A NEW STABILITY METHOD FOR SINGULARLY PERTURBED CONVECTION-DIFFUSION EQUATIONS

By

Viswanath Ramakkagari

An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Computer Science

The original of the complete thesis is on file
in the Rensselaer Polytechnic Institute Library

Examining Committee:

Joseph E. Flaherty, Thesis Adviser

Donald W. Schwendeman, Member

Kenneth E. Jansen, Member

Mark S. Shephard, Member

Rensselaer Polytechnic Institute
Troy, New York

May 2007
(For Graduation May 2007)

ABSTRACT

Singularly perturbed convection-diffusion problems are characterized by their dual behavior and the presence of sharp layers. These problems exhibit both convective and diffusive behavior. The convective and diffusive terms are in balance within the layers and the convective terms are dominant away from these layers. Standard methods perform poorly on coarse meshes because of the hyperbolic nature of the operator in a major part of the domain. To this end, we propose a new approach in the finite element framework to solve singularly perturbed convection-diffusion problems. In the proposed approach, the test function space is modified and hence, fits into the class of Petrov-Galerkin finite element methods.

This method is motivated by, and similar to one proposed by Hemker [**?**] for one-dimensional problems which was based on using Green's functions to motivate appropriate test spaces. For two-dimensional problems we no longer utilize Green's function, but rather we try to capture the solution of a suitably posed discretized dual problem with functions in the test space. The dual problem is posed in such a way that a weighted error at the interelement boundary is minimized. The basic idea is to control error growth across each element and thus, control the global error. We discuss the choice of the dual problem to achieve this and propose test spaces that provide a good approximation for the dual solution. Since the dual solution is exponential, it implies that the test spaces are exponentially fitted. The test spaces are designed in such a way that it is possible to easily develop a higher-order method in a hierarchical framework. Accurate numerical solutions are obtained and large errors are confined to the few elements in the vicinity of the layers.

Singularly perturbed convection-diffusion problems are ideal systems to be solved by adaptive methods. It is far more efficient to resolve the nonuniform behavior of the solutions with nonuniform meshes. Efficient and inexpensive error estimates are needed to achieve optimal refinement of the mesh. We propose an *a posteriori* error estimation technique where we solve element-wise auxiliary boundary value problems. We use the error estimate to adapt the meshes for singularly perturbed convection-diffusion problems. We discuss the performance of the error estimator and present the adapted meshes.