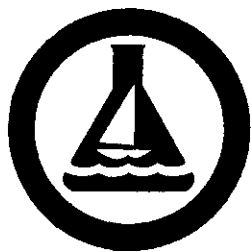


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RENSSELAER

Fresh Water Institute

AT LAKE GEORGE



LAKE GEORGE MODELING PROJECT
PRELIMINARY PROGRESS REPORT

By

R. A. Park and J. W. Wilkinson

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Rensselaer Polytechnic Institute

Troy, New York 12181

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PRELIMINARY PROGRESS REPORT

R. A. Park and J. W. Wilkinson

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June 1970

R. A. Park and J. W. Wilkinson

Introduction

The progress in developing a Lake George ecosystem model has been encouraging, particularly in the past three months. During the first phase of the project, simple models relating available diatom and environmental data were estimated and evaluated using methods of regression analysis; this led to a better understanding of biotic relationships specific to Lake George and resulted in several recommendations which have been implemented during subsequent data collection.

In the second phase of the project the modellers met with various representatives of the other Lake George projects in order to identify the form of the respective submodels. As a result of these meetings, several submodels have been developed: those of which, the hydrologic, nutrient, and phytoplankton submodels although requiring additional attention to algorithms, have been developed to the point where they may now be refined with the help of the project leaders and in light of data currently being collected. Two other submodels, the zooplankton and decomposer submodels, will be ready for experimentation shortly and programming on an integrative top-echelon model is well advanced.

In addition, in order to meet the immediate needs of the modelling group in synthesizing the many ecologic concepts, and interactive, free-format bibliographic is being developed and is almost complete. This will be available by means of Rensselaer's CPS time-sharing, as the first step in establishing an information system that is readily accessible for all Lake George investigators regardless of their location.

Programming Philosophy

All program development, with the exception of some of the hydrologic modelling, has been accomplished on-line using I.B.M. Conversational Programming System (CPS) PL/1. Time-sharing has significantly reduced development time and has permitted interactive experimentation with the models that otherwise would have been impossible. PL/1 has proven to be a very flexible language and is well suited to the demands of simulation. The hydrologic model, which was adapted "hands-on" with an I.B.M. 1130, is being translated to CPS PL/1 for compatibility.

The submodels are being written in modular form for ease in programming. Each submodel consists of numerous subroutine procedures which can be easily identified by any user and which can be expanded or changed without drastically affecting the rest of the program. At this stage, some of the procedures are null algorithms included only to indicate the logical flow of the simulation.

Within the next month, following the installation of additional core at Rensselaer's IBM 360/50, the submodels will be merged as external procedures which can be called from files and overlaid during execution. At that time the necessary arrays will be available to the various submodels and meaningful simulations can be performed - in contrast to the previous runs which have held key intrinsic variable arrays such as nutrient concentrations or plankton concentrations, constant for purposes of debugging.

The Submodels

We have tried to avoid using mechanistic or "black-box" submodels for any of the key functions of the ecosystem. Input variables are being kept to a minimum, and program logic is following physiologic principles and real-world processes as well as possible. However, it is anticipated that certain peripheral submodels, such as the physical limnologic submodel, will have to be greatly simplified in order to be consistent with overall modelling objectives.

The principal input variables are human population, rainfall temperature, and incident radiation. Population trends are predictive based on present trends and available land amenable to development. Rainfall and temperature follow seasonal trends, with a degree of uncertainty; these contribute appreciable stochastic elements to the model. Incident radiation can be calculated as a function of time of year and latitude, with a noticeable correction for the shading by surrounding mountains and with some variation imposed by cloud cover.




