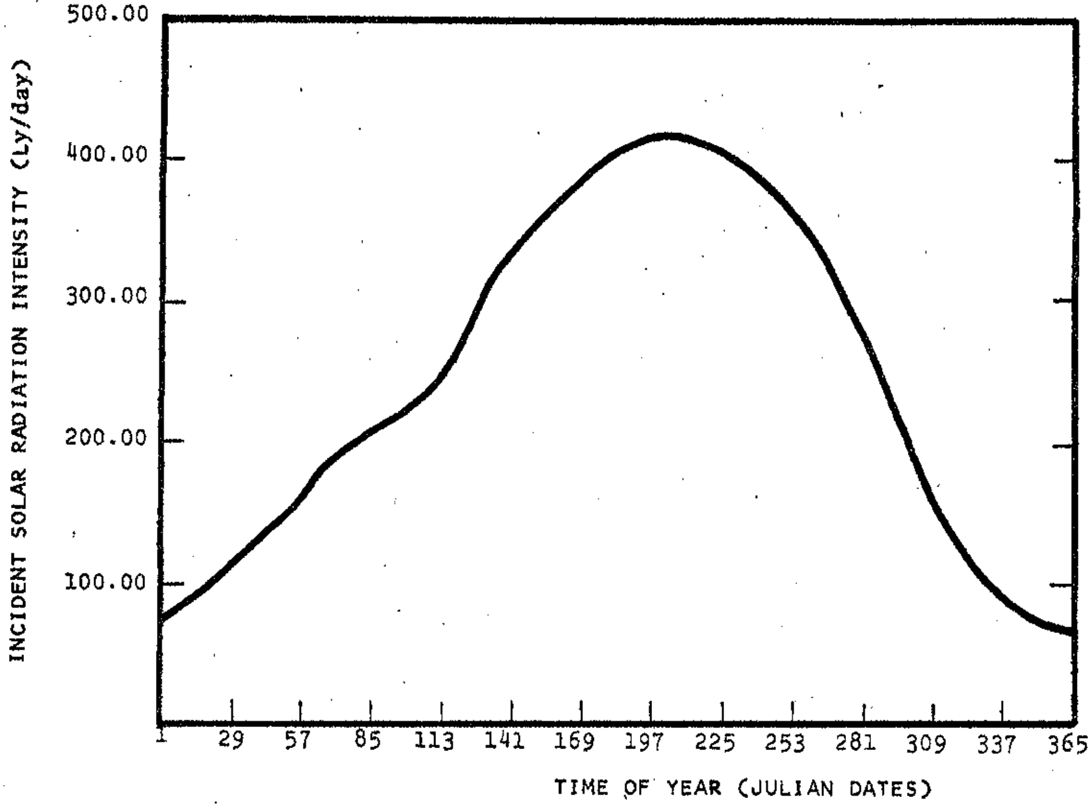

Figure 1 - Incident Solar Radiation Intensity at Water Surface (Weekly Average Value), Station 1, Lake George, N.Y. 1970. (Colon, unpublished data).

Nutrients

The nutrients considered for the phytoplankton model were orthophosphate, soluble ammonia, soluble nitrate, and soluble silica. Only phosphate and nitrate data are being used for validation presently. It is assumed that only 10% of the total phosphate is "active" (orthophosphate) and that the Michaelis-Menten constant for "active" phosphate is approximately equal to 1 μg-P/L (communication with Stross, primary productivity investigator, Lake George site). The Michaelis-Menten constant used for nitrate is 5 μg-N/L. These parameter values appear consistent with reported values from the literature as well as primary productivity studies in Lake George. The values of phosphate and soluble nitrate are shown in Figure 2.

Temperature

The derivation of the temperature equation has been discussed (Park and Wilkinson, 1971). Despite the fact that the prediction curve peaks a little too soon and its values are a little too low, the equation appears adequate to be used as a driving variable in the validation. Figure 3 shows the historic data and the prediction curve. The empirical temperature model is now operable for all depths and stations at Lake George.

Rate Constants and Coefficients

Warburg (1919) estimated a range for the Van't Hoff temperature coefficient, Q_{10}, for photosynthesis at high light intensity from 1.6 to 4.3. Presently work is being carried out to determine Q_{10}'s for both phytoplankton and zooplankton in Lake George. As a reasonable approximation, values for Van't Hoff temperature coefficients for phytoplankton growth, Q_{10}, phytoplankton respira-

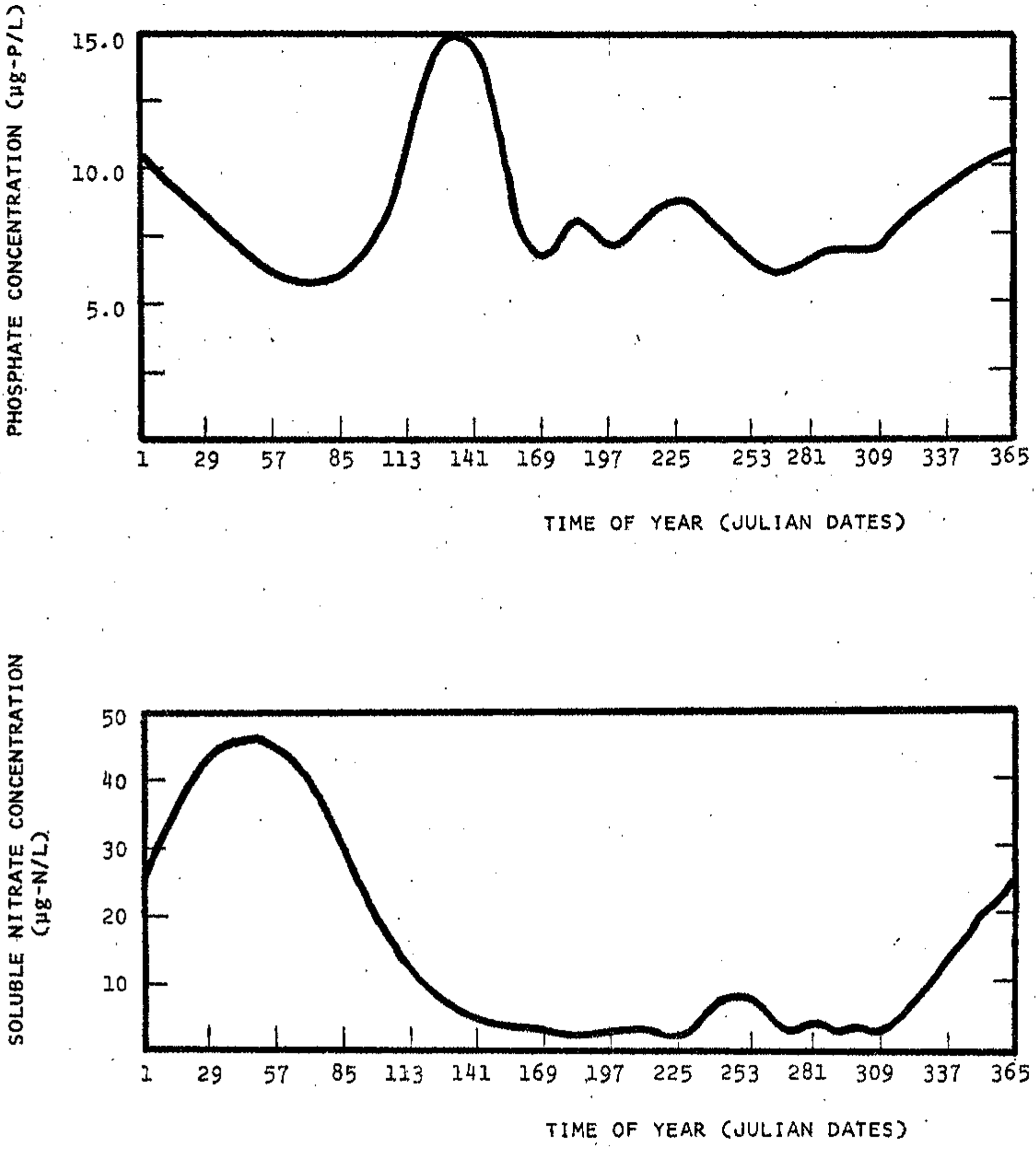

Figure 2 - Concentrations of phosphate and soluble nitrate, Station 1, Lake George, N.Y. 1970. (Williams, unpublished data).

