GALERKIN BOUNDARY INTEGRAL ANALYSIS OF THE GRAD-SHAFRANOV EQUATION

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ABSTRACT

The Magneto Hydrodynamic equilibrium in an axisymmetric plasma is described in terms of the magnetic flux by the Grad-Shafranov Equation. This equation is commonly solved via domain-based discretization approaches such as finite difference and finite element methods. However for plasma equilibrium analysis on actual fusion devices (e.g., Tokamaks) a boundary-only discretization technique can be very attractive. In a boundary integral equation framework, only the surface of the plasma is discretized and geometric modifications and updates of the plasma surface are inexpensive.

In what follows we solve the Grad-Shafranov Equation using the Galerkin Boundary Element Method. We solve this non-linear partial differential equation combining the Dual Reciprocity Method with an iterative scheme. An expansion of the nonlinearity is given in terms of particular solutions of the Grad-Shafranov equation. Then an iteration procedure is applied to solve the PDE with the source expanded in terms of particular solutions. We can simplify the PDE to a homogenous one. When we apply the iteration procedure we update the boundary conditions only. In this fashion the problem is solved by using Galerkin boundary integral analysis.