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SUPERSONIC VEHICLE SIMULATION STEADY-STATE ANALYSIS

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STEADY-STATE SIMULATION STUDY OF THE FLOW INDUCED BY AN INTERNALLY PROPELLED VEHICLE IN AN INFINITE TUBE -- SUPERSONIC VEHICLE

by

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ABSTRACT

The flow induced in the wake of an internally-propelled supersonic vehicle or of a disturbance moving supersonically through an infinite tube is analyzed as a steady one-dimensional flow in the frame of reference of the vehicle or disturbance, with full account of heat transfer and dissipative effects. The governing equations are solved numerically.

The results confirm that the flow can be steady if it is everywhere supersonic relative to the vehicle or disturbance.

NOMENCLATURE

a speed of sound, fps

C_p specific heat at constant pressure, ft-lb/slug ^oR

C_T nondimensional friction factor

$F = p\alpha + \dot{m}u$ stream force, lb

h convective heat transfer coefficient, lb/ft.sec. ^oR

$\dot{m} = \rho\alpha u$, mass flow rate in disturbance-fixed frame of reference, slugs/sec.

M Mach number in disturbance-fixed frame of reference

$M_o = u_w/a_o$, disturbance propagation Mach number

p static pressure, psf

p^o stagnation pressure in disturbance fixed frame of reference, psf

q heat input per unit mass, ft-lb/slug

r radius of tube, ft.

R gas constant, ft-lb/slug ^oR

T static temperature, ^oR

T^o stagnation temperature in disturbance fixed frame reference, ^oR

T^{o*} stagnation temperature in wall-fixed frame of reference, ^oR

u flow velocity relative to the disturbance, fps

v velocity of the disturbance relative to the tube walls, fps

$u_w = -v$, velocity of the tube walls in the disturbance-fixed frame of reference, fps

x distance from the disturbance, ft.

y nondimensional velocity parameter $= 1 - u/u_w$

α tube cross-sectional area, ft^2

γ ratio of specific heats

ρ gas density, slugs/ft^3

ξ = $F/\dot{m}u_\omega$, nondimensional stream force

η = x/r , nondimensional distance

Subscripts

0 conditions in the undisturbed region ahead of the disturbance.

1 conditions immediately behind the disturbance.

I. INTRODUCTION

The flow induced by a body moving in a tube, or by a disturbance propagating through it, can be analyzed, in the frame of reference fixed to the tube walls, by the laborious method of characteristics for nonsteady one-dimensional flow. The most comprehensive analysis of this problem so far is one that has just been completed by Skinner[1]. On the other hand, if the body or disturbance is moving at constant speed (in an inertial frame of reference) relative to the tube wall, and if the length of the tube is very great, the analysis can be simplified by viewing the process in the only frame of reference in which it can be steady - viz., the frame of reference of the moving body or disturbance.

This approach was first used, in radically different manners but with similar results, by Hagerup[2] and Foa[3]. Hagerup worked out a two-dimensional small perturbation analysis without consideration of viscous dissipation, and obtained closed-form solutions. He found, however, that steady-flow solutions satisfying the boundary conditions could be obtained only for situations of supersonic travel or propagation speed. Foa developed the general equations for one-dimensional flow with full consideration of dissipative effects, and found that steady-flow solutions could be obtained only for supersonic situations and over a limited range of subsonic conditions. He obtained closed-form solutions for an arbitrarily prescribed schedule of heat transfer to the surroundings. The same equations are applied here to situations involving a more realistic schedule of heat transfer.

Because of their complexity, the governing equations cannot be solved except numerically, and it is not apparent by inspection that they admit meaningful solutions at all. It is the purpose of this paper to test the prediction of previous theories[2,3] that steady-flow solutions are possible over certain ranges of operating conditions, by actually obtaining such solutions and determining the detailed wake behavior for specific cases in these ranges.

The particular case of a gaseous detonation propagating through a tube is considered to permit direct comparison with the results of the nonsteady flow analysis of Ref. 1. The case of a supersonic internally-propelled vehicle is also considered.

The equations are solved numerically on an IBM digital computer using a Runge-Kutta forward integration technique.

II. THE EQUATIONS OF MOTION

The present analysis considers a disturbance propagating at a constant supersonic velocity v through a constant area tube. In the disturbance-fixed frame of reference (see Fig. 1) the flow is assumed to be steady. In this frame of reference the tube walls are moving at a velocity $u_w = -v$. The flow induced by the disturbance is treated as one-dimensional, fully developed and turbulent throughout.