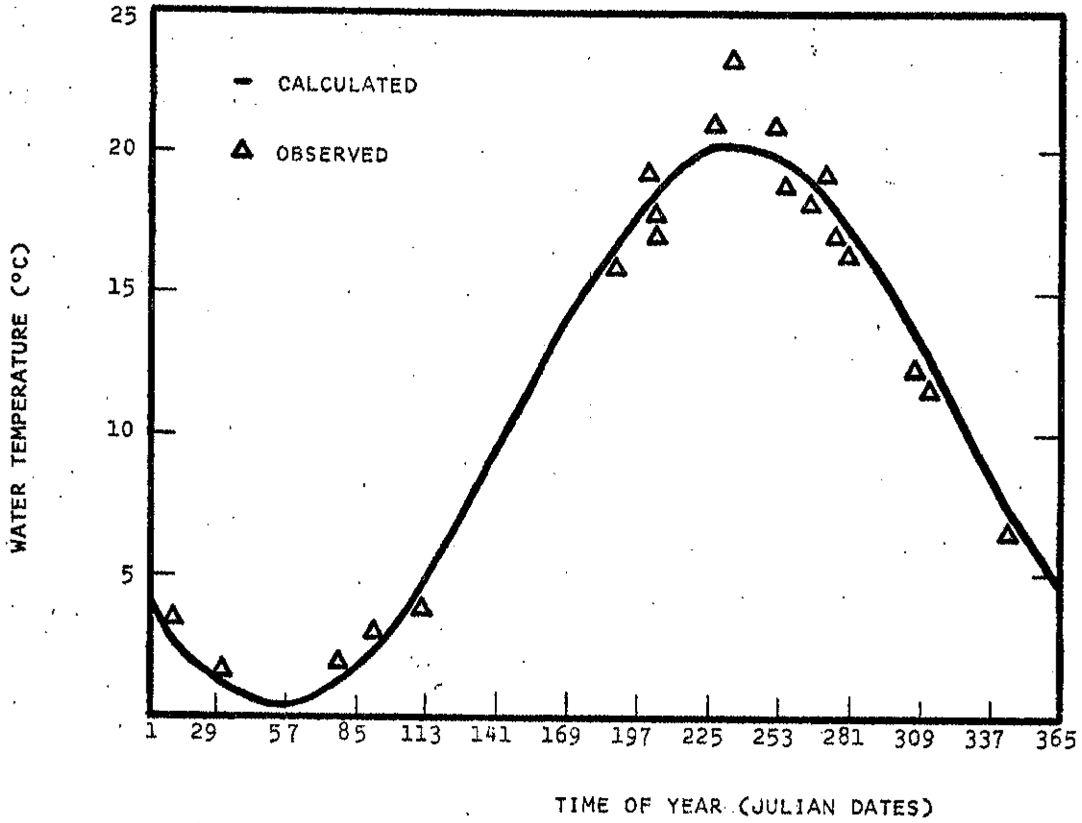

Figure 3 - Water temperature profile, calculated vs. observed, at 9 m, Station 1, Lake George, N.Y. 1970. (Williams, unpublished data).

tion, Q_{po_2}, and zooplankton respiration, Q_{hzo_2}, are assumed to be equal to 2.0.

A value of 0.22 in m^{-1} is used for the extinction coefficient, K_e. The value is adopted from Stross (1970); it is based on the mean for the water column from 2.0 meters to a depth of 1% surface intensity, in Lake George. If sensitivity analysis indicates that the model is very sensitive to light, then this coefficient can be made variable; it can be considered a function of turbidity due to phytoplankton (model feedback) and suspended clastic sediments.

The phytoplankton saturated growth rate constant, K_{sgp}, and the herbivorous zooplankton grazing rate constant, g_{hzg}, are approximately 2.5, and 2.0 in day^{-1} for K_{sgp}, and g_{hzg}, respectivel
These may or may not be valid and need to be evaluated in light of data now becoming available at Lake George.

Numerical values for all constants and parameters used for the simulation are summarized in Table 1. Part of these values were obtained from site investigators and part from literature (Burns, 1969; DiToro et. al., 1970; MacIsaac and Dugdale, 1969).

Numerical Results and Discussion

In order to obtain a solution for the model, the Runge-Kutta Method of numerical integration was used. Numerical values which have been discussed in the previous section are used as input for model simulation. Numerical results are presented in Table 2 and Figure 4. The model is now operable in both CPS (time-sharing) PL/1 and batch-mode FORTRAN IV.

TABLE 1

Numerical Values Used for Validation

Constants or Parameters	Values	Units
t_o	1	day
t_f	365	day
K_{sgp}	2.55	day^{-1}
Q_{10}	2.0	
Q_{po_2}	2.0	
Q_{hzo_2}	2.0	
K_e	0.22	m^{-1}
I_s	300	LY/day
K_{mnp}	.001	mg-P/L
K_{mnn}	.005	mg-N/L
K_{rp}	.025	day^{-1}
g_{hzg}	2.0	L/mg-C-day
K_{sp}	.008	day^{-1}-$^oC^{-1}$
dt	7	day
D	9	m
A_{hzp}	.75	
f_{hz}	2.0	L/mg-C-day
K_{mp}	.9	mg-C/L
K_{hzr}	.04	day^{-1}
$L_{hzg} + S_{hz}$	.025	day^{-1}
p_o	.04	mg-C/L
Z_{h_o}	.01	mg-C/L

TABLE 2

Numerical Results: Validation of Phytoplankton-Herbivorous Zooplankton Model, at 9 m, Station I, Lake George, N.Y. 1970

DAY	TM	NP	I_o	NN	P	Z
1	4.147	.00106	72.00	.0250000	.0400000	.0100000
15	2.499	.00094	88.08	.0340000	.0451466	.0097116
29	1.297	.00082	112.16	.0440000	.0543241	.0097051
43	.613	.00071	135.76	.0460000	.0684994	.0100606
57	.485	.00062	161.28	.0450000	.0895623	.0109342
71	.920	.00059	192.24	.0400000	.123294	.0126813
85	1.895	.00062	206.00	.0310000	.177703	.0161727
99	3.352	.00074	218.72	.0210000	.273141	.0237438
113	5.207	.00107	238.32	.0120000	.456968	.0432782
127	7.353	.00147	304.96	.0070000	.845294	.109636
141	9.665	.00146	337.12	.0040000	1.035840	.377923
155	12.011	.00102	356.56	.0030000	.159870	.842426
169	14.255	.00066	385.04	.0030000	.161469	.537744
183	16.267	.00081	404.80	.0020000	.0513492	.510734
197	17.930	.00070	414.32	.0030000	.0156153	.403374
211	19.149	.00080	414.64	.0030000	.0123796	.298518
225	19.852	.00089	406.00	.0010000	.0088009	.215101
239	20.001	.00079	386.56	.0070000	.0092880	.152146
253	19.585	.00068	364.32	.0080000	.0263034	.110694
267	18.629	.00060	336.48	.0030000	.0509625	.0874138
281	17.188	.00067	280.80	.0040000	.0544099	.0730609
295	15.345	.00070	232.64	.0030000	.0608135	.0640081
309	13.208	.00073	168.24	.0030000	.0545132	.0578107
323	10.899	.00084	120.24	.0070000	.0482514	.0528195
337	8.551	.00093	90.80	.0130000	.0458323	.0490957
351	6.302	.00100	78.00	.0200000	.0430136	.0463771

TM = Water temperature (°C)

NP = "Active" phosphate (mg-P/L)

I_o = Incident solar radiation intensity at water surface (LY/day)

NN = Soluble nitrate (mg-N/L)

P = Phytoplankton concentration (mg-C/L)

Z = Herbivorous zooplankton concentration (mg-C/L)