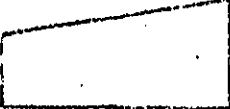
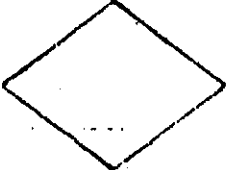



With these considerations in mind, the hydrologic submodel has incorporated a random component to help to mimic the uncertainty of the real-world. Currently the nutrient submodel is inherently deterministic assuming predictable population trends disregarding noise; however, it is sensitive to the fluctuations of the hydrologic submodel because of the input of nutrients by way of rainfall and runoff and because of significant changes in the volume of the Lake

The phytoplankton submodel, when more fully developed, will probably be sensitive to the changes in nutrient concentrations produced by the nutrient submodel and to the fluctuations in temperature (input directly to the submodel) and the seasonal change in incident radiation. The phytoplankton submodel, which is based primarily on Riley's work, utilizes difference equations to approximate phytoplankton distributions with respect to depth and time. It will soon be expanded to compute available nutrients with depth and will then be complementary to the nutrient submodel, which computes total nutrient concentrations. It is hoped that a short-term version with provision for diurnal variations in photosynthesis and grazing rates can be developed to meet the needs of some of the other Lake George investigators.

We have considered including spatial variations in the model as well as depth and time variations; however, it was felt that the computational load would be prohibitive. Instead, we propose to subdivide the lake into "cells" based on data on diatom heterogeneity now being collected. The nutrient and phytoplankton submodels have been programmed with this in mind, and the hydrologic submodel is being rewritten to yield continuous estimates of rainfall instead of the discrete Thiessen-Polygon estimates. Each cell would be simulated as a separate run, and the output variables (rates of productivity, nutrient concentrations, etc.) would be synthesized by an integrative top-echelon model. This model, which is currently being developed, will be essentially a response-surface model with the independent variables being the map coordinates and each of the dependent variables, the output variables from the simulation for each of the cells. The principal output will be contour maps of the lake and, where applicable, lake basin; these will probably be by means of an analog plotter for greater resolution.

At this stage the modelling activity has achieved an encouraging level. Part of this is due to the interest developed by some of the authors' students, namely, Paul Beck, William Caswell, James Hill and Curtis Norwood. Their participation is gratefully acknowledged, and it is hoped that appropriate funding will permit their continued involvement.

FLOWCHART SYMBOLS

SYMBOL	REPRESENTS
	The beginning, end, or a point of interruption in a program.
	Processing functions of the program.
	Output to the remote terminal.
	Input from the remote terminal.
	Decision function indicating branches to alternate paths based on variable conditions.
	Offpage connector indicating an entry or an exit from a page.
	Connector indicating entry from, or an exit to another part of the program.
	The direction of processing or data flow.

APPENDIX A

PART I

Hydrologic Submodel (HYDRØL)