The governing equations, in the disturbance-fixed frame of reference, are[3]

$$\dot{m} C_p dT^\circ = u_w dF + \dot{m}\, dq \quad (1)$$

$$dF = -\rho \alpha \frac{C_T}{r} (u - u_w) |u - u_w| dx \quad (2)$$

T° is the stagnation temperature relative to the disturbance,

hence $$T^\circ = T + u^2/2C_p$$

and $$dT^\circ = dT + \frac{u}{C_p} du \quad (3)$$

Also $$\dot{m} = \rho \alpha u = (F - \dot{m} u)\, u/RT$$

Thus $$\dot{m} R dT = u dF + F du - 2\dot{m} u\, du \quad (4)$$

Heat transfer is assumed to take place by convection and conduction. Thus dq, the heat transfer rate per unit mass, is approximated as

$$dq = -K\left(T^{o*} - T_o\right)\, dx \qquad (5)$$

where T^{o*} is the stagnation temperature of the flow relative to the wall,

$$T^{o*} = T + (u - u_w)^2 / 2C_p$$

T_o is the temperature in the undisturbed region ahead of the disturbance and is assumed to be constant and equal to the wall temperature. K is the sum of a constant for the conduction and a term which includes the convective heat transfer coefficient. Thus K may be written as

$$K = \frac{C_1}{r} + \frac{2\pi r h}{\dot{m}}$$

or

$$K = \frac{C_1}{r} + \frac{2h}{\rho u r}$$

Upon rearranging, equation (5) becomes

$$dq = -\left[C_1 + 2C_p S_t \frac{|u - u_w|}{u}\right]\left[T + \frac{(u-u_w)^2}{2C_p} - T_o\right]\frac{dx}{r} \qquad (6)$$

where $S_t = \dfrac{h}{C_p \rho |u - u_w|}$ is the Stanton Number. Substitution of equations 2,3,4 and 6 into 1 and rearranging gives

$$\left[\frac{F}{\dot{m} u_w} - \left(\frac{\gamma+1}{\gamma}\right)u\right]\frac{du}{dx} = \frac{C_T}{r}\left[1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{u_w}{u}\right](u - u_w)|u - u_w|$$

$$-\left[\frac{C_2}{r} + \frac{2S_t}{r}\frac{|u-u_w|}{u}\right]\left[\frac{Fu}{\dot{m}} - u^2 + \left(\frac{\gamma-1}{2\gamma}\right)(u-u_w)^2 - RT_o\right] \qquad (7)$$

As a check, it can be verified that for the case in which $u_w = 0$ and the heat transfer term is zero, equation (7) reduces to

$$\frac{du}{u} = \frac{\gamma M^2}{(1-M^2)} \frac{C_\tau}{r} dx$$

which corresponds to equation (4e-9) of Ref. 4, representing a Fanno process. For the case of a Rayleigh process ($u_w = 0$ and $C_\tau = 0$), equation (7) reduces to

$$\frac{du}{u} = \left[\frac{1 + \left(\frac{\gamma-1}{2}\right)M^2}{(1-M^2)}\right] \frac{dq}{C_p T^o}$$

which corresponds to the Rayleigh process equation (4b-7) of Ref. 4.

Equations (2) and (7) are two equations in the two unknowns F(x) and u(x). In nondimensional form, these equations become

$$\frac{d\xi}{d\eta} = C_\tau \frac{y|y|}{(1-y)} \tag{8}$$

and

$$\frac{dy}{d\eta} = \frac{1}{\left[\xi - \left(\frac{\gamma+1}{\gamma}\right)(1-y)\right]}\left\{C_\tau y|y|\left[1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{(1-y)}\right]\right.$$

$$\left. + \left[C_2 + \frac{2S_t|y|}{(1-y)}\right]\left[\xi(1-y) - (1-y)^2 + \left(\frac{\gamma-1}{2\gamma}\right)y^2 - \frac{1}{\gamma M_o^2}\right]\right\} \tag{9}$$

III. SOLUTION OF THE EQUATIONS

The fundamental assumption of this analysis is that for a disturbance moving at a constant velocity the induced flow is steady in the disturbance-fixed frame of reference. Physically this means that, through steady-flow dissipative process, the flow velocity relative to the disturbance, at $\pm\infty$, must become equal to the velocity of the wall. Furthermore, the state of the gas must also return to the ambient or undisturbed state. Also, since the propagating disturbance or self-propelled vehicle is stipulated to move at a constant velocity, the overall change of stream force across it must be zero, i.e. $\xi/\xi_o = 1$. (The above conditions are shown graphically in Figure 2).

Therefore, the above assumption means that any physically meaningful solution to the steady-flow equations 8 and 9 must result in y going to zero and the non-dimensional stream force ξ returning to the initial value, ξ_o, as $x/r \to \infty$.

In Eq. 8, the nondimensional momentum equation, as y approaches zero, $d\xi/d\eta$ becomes zero and ξ tends to a constant. From the energy equation (Eq. 9), the value of this constant must be $\xi = \xi_o$ (or $F = F_O$) for $dy/d\eta = 0$ and $y = 0$. Note that the denominator of Eq. 9 is not zero for this asymptotic value of ξ. Thus Eq. 8 and 9 satisfy the steady-flow wake conditions of $dy/d\eta$ and y becoming zero and the stream force returning to its initial value as indicated in Figure 2.

Because of their nonlinearity, Eqs. 8 and 9 must be solved numerically. The Runge-Kutta forward integration scheme has been used to obtain numerical solutions. The technique involves solving the two equations simultaneously for specified initial values of y and ξ.

The initial values of $y = y_1$ and $\xi = \xi_1$ depend on the nature of the disturbance. Furthermore, the existence of physically meaningful steady flow solutions depends on the values of y_1 and ξ_1. Some insight into these values can be gained through consideration of specific cases.