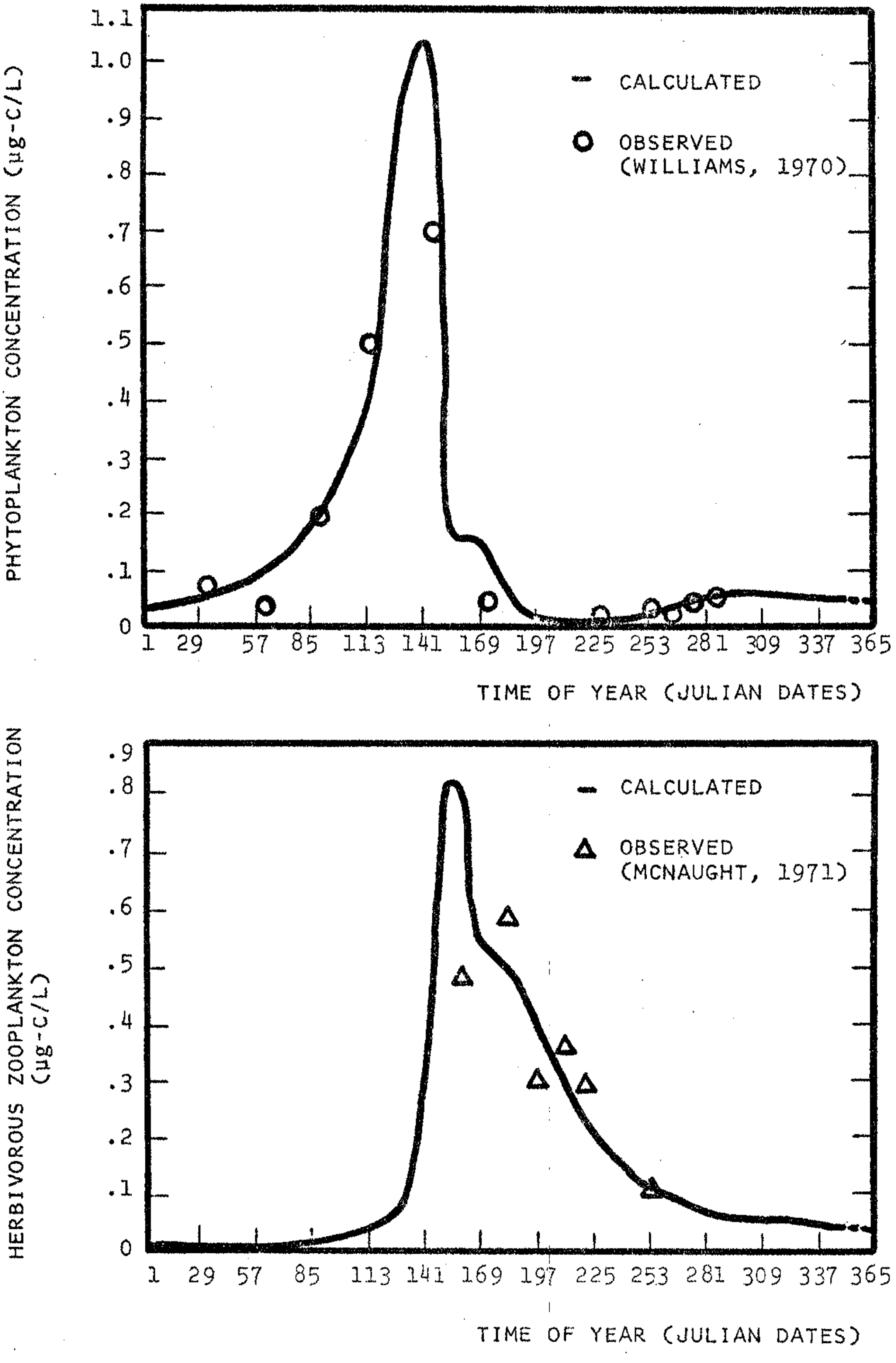

Figure 4 - Concentrations of phytoplankton and herbivorous zooplankton. Calculated vs. observed data, at 9 m, Station 1, Lake George, N.Y. 1970.

It can be seen from examination of Table 2 and Figure 4 that the solution of the model exhibits a good fit to the available diatom and herbivorous zooplankton data. The phytoplankton profile peaks at day 141 (late May), drops to a minimum between days 197 and 239 (July-August), and has a small peak at day 295 (late October). The zooplankton profile shows a lag of about two weeks; it peaks at day 155 (early June) and recedes gradually for the remainder of the year. In constrast, the phosphate maximum occurs at day 120 (early May), the nitrate maximum appears at day 40 (February), the temperature peak occurs at day 200 (mid-July), and the solar radiation maximum occurs at day 180 (early July).

The combined phytoplankton-herbivorous zooplankton model in its present state not only accounts for variations in the observed data, but also follows theoretical physiological and ecological relationships previously proposed for the coupled phytoplankton-zooplankton system.

Validation and Sensitivity Analysis

Content validity of the model appears to be good. Construct validity needs to be established more rigorously using detailed data from the process studies and using objective parameter identification techniques. Concurrent validity is presently being determined for 1) depths other than 9 m, 2) stations other than Station 1, 3) year 1969. Predictive validity can soon be tested using 1971 data.

Sensitivity analysis of the model is presently under way. It appears that nutrients, rather than solar radiation and water temperature, are more responsible for the spring peak in the phytoplankton population. The sudden decrease of phytoplankton between day 141 and 197 (late May to mid-June) is primarily due to the increase in the herbivorous zooplankton population, and secondarily to the decrease in the nutrients. Hopefully the relative contributions of the exogenous variables can be determined with sufficient precision to guide further data collection and model simplification or elaboration.

Model Extensions

The model is primarily a biomass or standing crop model because that is the most expedient form for incorporation in an ecosystem macromodel. However, it can also be considered as a model for predicting productivity, which is of particular interest because of the ongoing research on temporal and spatial productivity patterns in Lake George.

The model in its present form does not yield a good fit to the observed phytoplankton net productivity in Lake George (see Stross, 1971 final report). The model has been calibrated for diatoms, which account for the spring bloom; in so doing, the predicted fall bloom, which is principally composed of chrysophytes (sensu stricto), has been suppressed. A calibration for all phytoplankton, including flagellates and blue-green algae, is being attempted; however, it is unreasonable to suppose that precision similar to that already attained with diatoms can be attained by lumping together all phytoplankton.

It seems that the best strategy for phytoplankton modeling would be to disaggregate the present model so that each of the major bionomic groups of phytoplankton can be treated separately. In that way varying Michaelis-Menten nutrient constants, intrinsic growth rates, temperature coefficients, sinking rates, and grazing rates can be used. This is of particular interest in that Lake George is representative of oligotrophic lakes which fluctuate between diatom and chrysophyte populations. However, in order to achieve true generality, the model also should have the capability of simulating blue-green algae blooms, thus bridging the critical stage in eutrophication. Hopefully a preliminary disaggregation can be achieved this year, although a realistic model will not be possible until standing crop data are available for each major taxonomic group.

The time-step of the model can be shortened so that daily synchromy among the phytoplankton groups can be simulated. This is not feasible for long runs of the ecosystem macromodel. However, this type of specialized experimentation with individual models is readily accomplished; this promises to yield valuable insight into the dynamics of the ecosystem components and is further justification for the modeling approach.

Conversion of the model from a point model, capable of simulating a given depth at a given station, to a "profile" model, capable of integrating over depth, is dependent on development of a realistic mixing model. Research on this is in progress and the formulation is encouraging (see "MIXING MODEL"). However, much work remains before the results of the physical limnologic studies are fully incorporated into an operational biologic model.

