SHEAR WAVE SPEED RECOVERY IN CRAWLING WAVE SONOELASTOGRAPHY

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ABSTRACT

In this thesis, we accomplish the following tasks.

First, we consider the forward problem for wave propagation under the generalized linear solid viscoelastic model, and: (a) prove existence, uniqueness, and regularity results for the solution of the initial-boundary value problem with a Lipschitz domain, L^∞ coefficients, and initial and source functions belonging to the proper Sobolev spaces; (b) establish higher regularity results in the case that the domain, coefficients, and initial and source functions are sufficiently smooth; and (c) prove finite propagation speed for the solution.

Second, we: (a) develop an algorithm to recover the shear wave speed from spectral variance data collected in the crawling wave experiment, where two sources oscillate at slightly different frequencies to create a moving interference pattern of shear waves; (b) demonstrate our algorithm by presenting reconstructed shear wave speed images from data simulated using the generalized linear solid wave equation model, showing that our algorithm produces better images than the shear wave speed estimate that was imaged in the past; and (c) show shear wave speed images reconstructed from *in vitro* prostate crawling wave data, where our results are confirmed by pathology.

Third, we: (a) reconstruct the shear wave speed from synthetic interference patterns created by combining two data sets of acoustic radiation crawling wave (ARC) data in homogeneous and inhomogeneous phantoms, successfully identifying the inclusion in the inhomogeneous phantom; and (b) consider several inversion methods for ARC data where the given data is two data sets, each consisting of two components of the displacement in a homogeneous medium, and compare the results from each method to determine which models are appropriate for this experiment.