Figure 3 defines regions of y in terms of the disturbance propagation Mach number $M_o = u_w/a_o$, for values of $M_o \geq 1$. The curve of

$$y = \frac{M_o^2 - 1}{(\gamma+1) M_o^2}$$

corresponds to values of y and M_o such that the denominator of Eq. (9) becomes zero, hence $dy/d\eta$ becomes infinite. From the definition of ξ as

$$\xi \equiv \frac{F}{\dot{m} u_w} = \left[\frac{1}{\gamma M^2} + 1\right] \frac{u}{u_w} = \left[\frac{1}{\gamma M^2} + 1\right](1-y)$$

it is obvious that for the denominator of Eq. (9) to be zero, the local Mach number of the flow must be unity. Thus the curve of Figure 3 represents points of sonic flow relative to the disturbance.

Points above this curve (region 1, for $M_o \geq 1$) correspond to initial points of a subsonic flow which must eventually return to the supersonic wall velocity for steady flow to exist. Numerical computations have shown that solutions of the steady flow equations do not exist in this region. What actually happens is that, since the flow is subsonic, pressure waves propagating upstream continually change the conditions immediately behind the disturbance causing the flow to be unsteady.

When the initial point is below the sonic curve (regions 2 and 3 for $M_o \geq 1$), the flow in the wake is everywhere supersonic and pressure waves cannot propagate upstream. Thus the flow conditions immediately behind the disturbance remain constant and numerical computations show that solutions to the steady flow equations do exist. In region 2, the flow velocity is less than the wall velocity, whereas in region 3 it is greater.

Region 1 in Figure 3, where the flow is subsonic relative to the disturbance, corresponds to the flow behind a strong detonation. Regions 2 and 3, where the flow is supersonic, correspond to the flow behind a weak detonation or to a supersonic internally-propelled vehicle.

The sonic curve in Figure 3, separating the regions of flow behind the strong and weak detonations, represents the Chapman-Jouguet condition.

IV. RESULTS

Numerical results have been obtained for several specific cases and are presented in Figures 4 through 7.

Chapman-Jouguet Detonation

One of the situations considered here is that of the particular Chapman-Jouguet detonation that was investigated by Skinner[1] by the method of characteristics. The numerical results are shown in Figure 4 for u/u_w and ξ / ξ_o and in Figure 5 for p/p_o.

The Chapman-Jouguet condition, being represented by points on the sonic curve of Figure 3, cannot be used as the initial condition in this computation. Initial values of y_1 and ξ_1 for this analysis were taken from Skinner's results at a point away from the sonic condition.

The curve representing the flow in the wake of the Chapman-Jouguet detonation and including the effects of conduction, convection and friction shows the wake disturbances to decay rapidly immediately behind the detonation until the flow velocity becomes equal to the wall velocity. Then, as heat conduction causes a further decrease of temperature, the flow velocity at first becomes greater than the wall velocity, and then returns to the undisturbed value (equal to the wall velocity) as the stream force gradually returns to its initial value. This overshoot in the velocity is necessary in order for the stream force ξ, which at first becomes greater than ξ_o, to return to ξ_o. Temperature and pressure also return to their ambient levels. The individual points marked by x on this curve are taken from Skinner's nonsteady flow analysis to show the correlation between the results of the two analyses in the near field of the wake. However, the complete decaying of the flow properties to the ambient conditions requires several orders of magnitude greater distance than was required for Skinner's nonsteady flow analysis to reveal a clear trend toward steady flow.

Peculiar results are obtained if no consideration is given to heat transfer by conduction, i.e., if C_2 is taken to be zero. If $C_2 = 0$, then when $y = 0$ the flow cannot undergo further change. Consequently, whereas the velocity of the flow relative to the wall has become zero, the pressure, temperature and stream force are not reduced to their respective ambient levels. The numerical results for this case are also shown in Figures 4 and 5.

Numerical results have also been obtained both for frictionless and for adiabatic flow. These results are also shown in Figures 4 and 5 for comparison. The velocity ratio in the frictionless flow is seen to tend to a value other than unity while the stream force is, of course, constant throughout.

In the adiabatic case, the stream force diverges rapidly to a non-unity asymptote since the velocity ratio only tends to one without the overshoot needed to return the stream force to its initial value.

Supersonic Vehicle

Figures 6 and 7 show the behavior of the flow induced in the wake of an internally propelled supersonic vehicle. The vehicle has a low rate of heat release, hence the pressure and temperature increments across the vehicle are much less than for the detonation. Thus the rate of decay of the disturbances is much slower than for the detonation.

Fig. 6 shows u/u_w equal to one at a distance of about 2×10^5 feet behind a Mach 2 vehicle in a 10 foot diameter tube. This means that with a vehicle headway of only 2 minutes, each vehicle would find an essentially undisturbed medium.

V. CONCLUSIONS

The results of this analysis confirm that the flow in the wake of a disturbance propagating supersonically through an infinite tube can be treated as steady in the disturbance-fixed frame of reference if the full effects of dissipation are considered. Thus, the detailed behavior of the wake can be determined by the numerical method of this report for those cases when the flow relative to the disturbance is supersonic throughout. If the flow relative to and immediately behind the disturbance is sonic, as in the case of a Chapman-Jouguet detonation, then the values of velocity and stream force at some point away from the sonic station, as determined by the method of characteristics, must be used as initial values for the numerical integration of the steady flow equations.