Appendix

Phytoplankton Model

1. Model Equation

$$dP/dt = (G_p - R_p - L_{hzg} - S_p) * P \qquad \text{(A-1)}$$

where

$$\begin{aligned} G_p + K_{sgp}&*(Q_{10}**(T_m/10))*(2.718 * F_p/(K_e*D) * \\ &(EXP(-A_1)-EXP(-A_o)))*(N_p/K_{mnp}+N_p))* \\ &(N_n/(K_{mnn} + N_n)) \end{aligned} \qquad \text{(A-1.1)}$$

$$R_p = K_{rp} * (Q_{po_2} ** (T_m/10)) \qquad \text{(A-1.2)}$$

$$L_{hzg} = g_{hzg} * Z_h \qquad \text{(A-1.3)}$$

$$S_p = K_{sp} * T_m \qquad \text{(A-1.4)}$$

$$F_p = .04167 * (12.1 + 3.15 * SIN(.017262*(t-81)) \qquad \text{(A-1.5)}$$

$$T_m = 10.24 - 5.96 * COS(2\Pi t/365) - 7.57*SIN(2\Pi t/365) \qquad \text{(A-1.6)}$$

$$A_o = I_1/I_s \qquad \text{(A-1.7)}$$

$$A_1 = A_o * EXP(-K_e*D) \qquad \text{(A-1.8)}$$

2. Notation

P = phytoplankton concentration (mg-C/L)

G_p = phytoplankton growth rate (day^{-1})

R_p = phytoplankton endogenous respiration rate (day^{-1})

L_{hzg} = phytoplankton loss rate due to grazing by herbivorous zooplankton (day^{-1})

S_p = phytoplankton sinking rate (day^{-1})

t = time of year (day)

K_{sgp} = phytoplankton saturated growth rate constant (day^{-1})

Q_{10} = Van't Hoff temperature coefficient for phytoplankton growth (unitless)

T_m = water temperature (^oC)

F_p = photoperiod (day)

K_e = extinction coefficient (m^{-1})

D = chosen depth in euphotic zone (m)

I_o = incident solar radiation intensity at water surface (Ly/day)

N_p = nutrient concentration (total active phosphate) (mg-P/L)

K_{mnp} = Michaelis-Menten constant for orthophosphate (mg-P/L)

N_n = nutrient concentration (total soluble nitrate) (mg-N/L)

K_{mnn} = Michaelis-Menten constant for nitrate (mg-N/L)

K_{rp} = phytoplankton endogenous respiration rate constant (day^{-1})

Q_{po_2} = Van't Hoff respiration coefficient for phytoplankton (unitless)

g_{hzg} = grazing rate constant of herbivorous zooplankton (L/mg-C-day)

Z_h = herbivorous zooplankton concentration (mg-C/L)

K_{sp} = phytoplankton sinking rate constant (day^{-1}-$^oC^{-1}$)

Herbivorous Zooplankton Model

1. Model Equation

$$dZ_h/dt = (G_{hz} - R_{hz} - L_{pzg} - S_{hz}) * Z_h \tag{A-2}$$

where

$$G_{hz} = A_{hzp} * f_{hz} * K_{mp} * (P/(K_{mp}+P)) \tag{A-2.1}$$

$$R_{hz} = K_{rhz} * (Q_{hzo_2} ** (T_m/10)) \tag{A-2.2}$$

$$L_{czg} = g_{pzg} * Z_p \tag{A-2.3}$$

2. Notation

Z_h = herbivorous zooplankton concentration (mg-C/L)

G_{hz} = herbivorous zooplankton growth rate (day^{-1})

R_{hz} = herbivorous zooplankton respiration rate (day^{-1})

L_{pzg} = herbivorous zooplankton loss rate due to grazing by predatory zooplankton (day^{-1})

S_{hz} = herbivorous zooplankton loss rate due to nromal mortality (day^{-1})

t = time of year (day)

A_{hzp} = biomass conversion factor between herbivorous zooplankton and phytoplankton (mg-C/mg-C)

f_{hz} = filtering rate of herbivorous zooplankton (L/mg-C-day)

K_{mp} = Michaelis-Menten constant for phytoplankton (mg-C/L)

P = phytoplankton concentration (mg-C/L)

K_{rhz} = herbivorous zooplankton respiration rate constant (day^{-1})

Q_{hzo_2} = Van't Hoff respiration coefficient for herbivorous zooplankton (unitless)

T_m = water temperature (^{o}C)

G_{pzg} = grazing rate constant of predatory zooplankton (L/mg-C-day)

Z_p = predatory zooplankton concentration (mg-C/L)

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MACROPHYTE MODEL

by

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and

Irving K. Hwang

The macrophyte model for Lake George was developed from the Lake Wingra group's seasonal macrophyte model, the Lake George phytoplankton model (Park and Wilkinson, 1971), and from Westlake (1966). Its general form is as follows:

$$\frac{dM}{dt} = (G_m - R_m - L)*M \tag{1}$$

where: M = Macrophyte Biomass
t = Time (days)
G_m = Photosynthetic Growth Rate (days^{-1})
R_m = Loss rate due to respiration (days^{-1}).

The photosynthetic growth rate, G_m is a function of water temperature, incident radiation, nutrient concentrations in the bottom water and current action:

$$G_m = \mu * \left(\frac{2.718}{K_e \cdot D} \right) * \left[e^{-\left(\frac{I_o}{I_s}\right) e^{-K_e D}} - e^{-\left(\frac{I_o}{I_s}\right)} \right] * Q_{10}^{\frac{T_m}{10}} * \prod_{i=1}^{n} \left(\frac{N_i}{K_{mni} + N_i} \right)$$

where: F_p = Photoperiod (day^{-1})
K_e = Extinction Coefficient (meter^{-1})
D = Depth (meter)
I_s = Saturation Solar Radiation Intensity (Langley-Day^{-1})
I_o = Incident Solar Radiation Intensity at Water Surface (Langley-Day^{-1})
Q_{10} = Van't Hoff Temperature Coefficient for Photosynthesis

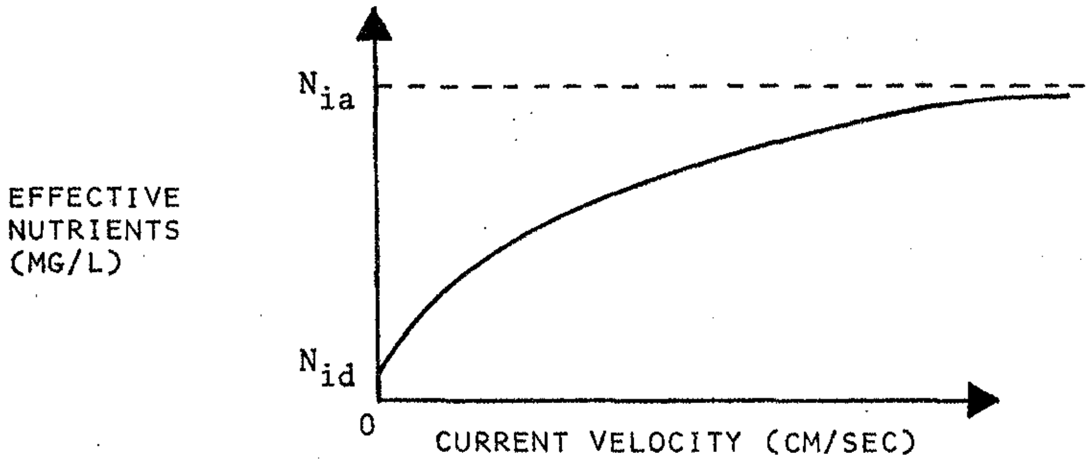
T_m = Water Temperature (^{o}C)

N_i = Effective Nutrient Concentration in Water (mg-liter^{-1})

K_{mni}= Michaelis Menten Constant for Nutrient (mg-liter^{-1})

μ = Saturated Growth Rate (Day^{-1})

The effective nutrient concentration N_i is a function of current velocity,V, and actual nutrient concentration outside of the macrophyte beds, N_{ia}. Sculthorpe (1967) cites studies on Potamogeton and Ranunculus in which significantly higher rates of photosynthesis were attained in flowing water. Preliminary studies at Lake George produced similar results (R. Stross, personal communication, 1971). The hypothetical relationship between current velocity and effective nutrient concentration is shown below



The effective nutrient concentration can be approximated by:

$$N_i = N_{ia*} \left(1 - e^{-k_1 v} \right) + N_{id} \qquad (3)$$

where: k_1 is a constant,

N_{id} = Nutrient conentration with only diffusion mixing (mg-liter^{-}

The saturated growth rate,μ, may be considered as a state variable related to the time of day, and the age of the plants, and amount of self-shading.

The nutrients to be considered may be classified into four categories:

1) Macronutrients (P,N,...)
2) Trace nutrients (S_i,F_e,Z_n,...)
3) Dissolved gases (O_2,CO_2)
4) Auxiliary growth factors

The respiration term, R_m is a function of water temperature:

$$R_m = k_r \, Q_{O_2m}^{\frac{T}{10}} \tag{4}$$

Q_{O_2m} = Van't Hoff Temperature coefficient for respiration
T_m = Water temperature (°C)
K_r - Respiration coefficient (Day^{-1})

The loss due to grazing, L can be considered as a function of consumer biomass and grazing rate.