EMILI,H2.

```
1. DECLARE P(6,12), A1(6), A2(6), DIS(12), GDIS(12), DAY(12);
2. DECLARE TEMP(12), DELH(12), WP1(6), WP2(6), AVEP1(12);
3. DECLARE AVEP2(12), TDIS(12), WL(13), VOL(13), VOLL(13);
4. DECLARE RUNOF(12), PE(12), ACUSE(12);
5. DECLARE PREC(12), MONTH(12) CHAR(3);
6. MONTH(1)='OCT';
7. MONTH(2)='NOV';
8. MONTH(3)='DEC';
9. MONTH(4)='JAN';
10. MONTH(5)='FEB';
11. MONTH(6)='MAR';
12. MONTH(7)='APR';
13. MONTH(8)='MAY';
14. MONTH(9)='JUN';
15. MONTH(10)='JUL';
16. MONTH(11)='AUG';
17. MONTH(12)='SEP';
18. PUT LIST('STATE NUMBER OF YEARS');
19. GET LIST(MOYRS);
20. PUT LIST('INITIALIZE WATER LEVEL');
21. GET LIST(WL(1));
22. PUT LIST('INITIALIZE SOIL MOISTURE');
23. GET LIST(ASTOR);
24. PUT LIST('STATE CORRECTION FOR RUNOFF');
25. GET LIST(PORR);
26. L7: DO KYRS=1 TO MOYRS;
27. GET LIST(STATE);
28. GET LIST(P);
29. GET LIST(A1,A2);
30. GET LIST(DIS);
31. GET LIST(GDIS);
32. GET LIST(DAY);
33. GET LIST(TEMP);
34. H1=45.16;
35. A=.00000675*H1**3-.0000771*H1**2+.0179*H1+.49;
36. L1: DO M=1 TO 12;
37. TDIS(M)=GDIS(M)*.03719/44;
38. END L1;
39. L0: DO M=1 TO 12;
40. N=M+1;
41. CALL PREC;
42. CALL RUNOFF;
43. CALL BAL;
44. DELH(N)=(RUNOF(N)+AVEP1(N)-PE(N)-TDIS(N))/12;
45. WL(N)=WL(N)+DELH(N);
46. VOL(N)=2.1*((WL(N)-301)/18);
47. END L0;
48. CALL SKIPL(2);
49. PUT LIST('-----');
50. CALL SKIPL(2);
51. PUT LIST(' WATER YEAR',DATE);
52. CALL SKIPL(2);
53. PUT IMAGE(WL(1))(F3);
54. F3: IMAGE;
55. INITIAL WATER LEVEL -----
56. CALL SKIPL(2);
57. PUT LIST(' LAND SURFACE WATER BALANCE');
58. CALL SKIPL(1);
59. PUT LIST('MTH. PRECIPITATION RUNOFF AVAILABLE P. ACT. ');
60. PUT LIST(' N LAKE PPTN. EVAPT. EVAPT.');
```

```

61. PUT IMAGE(MONTH(I),AVEP2(I),RUNOF(I),PREM(I),PE(I),ACUSE(I))(F1);
62. F1: IMAGE;
-----
63. END L2;
64. CALL SKIPL(2);
65. PUT LIST(' LAKE WATER BALANCE');
66. CALL SKIPL(1);
67. PUT LIST('MTH. PRECIPITATION INPUT LAKE LAKE CHANGE');
68. PUT LIST(' RUNOFF RUNOFF EVAPO DIS. HEIGHT');
69. L3: DO I=1 TO 12;
70. PUT IMAGE(MONTH(I),AVEP1(I),RUNOF(I),PE(I),TDIS(I),DELH(I))(F1);
71. END L3;
72. CALL SKIPL(2);
73. PUT LIST('CHANGE HEIGHT=CHANGE LAKE ELEVATION IN FEET');
74. PUT LIST('ALL OTHER UNITS IN INCHES');
75. PUT LIST('MTH. WATER LEVEL VOLUME');
76. F2: IMAGE;
-----
77. L5: DO I=1 TO 12;
78. J=I+1;
79. PUT IMAGE(MONTH(I),WL(J),VOL(J))(F2);
80. END L5;
81. PUT LIST('WATER LEVEL IN FT. ABOVE MEAN SFA LEVEL,');
82. PUT LIST('VOLUME IN CUBIC KM. ');
83. L6: DO I=2 TO 13;
84. VOL(I)=VOL(I)*.1E13;
85. END L6;
86. CALL SKIPL(2);
87. PUT LIST('-----');
88. WL(I)=WL(I3);
89. END L7;
90. STOP;
91. PRFC: PROCEDURE;
92. AVEP1(M)=0;
93. AVEP2(M)=0;
94. L4: DO L=1 TO 6;
95. WP1(L)=A1(L)*P(L,M);
96. WP2(L)=A2(L)*P(L,M);
97. AVEP1(M)=AVEP1(M)+WP1(L);
98. AVEP2(M)=AVEP2(M)+WP2(L);
99. END L4;
100. RETURN;
101. END PRFC;
102. RUNOFF: PROCEDURE;
103. RUN=DIS(M)/23.4*.03719;
104. PRE1=P(2,M)*.862;
105. PRE2=P(3,M)*.133;
106. RUNCO=RUN/(PRE1+PRE2);
107. RUNOF(M)=AVEP2(M)+RUNCO*190/44*CORR;
108. RETURN;
109. END RUNOFF;
110. BAL: PROCEDURE;
111. TEMPO=TEMP(M)-32;
112. IF TEMPO<=0 THEN GO TO SKIP;
113. TEMPO=(59/9*TEMPO/M)**A;
114. PE(M)=DAY(M)=(1.6/2.54)*TEMPO;
115. GO TO SKIP1;
116. SKIP: PE(M)=0;
117. SKIP1: PREM(M)=AVEP2(M)-RUNOF(M)*(44/190);
118. IF PREM(M)<0 THEN PREM(M)=0;
119. ASTOR=ASTOR+PREM(M)-PE(M);
120. IF ASTOR<=0 THEN ASTOR=0;
121. AVAIL=PREM(M)+ASTOR;
122. IF AVAIL-PE(M)>=0 THEN ACUSE(M)=PE(M); ELSE ACUSE(M)=AVAIL;
123. RETURN;
124. END BAL;