The wake decays most rapidly immediately behind the disturbance because of the high rate of energy loss and dissipation due to friction and convective heat transfer. At greater distances, where energy losses are limited primarily to heat conduction, further decaying of the wake is relatively slow.

Solutions of the steady flow equations for a Chapman-Jouguet detonation confirm both qualitatively and quantitatively the trend towards steady flow obtained by Skinner[1] in a nonsteady analysis for the same situation.

VI. REFERENCES

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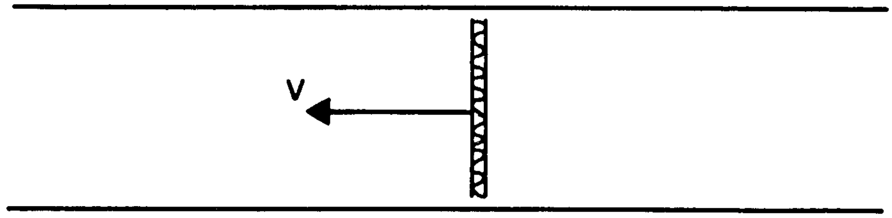
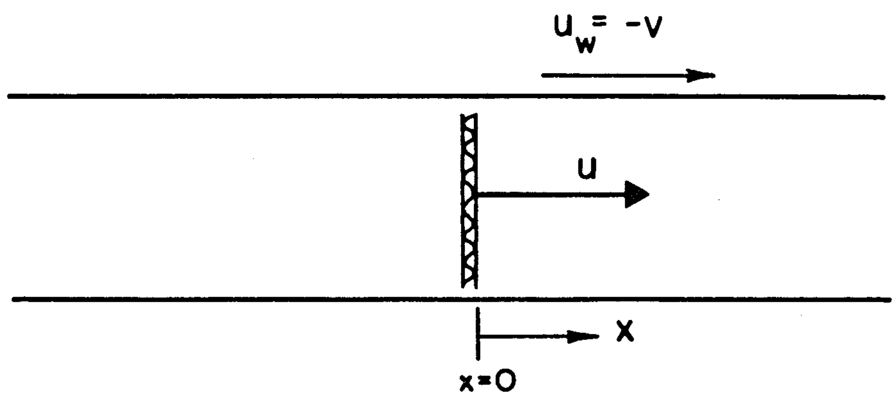