It is anticipated that during 1972, more data will become available from ongoing research at Lake George and Lake Wingra, Wisconsin; these data will be used to quantify the various coefficient in the model and develop μ as a state variable.

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DECOMPOSER MODELING*

by

Richard A. Park

and

Lenore S. Clesceri

The first step in the Lake George decomposer model development has been to construct a conceptual model (Fig. 1). This is expressed graphically as compartments, representing nutrient pools, and biomass, connected by arrows, representing transfers. This phase of modeling has been dependent on a high degree of interaction between the modeling team and the principal investigator for the decomposer system (L.S.C.). The model has not yet been expressed in terms of mathematical equations and computer logic; however, the development of a conceptual model represents a major step in that it has served to formalize our ideas concerning the more important components of the system and has given direction to the data collection effort.

The Lake George model takes into consideration the various forms of organic inputs which are either quantitatively important or which are of special interest (such as pesticides). The model also considers several classes of organic material derived from higher organisms in the system. The enumeration of all these forms of organics is thought to be important because of differential degradation rates, especially under the variety of environmental conditions which may be obtained as eutrophication proceeds. In order to achieve both resolution and robustness (generality), the decomposers are subdivided into cellulolytic, other hydrolytic, and non-hydrolytic groups, each of which may be either aerobic or anaerobic.

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*Presentation given at IBP Eastern Deciduous Forest Biome Decompositio Workshop, Raleigh, North Carolina, February 19 and 20, 1971.

The decomposer model will be treated as a submodel in the overall ecosystem simulation. Like the other submodels, this will eventually be expressed as a series of mathematical equations and will then be programmed as an interactive computer simulation model using time-sharing PL/1. It will simulate ecosystem dynamics for each of a number of locations in Lake George, each representing a segment of the lake with its associated drainage basin.

At this time estimates are needed for each of the transfers indicated in the model. These will be taken from the literature where possible; however, we anticipate that most of the relationships, including an assessment of important controlling factors, will have to be determined as a part of the continuing Lake George decomposition study.

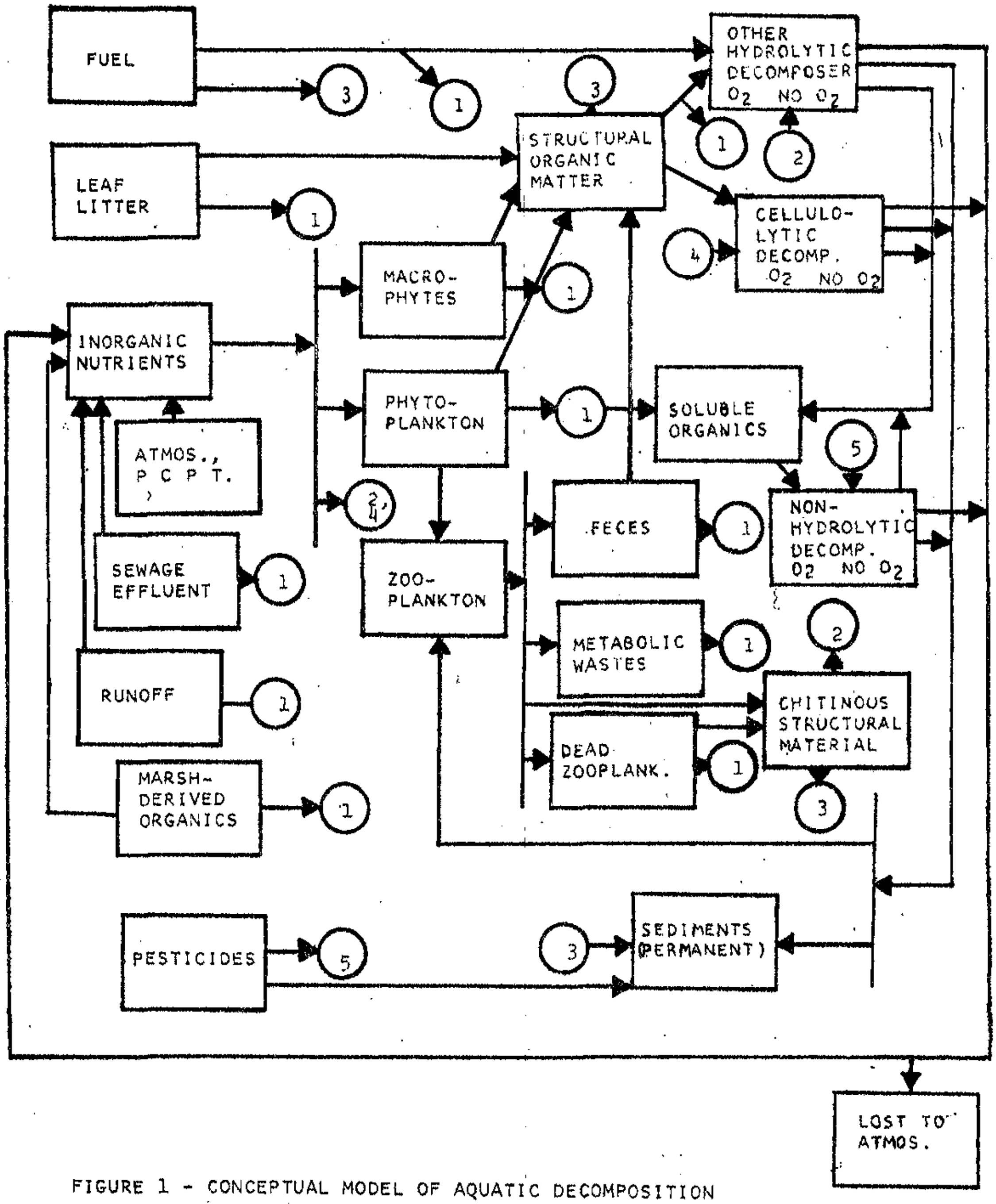

FIGURE 1 - CONCEPTUAL MODEL OF AQUATIC DECOMPOSITION

VERTICAL MIXING MODEL

by

Mitchell Silver

and

Samuel Katz

Introduction

A mathematical model of vertical mixing of phytoplankton has been developed so that the phytoplankton point model can be converted into a "profile" model. The model is based on one presented by Bella (1970), with two major differences: we use a mass-transport term based on wind mixing instead of conduction (which seems to be an oversimplification), and we incorporate the terms of the Lake George phytoplankton model (see section on "Phytoplankton-Herbivorous Zooplankton Model").

Many factors contribute to lake mixing; however, only a few factors are of importance. Therefore, it is possible to make a number of simplifying assumptions without decreasing the predictive value of the resulting model. We can assume that all mixing significant to phytoplankton production occurs in the epilimnion. This assumption appears to hold for most situations, although in areas affected by large seiche nodes, such as the Lake George Narrows (see Stewart, 1971) there may be important mixing at depth.

We also assume that most mixing in the epilimnion is a direct or indirect effect of wind, as shown by several cognate Lake George studies (Scott and Stewart, 1968; Stewart, 1971). Specific mixing processes in the epilimnion include Langmuir circulation, seiches, and wind-induced vertical turbulent diffusion. Langmuir circulation requires a minimum wind velocity of approximately 3m/sec. and certain thermal conditions, seiches require winds of certain minimum duration, and vertical turbulence due to wind shear is of questionable importance (Scott and Stewart, 1968; Stewart, 1971). At this time we are developing an empirical model that combines the effects of these processes. If this proves to be an oversimplification, then the mixing construct will be disaggregated.

The deepening of the epilimnion is correlated with the action of wind over time. However, under certain conditions, particularly prevalent in the vicinity of Lake George Station 1 (see Park and Wilkinson, 1971, Fig. 5), the upper part of the epilimnion may be stripped off by a prevailing wind and the thermocline is then brought closer to the surface. These effects can be simulated so that the depth of the epilimnion can be predicted. In this way, for ease in modeling, we can readily subdivide the water column into two separate mixing regimes, epilimnion and thermocline-hypolimnion. Turnover can be considered as an effect of mixing when all the water column is included in the epilimnion (no thermocline-hypolimnion).