```

```
125. MAIN: END ;
126. SKIPL: PROCEDURE (NSP);
127. L10: DO J=1 TO NSP;
128. PUT LIST(' ');
129. END L10;
130. RETURN ;
131. END SKIPL;
```

1. b) Input Information

All input is keyed in from the remote terminal in free format.
CPS will request input by outputting the variable's name.

See the sample input for this model below.

STATE NUMBER OF YEARS

NYRS

23

INITIALIZE WATER LEVEL

W(1)

2319.45

INITIALIZE SOIL MOISTURE

ASTOR

73.

STATE CORRECTION FOR RUNOFF

CORR

21.00

DATE

21966

P(1, 1)

25.11 6.0 2.91 3.39 3.9 5.7 1.25 3.54 2.49 2.7 5.28 4.55

P(2, 1)

23.02 3.69 1.58 1.79 2.92 3.89 .74 4.71 1.86 2.15 4.56 4.55

P(3, 1)

23.36 3.18 .91 1.14 2.24 2.54 .64 1.79 3.26 2.41 3.17 3.16

P(4, 1)

22.93 3.36 1.57 1.23 2.58 2.75 .97 2.78 2.78 2.46 6.11 3.99

P(5, 1)

22.61 3.39 1.26 1.75 2.76 3.29 1.11 3.43 2.30 2.79 4.06 5.15

P(6, 1)

23.26 4.01 1.53 2.2 3.85 4.54 1.58 5.47 4.22 3.58 6.2 6.88

AI(1)

2.35 .24 .24 .15 0. .04 .23 .35 .21 .12 .04 .05

DIS(1)

7601. 1428. 970. 452. 416.6 2595. 1197. 1532. 356.8 55.4 64.2 154.

SDIS(1)

20482. 5929. 6299. 8345. 9833. 12085. 12779. 16563. 8567. 2407. 12

ODIS(12)

23055.

PAY(1)

2.92 .81 .74 .78 .86 .99 1.12 1.22 1.23 1.25 1.16 1.05

TEMP(1)

27.8 37.3 30.1 20.9 21.9 34.1 44. 54.7 68.2 73. 70.8 59.1

1. c) Sample Output

WATER YEAR 1966
 WL(1) = 319.45

LAND SURFACE WATER BALANCE

| MTN. | PRECIPITATION | RUNOFF
IN LAKE | AVAILABLE
PPTH. | P.
EVAPT. | ACT.
EVAPT. |
|------|---------------|-------------------|--------------------|--------------|----------------|
| OCT | 3.10 | 4.16 | 2.13 | 1.49 | 1.49 |
| NOV | 4.08 | 11.04 | 1.52 | .30 | .30 |
| DEC | 1.70 | 7.63 | 0. | 0. | 0. |
| JAN | 1.97 | 3.60 | 1.14 | 0. | 0. |
| FEB | 3.00 | 3.04 | 2.30 | 0. | 0. |
| MAR | 3.88 | 18.83 | 0. | .12 | 0. |
| APR | .92 | 10.42 | 0. | 1.13 | 0. |
| MAY | 3.58 | 8.75 | 1.56 | 2.67 | 1.56 |
| JUN | 2.54 | 3.04 | 1.84 | 4.86 | 1.84 |
| JUL | 2.47 | .43 | 2.37 | 5.52 | 2.37 |
| AUG | 4.68 | .47 | 4.57 | 4.79 | 4.57 |
| SEP | 4.33 | 1.06 | 4.09 | 2.82 | 2.82 |