WALL - FIXED FRAME OF REFERENCE



DISTURBANCE - FIXED FRAME OF REFERENCE

FIGURE 1

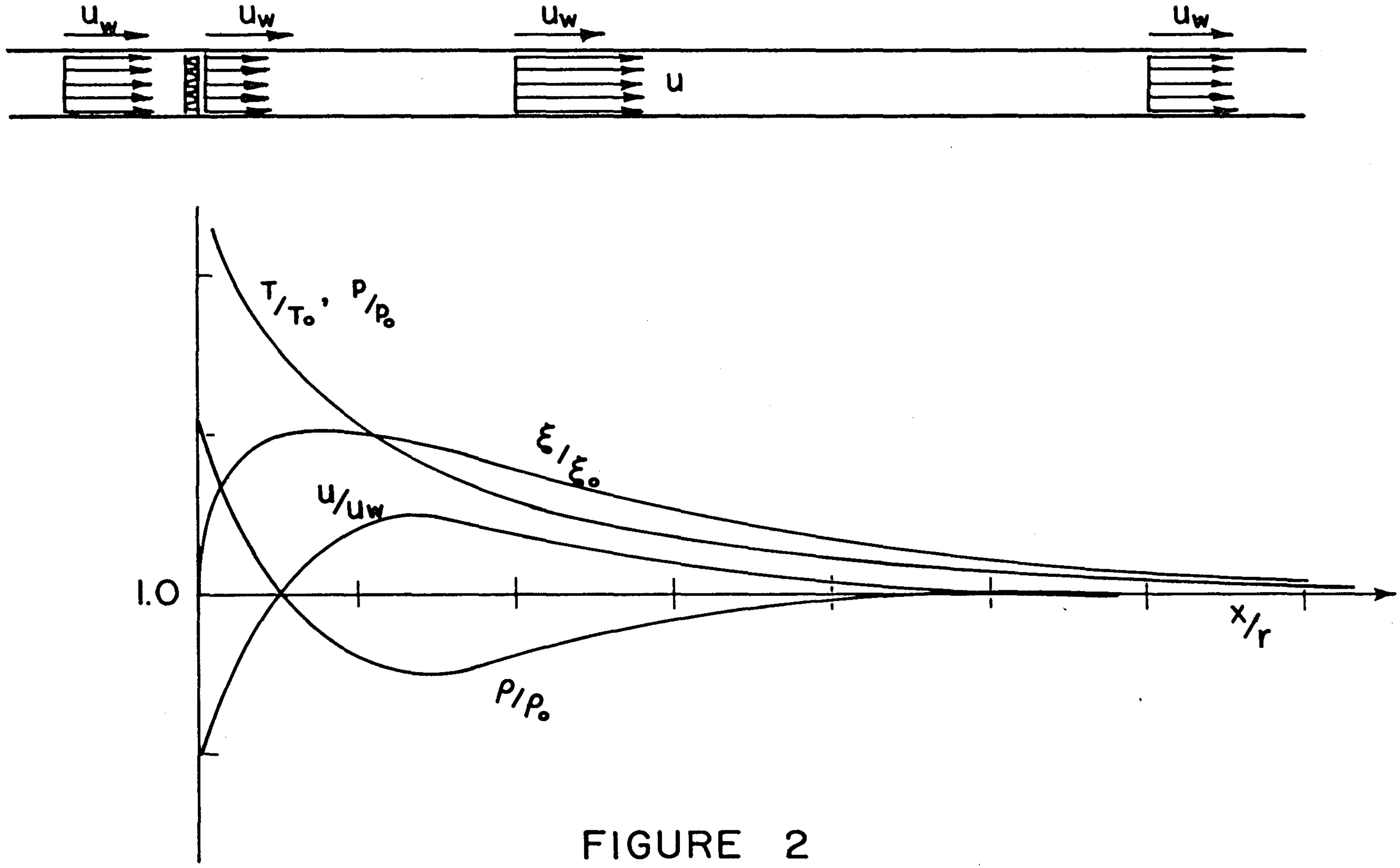
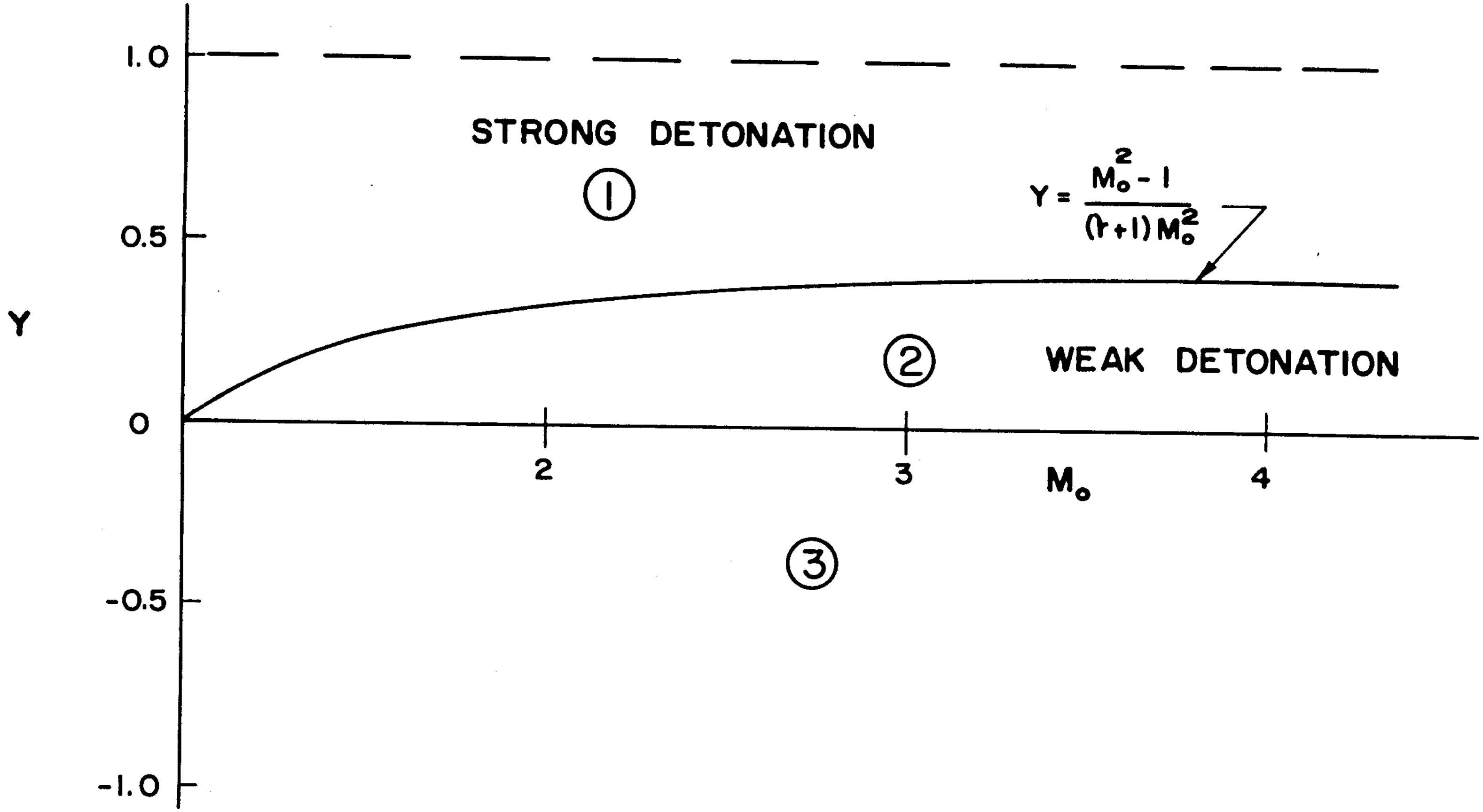