Another simplifying assumption would be that phytoplankton sinking, growth, and grazing loss rates are independent of depth. If this were true, the model would be much simpler. However, sinking is temperature dependent, growth is dependent on solar radiation, and grazing is dependent on the vertical dispersion of zooplankton; all three processes vary with depth-related factors. Therefore, we are presently developing the capability of modeling these effects. If sensitivity analysis indicates that one or more factors are unimportant operationally, we then will have an objective rationale for simplifying the model.

Mathematical Model

$$\frac{\alpha P}{\alpha t} = \frac{1}{A} \quad \frac{\alpha(D_z A\frac{\alpha P}{\alpha z})}{\alpha z} - \frac{\alpha(S_p AP)}{\alpha z} + \frac{\alpha[AP(G-L_{hzs})]}{\alpha z}$$

where

A = horizontal area

P = phytoplankton concentration

t = time

D_z = empirical coefficient combining the effects of wind-induced mixing produced by Langmuir circulation, seiches, and turbulent vertical diffusion

z = depth

S_p = phytoplankton sinking rate

G = phytoplankton growth rate

L_{hzs} = phytoplankton loss rate due to grazing by herbivorous zooplankton.

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ENVIRONMENTAL MANAGEMENT MODELING FOR LAKE GEORGE BASIN

William A. Wallace, John P. Weyant
and Frank DiCesare

Research activities in this component of the program have focused on (1) the development of a data base containing information on the socio-economic characteristics of the Lake George Basin and (2) formulation of optimization models that seek to prescribe ways in which Man should manage his impact on the lake basin.

Descriptive Approaches

As previously noted (Park and Wilkinson, 1971) tapes containing the data obtained from the Land Use and Natural Resource Inventory (LUNR), an aerial survey conducted by New York State, are being made operational. In addition, 1970 census data for New York State were obtained; the population data for the counties that contain the Lake George basin were processed as a test (see Table I).

The major thrust of the work to date has focused on putting the LUNR data in useful form. It consists of:

1. two tapes which can be transferred to a 2314 disk pack by an IBM utilities program;
2. an access subprogram in FORTRAN which returns the data on one km square blocks when given coordinates according to the last four digits of the Standard Mecator Co-ordinate System (GETOPC);

TABLE I

1970 CENSUS OF POPULATION
U.S. DEPARTMENT OF COMMERCE
BUREAU OF THE CENSUS
NEW YORK - ADVANCE REPORT

Essex County	1970	1960	% Change
Total	34,631	35,300	- 1.9
Chesterfield Town	2,010	2,003	0.3
Keeseville Village (Part)	912	974	- 6.4
Crown Pt. Town	1,857	1,685	10.2
Elizabeth Town	1,284	1,328	- 3.3
Elizabethtown Village	607	779	-22.1
Essex Town	837	880	- 4.9
Jay Town	2,132	2,257	- 5.5
Keene Town	763	726	5.1
Lewis Town	763	803	- 5.0
Minerva Town	733	700	4.7
Moriah Town	5,244	5,837	-10.2
Mineville-Witherbee (U)	1,967	00000	00000
Port Henry Village	1,532	1,767	-13.3
Newcomb Town	957	1,187	-19.4
North Elba Town	5,776	6,005	- 3.8
Lake Placid Village	2,731	2,998	- 8.9
Saranac Lake Village (Part)	1,496	1,557	- 3.9
North Hudson Town	212	220	- 3.6
St. Armand Town	903	868	4.0
Bloomingdale Village	536	490	9.4
Saranac Lake Village (Part)	169	223	-24.2
Schroon Town	1,403	1,220	15.0
Ticonderoga Town	5,839	5,617	4.0
Ticonderoga Village	3,268	3,568	- 8.4
Westport Town	1,453	1,565	- 7.2
Westport Village	673	723	- 6.9
Willsboro Town	1,688	1,716	- 1.6
Wilmington Town	777	683	13.8

TABLE I (cont'd)

Warren County	1970	1960	% Change
Total	49,402	44,002	12.3
Bolton Town	1,589	1,417	12.1
Chester Town	2,330	1,974	18.0
Glen Falls City	17,222	18,580	- 7.3
Hague Town	910	771	18.0
Horicon Town	890	833	6.8
Johnsburg Town	2,377	2,250	5.6
Lake George Town	2,806	2,429	15.5
Lake George Village	1,046	1,026	1.9
Lake Luzerne Town	2,174	1,830	18.8
Queensbury Town	14,506	10,004	45.0
West Glens Falls (U)	3,363	2,725	23.4
Stony Creek Town	560	459	22.0
Thurman Town	708	548	29.2
Warrensburg Town	3,330	2,907	14.6
Warrensburg Center	2,743	2,240	22.5

TABLE I (cont'd)

Washington County	1970	1960	% Change
Total	52,725	48,476	8.8
Argyle Town	2,415	1,898	27.2
Argyle Village	392	355	10.4
Cambridge Town	1,702	1,610	5.7
Cambridge Village (Part)	493	519	- 5.0
Dresden Town	480	426	12.7
Easton Town	1,956	1,681	16.4
Greenwich Village (Part)	334	322	3.7
Fort Ann Town	3,749	3,124	20.0
Fort Ann Village	562	453	24.1
Fort Edward Town	6,719	6,523	3.0
Fort Edward Village	3,733	3,737	- 0.1
South Hudson Falls (U)	2,097	00000	00000
Granville Town	5,412	5,015	7.9
Granville Village	2,784	2,715	2.5
Greenwich Town	4,177	3,969	5.2
Greenwich Village (Part)	1,758	1,941	- 9.4
Hampton Town	464	469	- 1.1
Hartford Town	1,398	1,058	32.1
Hebron Town	1,212	1,026	18.1
Jackson Town	941	795	18.4
Kingsbury Town	11,737	11,012	6.6
Hudson Falls Village	7,917	7,752	2.1
Putnam Town	579	490	18.2
Salem Town	2,346	2,258	3.9
Salem Village	1,025	1,076	- 4.7
White Creek Town	2,644	2,365	11.8
Cambridge Village (Part)	1,276	1,229	3.8
Whitehall Town	4,794	4,757	0.8
Whitehall Village	3,764	4,016	- 6.3

3. three programs which use the access subprogram to access the disk to produce (a) unformated dump of a requested rectangular plot (DISKDUMP); (b) formulated print-out of raw data or calculated values based on raw data for an area bounded by up to fourteen straight lines (DATALIST); and (c) mapping routine permitting various presentations of raw or evaluated data (PLANMAP).

It was found that it was inefficient to use the complete diskpack for analysis of the Lake George basin. Therefore,

1. GETOPC was rewritten so that setting one flag would permit access to either the full data set or any square subset of it; the part that determines which data set to access and sets its parameters is a separate FORTRAN subroutine which can be modified by anyone with a basic knowledge of the language;
2. DISKDUMP was modified to produce its previous unformatted dump or generate a square subset of the full pack for storing on a normal systems pack; and
3. the data on the tapes were transferred to a 2314 Diskpack and, using the modified DISKDUMP, a data set comprising the information for the Lake George basin was generated; it takes 106 tracks on a 2311 standard system pack.

Immediate tasks will be to debug DATALIST and PLANMAP. Following that, efforts will be made to use the existing statistical analysis programs to begin identifying critical variables as an initial part of developing the descriptive model of the lake basin.

Optimization

As previously reported (Park and Wilkinson, 1971), researchhas been initiated on various mathematical programming formulations of the environmental management model of a lake basin. These representations depend upon the characteristics and attributes of the particular lake basin. Although the objective was to find an "optimal" plan for sewage collection and treatment, the methodology utilized permits sensitivity analyses that can present alternative plans for consideration by appropriate governing bodies.

Single-Component Effluent Regional Waste Treatment Models

The two models considered in this section are referred to as single-component effluent models, because only one effluent-component (usually B.O.D.) can be considered at one time; the percentages of this component that are removed are the decision variables.

