ERROR ESTIMATION FOR THE DIRECT INVERSION MODEL AND NUMERICAL SCHEMES FOR THE FULL INVERSION MODEL IN ELASTOGRAPHY

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ABSTRACT

In this thesis, we focus on reconstructing shear modulus in biological tissue from single frequency elastographic data. Our goal is to resolve the following two issues in the inverse problem: (1) when is an approximate mathematical model acceptable for computations; and (2) how to solve the first order partial differential equation inverse problem model accurately without making approximations.

With the successful imaging of shear stiffness in soft tissue, a new medical tool for cancer diagnosis can be developed based on the close relationship between the elastic parameter and the abnormality of tissue. In all the experiments developed so far, the tissue is mechanically displaced and the interior displacement tissue response is measured on a fine grid by either ultrasound or MR.

For given single component displacement data, the mathematical model to recover the shear modulus is a first order partial differential equation, called the Full Inversion, while a much simpler algebraic model, called the Direct Inversion, can be derived by eliminating the first derivative terms of the shear modulus from the p.d.e. model. In this thesis, we establish a theoretical bound on the relative difference between the true value of the modulus and the approximated value reconstructed from the Direct Inversion. We exhibit a quantitative estimate of the relative error and demonstrate numerically that the Direct Inversion Model is a good approximation model in many cases.

On the other hand, it is more consistent with the forward problem model to reconstruct the shear modulus from the Full Inversion Model in the inverse problem. To capture possible exponential growth and decay of the targeted parameter μ numerically in a stable manner, we develop a log-elastographic nonlinear scheme and a linear finite difference based elliptic scheme. Both methods are proved to be convergent at first order and their performances are compared with the performances of a semi-implicit upwind scheme and the Direct Inversion Model. Shear moduli are reconstructed from synthetic data and the superior performance of the proposed schemes is demonstrated in cases where the shear modulus is rapidly changing.