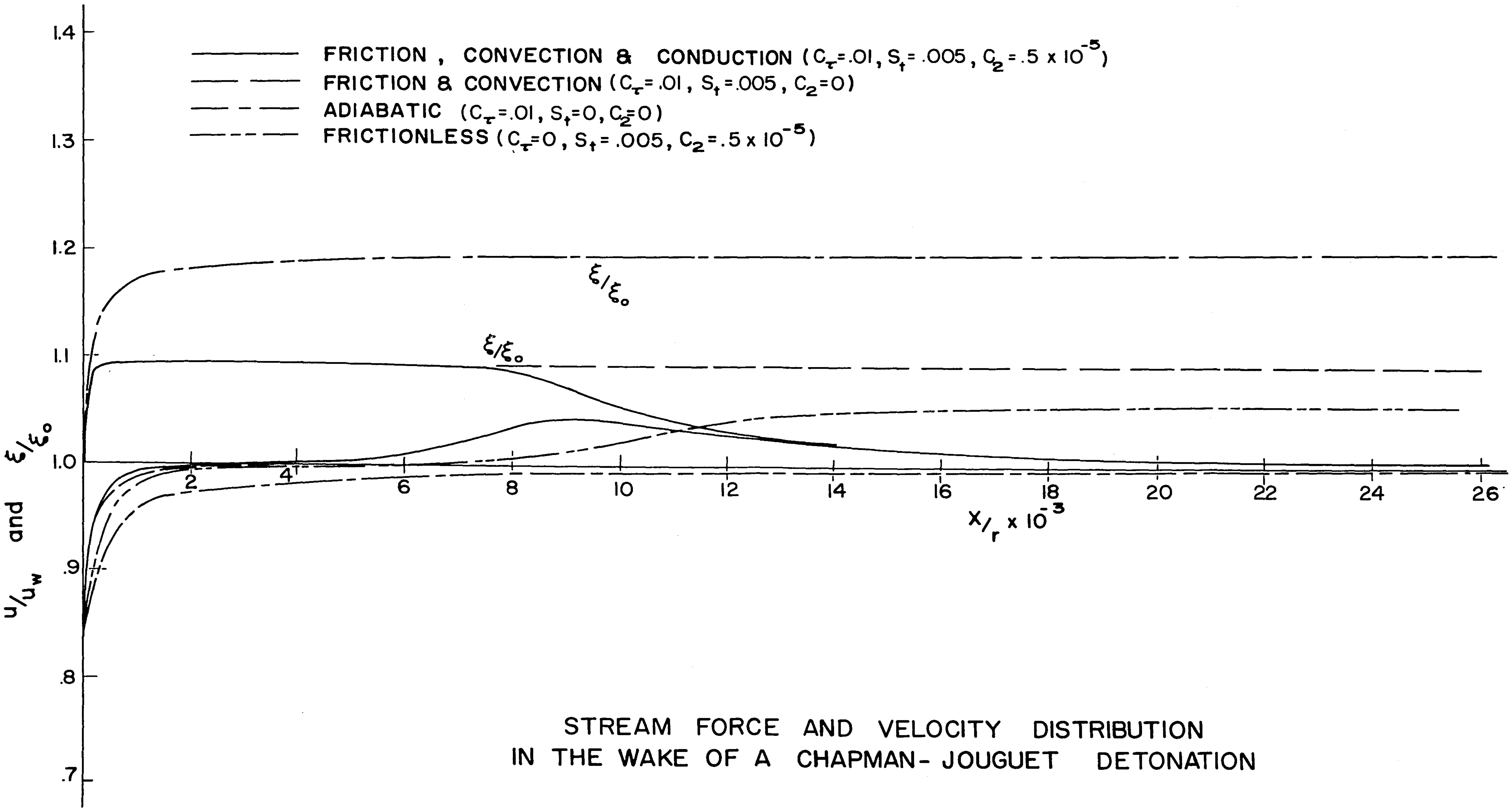
FIGURE 2

SUPERSONIC STEADY-FLOW WAKE BEHAVIOR



1.0
0.5
0
-0.5
-1.0
Y
STRONG DETONATION
①
$Y = \frac{M_o^2 - 1}{(\gamma + 1) M_o^2}$
②
WEAK DETONATION
2
3
M_o
4
③