A Linear Programming Model. The linear programming formulation of the problem of regional waste treatment is:

$$\min \sum_{i-1}^{n} C_i x_i$$

$$\text{subject to;} \quad \sum_{i-1}^{n} x_i V_i \geq K,$$

$$x_i \leq U_i, \text{ and}$$

$$x_i \leq l_i;$$

where i corresponds to which user in the region
is presently being considered (i=1,...,n),

C_i is the slope of the cost versus percent B.O.D. removal curve over its linear region for the ith user in dollars per percent removal per year,

x_i are the decision variables which denote the operating level of the ith user's treatment facility in percent B.O.D. removal required per year,

V_i is the volume of waste water treated by the ith user in m.g./day,

K is the water quality goal for the region defined as the percentage B.O.D. removal desired for the region multiplied by the total volume in m.g. of waste-water discharged into the region per day,

U_i is the upper bound of the linear region of the cost versus percent B.O.D. removal curve for the ith user, and

l_i is the lower bound of the linear region of the cost versus percent B.O.D. removal curve for the ith user.

Revelle, Loucks, and Lynn (1968) present typical cost curves, indicating how they are linear over a certain region of its extent. In order to use this model for determining the optimal regional waste-water treatment policy it is first necessary to specify all possible (perhaps probable would be more correct) collection policies for the region which would, in turn, yield the location and capacity of all required treatment facilities in the region. This procedure enables one to specify a collection policy and then use the model to determine the optimum treatment policy for this particular collection policy. Total costs could, by considering all feasible collection policies, be minimized.

A Geometric Programming Model. Ecker and McNamara (1971) propose a geometric programming approach to the preliminary design of an industrial waste treatment plant problem, which can be extended to the multi-user regional waste treatment problem as follows:

$$\text{min.} \quad \sum_{i=1}^{n} \sum_{j=1}^{p_i} c_{ij} \, x_{ij}^{a_{ij}}$$

$$\text{subject to;} \quad k^{-1} \sum_{i=1}^{n} v_i \prod_{j=1}^{p_i} x_{ij} \leq 1,$$

$$x_{ij} > 0, \text{ and}$$

$$a_{ij} \text{ real and } c_{ij} \text{ positive,}$$

where i corresponds to the user presently being considered $(i=1,\cdots,n)$,

j denotes which of the p_i possible treatment processes available to user i is presently being considered $(j=1,\cdots,p_i)$,

c_{ij} is the coefficient of cost of treating effluent from user i by process j in dollars per year,

x_{ij} is the percent B.O.D. of user i remaining after treatment by process j,

v_i is the amount of effluent produced yearly by user i, and

k is the water quality standard defined as the desired percent B.O.D. remaining for the region multiplied by the total volume of effluent discharged into the region per day.

Ecker and McNamara also show why the given forms of the cost functionals are realistic. This model is similar to the previous one in that it is better applied to a region where there are more industrial than municipal users, as collection systems for municipal users must be specified before the model can be applied. One obvious advantage of this model over the previous one, however, is the fact that staging at each facility can be considered. Another advantage is that non-linear portions of the cost curve can be considered. The main disadvantages are: (1) only one effluent component can be considered, (2) regional water quality optimization must be allowed, and (3) the required "posynomial" cost functionals may not be easy to approximate.

In developing environmental management policy for the Lake George basin, this formulation is not appropriate due to the lack of individual users.

Multi-Component Effluent Regional Waste Treatment Models. The models discussed in this section are termed multi-component effluent models since more than one effluent component can be constrained at a time. This formulation states that the amount of each user's effluent to be treated by each method available to that user become decision variables rather than percent treatment at each site by each method.

A Linear Programming Model. Carew and Van Slyke (1968) propose a linear programming model for the multi-component effluent regional waste-water treatment problem for the San Francisco Bay area. Only minor modifications are necessary to make this model applicable to the lake case as follows:

$$\text{min.} \quad \sum_{\ell=1}^{L} \sum_{k=1}^{k_\ell} c_{k\ell} \; x_{k\ell}$$

$$\text{subject to;} \quad \sum_{\ell=1}^{L} \sum_{k=1}^{k_\ell} a_{k\ell m} \, x_{k\ell} \leq b_m, \quad m=1,2,\ldots,M,$$

$$\sum_{k=1}^{k_\ell} x_{k\ell} = d_\ell, \text{ and}$$

$$x_{k\ell} \geq 0,$$

where ℓ denotes which of the L users is presently being considered ($\ell=1,\cdots,L$),

k denotes which of k_ℓ possible treatment methods available to user ℓ is presently being considered ($k=1,\cdots,k_\ell$),

m denotes which of M total effluent components is presently being considered ($m=1,\cdots,M$),

$a_{k\ell m}$ is the amount of effluent component m released per unit of untreated effluent by the kth process of industry ℓ,

b_m is the water quality standard given in terms of the maximum allowable discharge of effluent component m left after treatment totaled over all dischargers in pounds per day,

d_ℓ is the total amount of effluent before treatment from discharger ℓ in millions of gallons per day for a given level of activity,

$c_{k\ell}$ is the operating cost in dollars per million gallons of processing one unit of raw effluent from discharger ℓ by process k, and

$x_{k\ell}$ is the amount of untreated effluent in m.g.d. from user ℓ to be processed by method k.

This model requires that the collection policies to be used by each of the municipal users are specified. It does, however, have the advantage that more than one effluent component can be considered. Another advantage is that elementary by-pass piping can easily be implemented within the scope of the model.

A major short-coming of this formulation is that capital costs are not considered explicitly. One approach to this problem would be: if the number of effluent components being considered is small compared to the number of users, capital costs could be incorporated into the model by specifying the capacity of each user and prorate capital costs based on this capacity.

This formulation is not especially appropriate for the Lake George basin because a small number (usually) of users are considered, all are of the municipal type. These municipal users treat effluent of identical characteristics meaning that only the loading factor cost component is optimized rather than the composition factor cost component.

A Zero-One Integer Program. The Zero-One integer program has the following form:

$$\text{min.} \quad \sum_{k=1}^{k_\ell} \sum_{\ell=1}^{L} c_{k\ell}\, x_{k\ell},$$

$$\text{subject to:} \quad \sum_{\ell=1}^{L} \sum_{k=1}^{k_\ell} a_{k\ell m}\, x_{k\ell} \leq b_m \qquad m=1,2,\ldots,M,$$

$$\sum_{k=1}^{k_\ell} x_{k\ell} = 1, \; \ell = 1,\ldots,L, \text{ and}$$

$$x_{k\ell} = 0 \text{ or } 1,$$

where

ℓ	denotes which of L total users is presently being considered ($\ell=1,\cdots,L$),
k	denotes which of k_1 possible treatments available to user ℓ is presently being considered ($k+1,\cdots,k_\ell$),
m	denotes which of M total effluent components is presently being considered ($m=1,\cdots,M$),
$c_{k\ell}$	is the annual total cost (fixed and operating) of user ℓ using treatment k in \$/day,
$a_{k\ell m}$	is the amount of effluent component m left when user ℓ uses treatment k in lb./day,
b_m	is the water quality standard for the sector in terms of the maximum allowable influx of component m allowed in lb./day, and
$x_{k\ell}$	is zero-one variate whose value is one if treatment m is used by user 1 and zero otherwise.

This model has the same attributes as the linear programming model, but the cost functionals are more accurate; however some flexibility is lost by assuming only one treatment at each user site can be considered.

The Zero-One integer program is especially good for evaluating the relative merits of detailed pollution control proposals being considered for a region. In the case of the Lake George Basin proposals like those given by Metcalf and Eddy (1965) and Rist Frost Associates (1969), consulting engineers, could be evaluated along with other policies based on the results of the leaching field treatment facility studies, various town proposals, etc. The one minor shortcoming of this model as applied to the Lake George basin is that collection costs must be specified before the model can be used to optimize total treatment costs.

Network Flow Models. Deininger (1966) has considered the design of a regional treatment system as a network flow problem. Population centers (i.e., clusters of population), rather than communities, will be considered as individual contributors, following Deininger (1966). Figure 1 presents a schematic of the design problem. There are five population centers with their populations. The wastes from these centers are collected by a local network and transported to a central collection point. These collection points serve as inputs (with specified input volumes) to a regional waste treatment and collection system. The major assumption is that equity water quality standards are assumed to exist (i.e. the water quality standard is the same for every user discharging into any given receiving body of water). For example, in the Lake George basin all treatment plants in the region presently

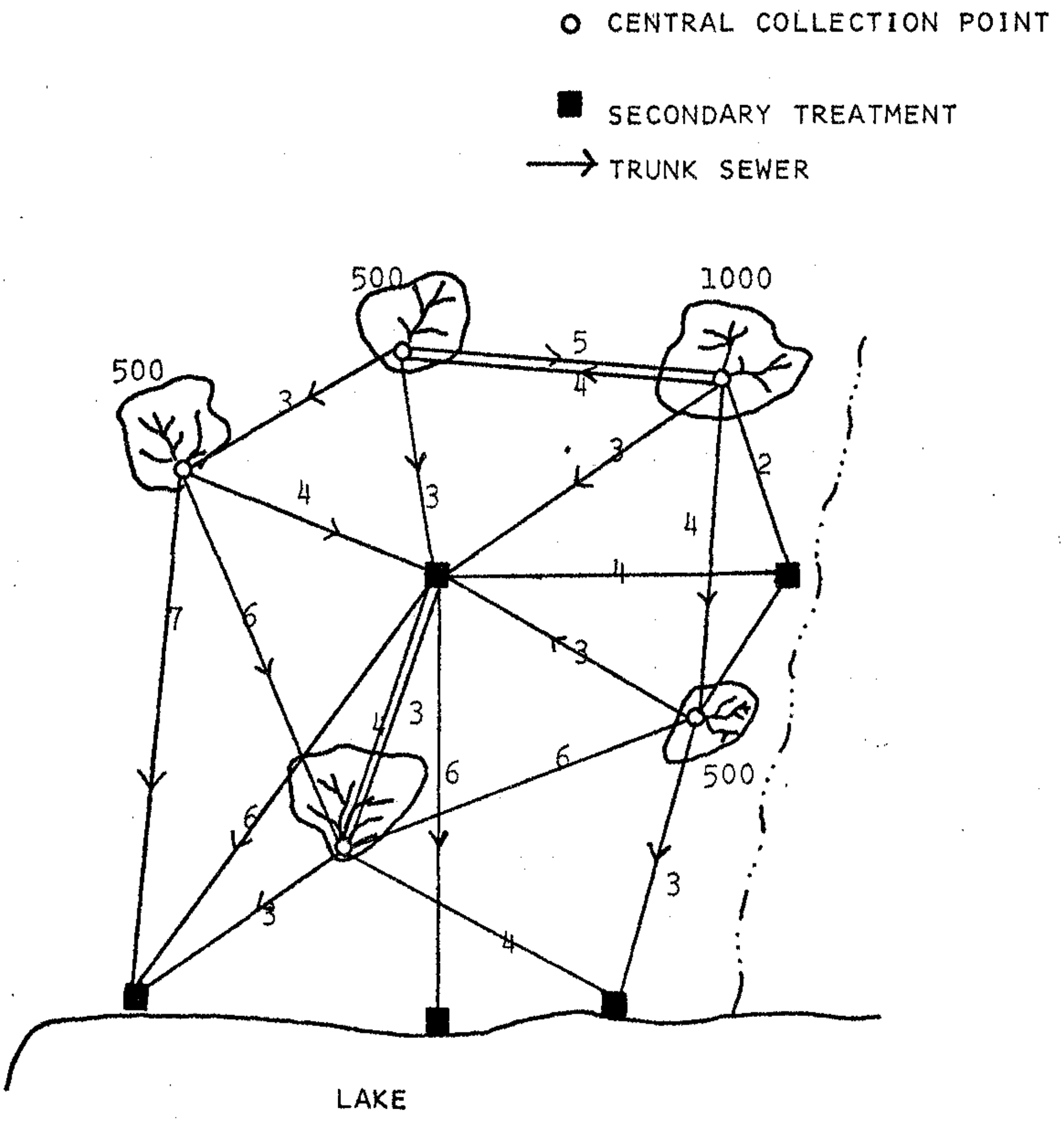

Figure 1 - Schematic layout of a regional waste water collection and treatment system.

under consideration for construction must treat to the secondary level. The lines between input points and treatment plants represent possible trunk sewers and the numbers indicate the effective directional distances between points. The unidirectional cost assumption is necessary since the distances (or cost) of transporting wastes from one point to another is not, in general, equal to the costs of flow in the other direction.

The network formulation for the problems:

$$\text{min.} \quad \sum_{x} \sum_{j} f_{ij}(x_{ij})$$

$$\text{subject to:} \quad \sum_{j} x_{ij} - \sum_{j} x_{ji} = \sum_{j} (x_{ij} - x_{ji}) = a_i,$$

$$x_{ij} \geq 0; \text{ and } \sum_{j} a_i = 0,$$

where x_{ij} is the amount of waste transported from node i to node j,

$f_{ij}(x_{ij})$ is cost of transporting x_{ij} units of waste from node i to node j,

$a_i \geq 0$ implies an input node,

$a_i = 0$ implies a transhipment node, and

$a_i \leq 0$ implies an output node.

Solutions of the problem depends largely on the nature of the functions $f_{ij}(x_{ij})$. The following sections deal with problem formulations that depend on the form of these "cost" functionals. Fortunately, recent developments in the field of mathematical programming make this problem more solvable for realistic cost functionals than when Deininger originally proposed the model.

A Linear Programming Model. Hatfield, Graves, and Whinston (1969) propose a linear approximation of the cost functionals based on Linaweaver and Clark's (1964) general cost of water transmission through pipe expression,

$$C_T = .00511Q^{-.402},$$

where C_T is the total annual cost in \$/1000 gal/mi, and

Q is the flow in m.g.d.

The approximation technique is based upon the maximum flow possible through a given branch of the network. A per unit cost is calculated as a linear fraction of this maximum capacity. These per unit costs are then

$$g_{ij}=1865\ L_{ij}Q^{-.402},$$

where g_{ij} is the per unit cost of transporting one unit from node i to node j per year,

L_{ij} is the distance from node i to node j in miles, and

Q_{ij} is the maximum expected flow from node i to node j.

Assuming appropriate linear approximations (probably of the same form as the pipe flow cost approximations) can be found for the treatment costs one can consider the problem as a linear programming problem as follows:

$$\text{min.} \quad \sum_i \sum_j g_{ij} x_{ij},$$

$$\text{subject to:} \quad \sum_j x_{ij} - \sum_j x_{ji} = \sum_j (x_{ij} - x_{ji}) = a_i,$$

$$x_{ij} \geq 0, \text{ and } \sum_j a_i = 0;$$

where the definitions are the same as before, except that the g_{ij}'s are constants. To refine the linear approximation, the model could be run and any decision variable (x_{ij}) with a low value could be used to compute new per unit cost based on new capacities. Using this approach a final solution could be found that involved g_{ij}'s, each based on a capacity approximately equal to its actual capacity (as the solution of an L.P. problem). Optimization, while not assured, might be approximated quite well.

Non-Linear Programming Formulations. The simplest non-linear programming formulation to the network problem would be to use a separable, piece-wise linear approximation to the objective function. However, the time spent on the tedious calculations necessary to formulate the problem in this manner might be excessive.

If the actual $f_{ij}(x_{ij})$'s in the problem are convex, then the $\sum_i \sum_j f_{ij}(x_{ij})$ will be a convex functional. The corresponding maximum problem may be solved by the method of linear manifold sub-optimization (Zangwill, 1969, p. 178). If the functional is not convex, but has continuous partial derivatives, the convex simplex method (Zangwill, 1969, p. 162) may be used effectively.

Concluding Remarks

It has been demonstrated that the form of the water quality standards existing for a certain region (equity type or regional type) have great bearing on the type of optimization model to be considered for the regional waste-water treatment problem. While the models given here are by no means exhaustive of the types of models that can be formulated for the regional waste-water treatment problem, they represent a starting point for an attack on the problem for any type of lake basin.

At present, the "best" approach for Lake George is a linearized network model. However, it is strongly recommended that a computer program utilizing the convex-simplex method be prepared. This method is directly applicable to regional waste-water treatment problems where there are a large number of municipal users. This is the situation in many lake basins, such as Lake George, that are feeling the impact of increased leisure-time and retirement village developments.

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