LAKE WATER BALANCE

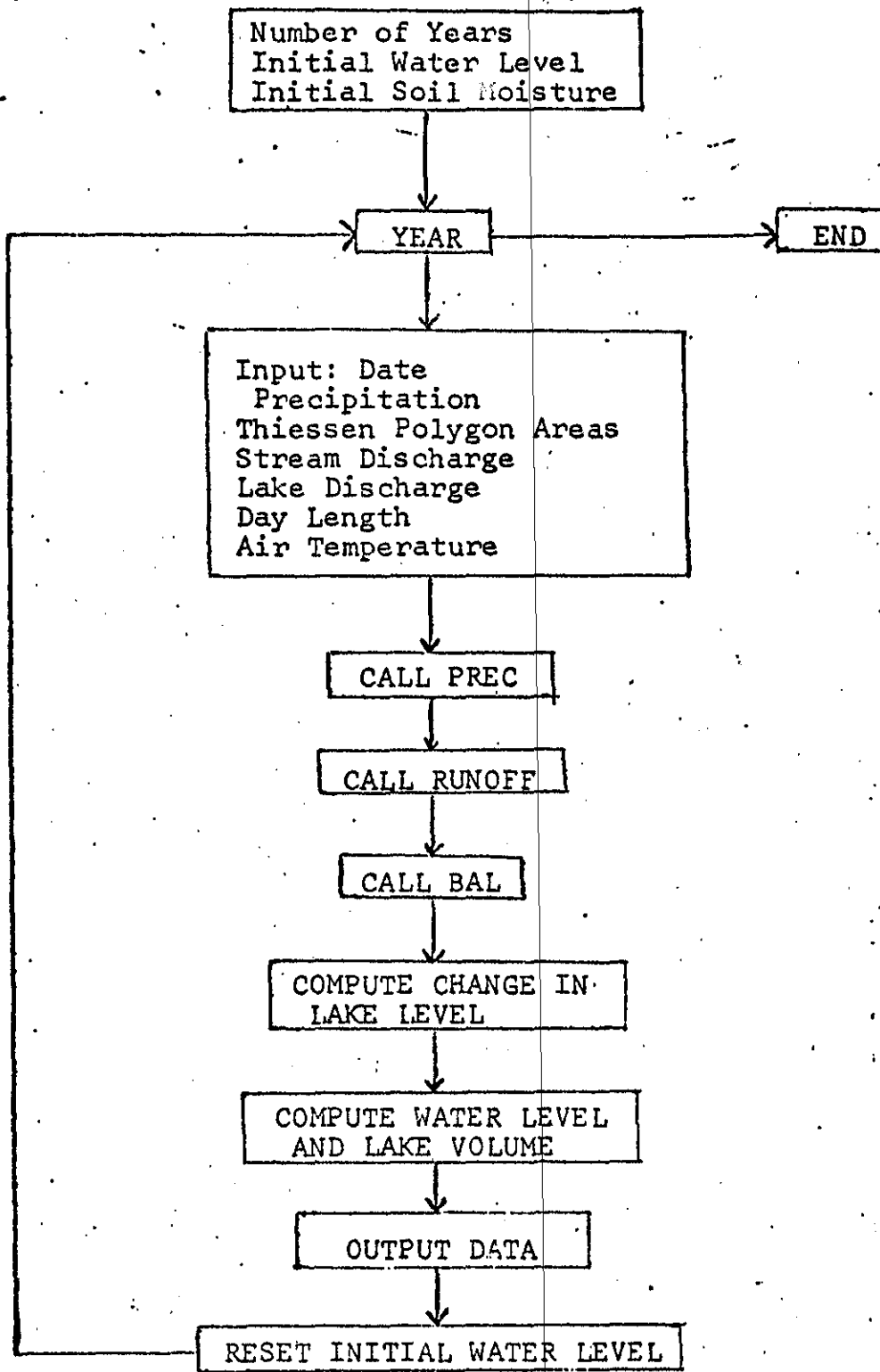
| MTN. | PRECIPITATION | LAKE WATER
INPUT
RUNOFF | LAKE
EVAPD | LAKE
DIS. | CHANGE
HEIGHT |
|------|---------------|-------------------------------|---------------|--------------|------------------|
| OCT | 3.13 | 4.15 | 1.49 | 2.03 | .31 |
| NOV | 4.29 | 11.04 | .30 | 5.01 | .83 |
| DEC | 1.82 | 7.63 | 0. | 7.01 | .20 |
| JAN | 2.09 | 3.60 | 0. | 7.05 | -.11 |
| FEB | 3.07 | 3.04 | 0. | 4.97 | .09 |
| MAR | 4.01 | 18.83 | .12 | 10.21 | 1.04 |
| APR | .95 | 10.42 | 1.13 | 10.80 | -.05 |
| MAY | 3.35 | 8.75 | 2.67 | 14.00 | -.38 |
| JUN | 2.64 | 3.04 | 4.86 | 7.24 | -.54 |
| JUL | 2.50 | .43 | 5.52 | 2.03 | -.39 |
| AUG | 4.76 | .47 | 4.79 | 1.09 | -.05 |
| SEP | 4.23 | 1.06 | 2.82 | 2.58 | -.01 |

CHANGE HEIGHT = CHANGE LAKE ELEVATION IN FEET
 ALL OTHER UNITS IN INCHES

| MTN. | WATER LEVEL | VOLUME |
|------|-------------|--------|
| OCT | 319.76 | 2.19 |
| NOV | 320.60 | 2.29 |
| DEC | 320.80 | 2.51 |
| JAN | 320.69 | 2.30 |
| FEB | 320.73 | 2.51 |
| MAR | 321.82 | 2.43 |
| APR | 321.77 | 2.42 |
| MAY | 321.39 | 2.39 |
| JUN | 320.86 | 2.32 |
| JUL | 320.47 | 2.27 |
| AUG | 320.42 | 2.27 |
| SEP | 320.41 | 2.26 |

WATER LEVEL IN FT. ABOVE MEAN SEA LEVEL,
 VOLUME IN CUBIC KW.

2. Hydrologic Submodel (HYDRØL) - Flowchart



3. Hydrologic Submodel (HYDRØL)

- a) This is a simple water budget and balance model to predict the water level of the lake and its corresponding volume. The average precipitation over the basin and over the lake, and the lake evaporation are also determined. The Hydrological Equation, $\text{Inflow} = \text{Outflow} \pm \Delta \text{Storage}$, is solved with change in storage as the unknown variable.
- b) CPS PL1.
- c) IBM 360/50.
- d) Type III, Class A Program, version 4 CPS.
- e) Algebraic functions are utilized.
- f) No unusual numerical methods.
- g) (i) The Hydrologic Equation
Line 46. $\text{DELH}(M) = (\text{RUNOF}(M) + \text{AVEP1}(M) - \text{PE}(M)).$
- (ii) Lake Volume
Line 50. $\text{VOL}(N) = 2.1 * ((\text{WL}(N) - 301)/18);$
- (iii) Runoff into Lake
Line 270. $\text{RUNØF}(M) = \text{AVDP2}(M) * \text{RUNCO} * (190/44);$
- (iv) Potential Evapotranspiration, Thornthwaite Method.
Line 296. $\text{TEMPØ} = (50/9 * \text{TEMPØ}/\text{HI})^{**A}$
Line 298. $\text{PE}(M) = \text{DAY}(M) * (1.6/2.54) * \text{TEMPØ}.$
- h) (i) P(6,12) Precipitation for each station, for one year, (inches).
- (ii) A1 Thiessen Polygon Areas over Lake, (%).
- (iii) A2 Thiessen Polygon Areas over Basin, (%).
- (iv) DIS(12) Total discharge from Northwest Bay Brook, (c.f.s. per month).
- (v) GDIS (12) Total discharge from Lake, (c.f.s. per month).
- (vi) DAY(12) Correction for day length, based on 12 hours of daylight.
- (vii) TEMP(12) Mean monthly air temperature.
- (viii) DELH(M) Change in Lake storage, (inches).
- (ix) VOL(N) Lake Volume, (Km^3).
- (x) AVEP1(M) Average Precipitation over Lake, (inches).
- (xi) AVEP2(M) Average Precipitation over Basin, (inches).
- (xii) RUNCO Runoff coefficient, determined for Northwest Bay Brook.

- (xii) RUNØF(M) Runoff into Lake, (inches)
- (xiv) PE(M) Potential Evapotranspiration and Lake Evaporation, (inches).
- (xv) PREM(M) Available precipitation for Consumptive use, (inches).
- (xvi) ACUSE(M) