SYSTEMATIC ASYMPTOTIC APPROXIMATIONS TO FORWARD KOLMOGOROV EQUATIONS FOR STOCHASTIC NEURONAL MODELS

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An Abstract of a Thesis Submitted to the Graduate

Faculty of Rensselaer Polytechnic Institute

in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: MATHEMATICS

The original of the complete thesis is on file
in the Rensselaer Polytechnic Institute Library

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Troy, New York

July 2011
(For Graduation August 2011)

ABSTRACT

In this thesis several problems in stochastic neuronal models will be addressed. The focus will be on improving solution methods for Forward Kolmogorov Equations (FKEs) that arise from stochastic neuronal models.

The first chapter contains a brief introduction to stochastic neuronal models. An overview of the results in this thesis and its relation to the relevant current literature will also be provided.

In the second chapter some problems in the current based stochastic neuronal model will be addressed. First it will be shown that provided the voltage process is ergodic, the flux based and mean exit time based definitions of stationary firing rate are equivalent. Second a formal limiting argument on the associated FKE will be used to derive the exact probability distribution in the asymptotic limit of large input Poisson rate. This result can be used to compute approximate firing rates when the input Poisson rate is large. Finally a simple correction to the usual implementation of the diffusion approximation on the Forward Kolmogorov Equation of the current based model will be shown to increase its accuracy.

The third chapter of the thesis is devoted to the study of the stochastic conductance based neuronal model. Only an approximate solution to the marginal density of the conductance variable is currently available [1]. First integral transform methods will be used to derive the exact solution for the distribution of conductance. Then that solution will be used to devise a systematic justification for moment closure dimensional reduction methods [1, 13, 2]. The systematic justification gives sufficient conditions on problem parameters for moment closure methods to be accurate, which in turn will be used to explain some observations in the numerical simulations in the current literature (Figure 12, [2], Figure 4.3 [1]). Finally a correction on the reduced trajectory dynamics in the small conductance time constant limit of the conductance based model will be presented. It is pointed out that this correction can be extended to devise a more biologically realistic one dimensional integrate and fire neuronal model that exhibit multiple spiking behavior (bursting)

in response to a large impulse.