FIGURE 3

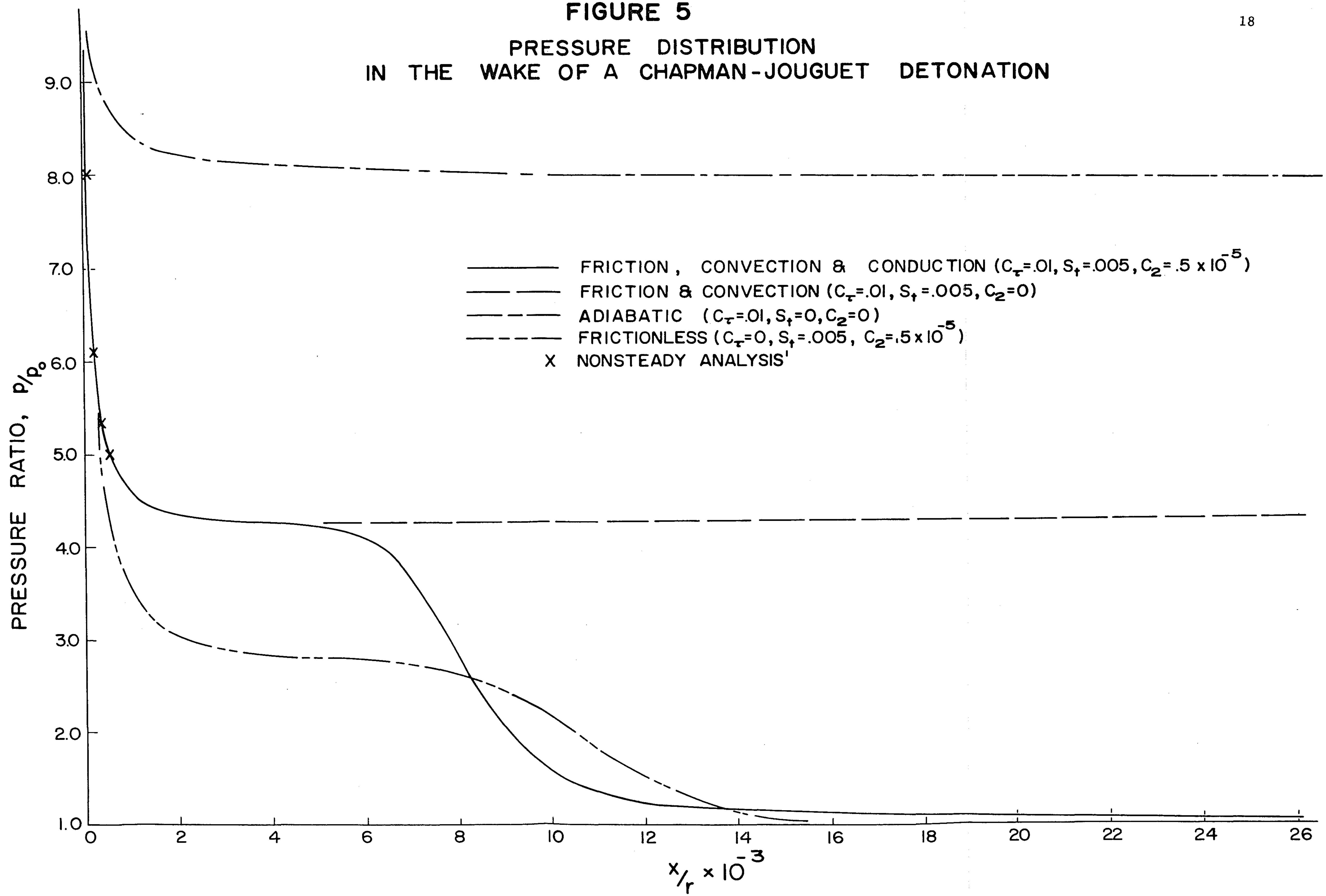


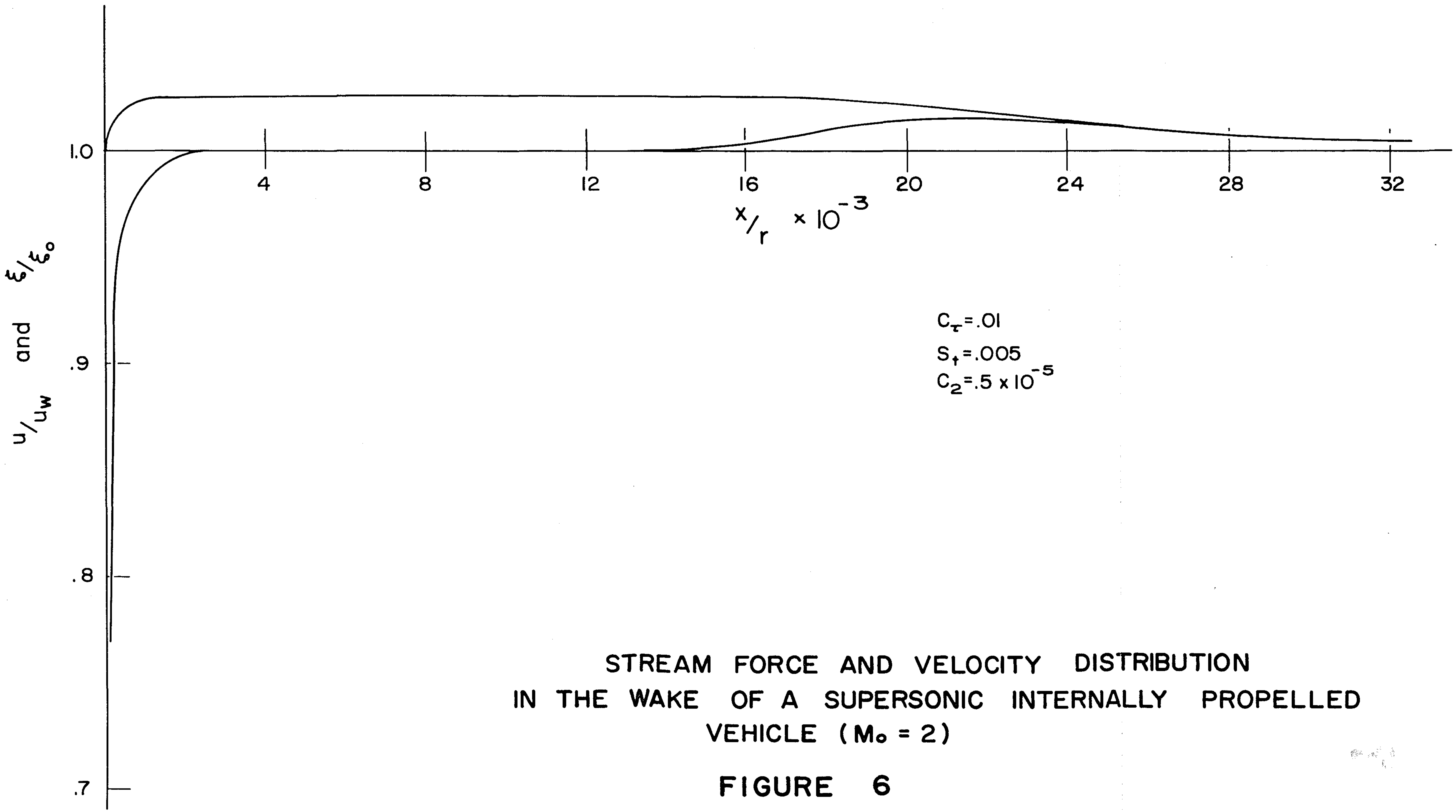
STREAM FORCE AND VELOCITY DISTRIBUTION
IN THE WAKE OF A CHAPMAN- JOUGUET DETONATION

FIGURE 4

FIGURE 5

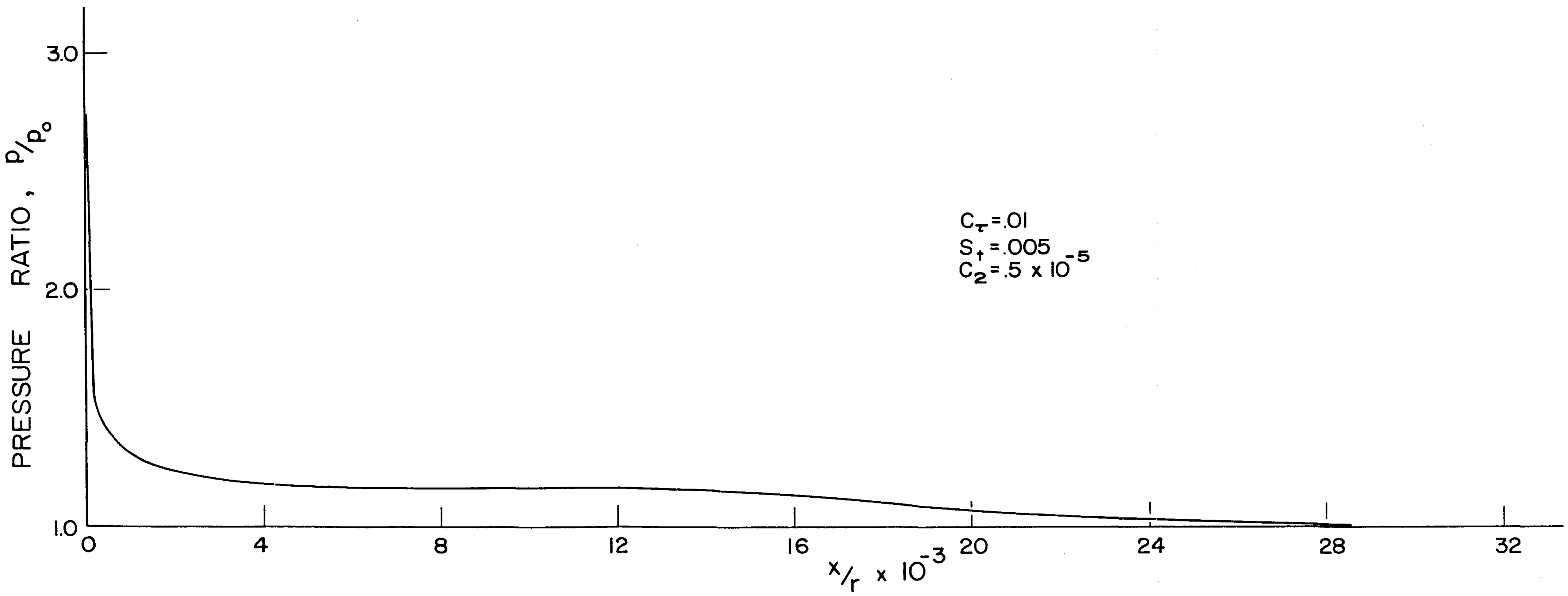
PRESSURE DISTRIBUTION IN THE WAKE OF A CHAPMAN-JOUGUET DETONATION





STREAM FORCE AND VELOCITY DISTRIBUTION IN THE WAKE OF A SUPERSONIC INTERNALLY PROPELLED VEHICLE ($M_o = 2$)

FIGURE 6



PRESSURE DISTRIBUTION

IN THE WAKE OF A SUPERSONIC INTERNALLY PROPELLED

VEHICLE ($M_o = 2$)

FIGURE 7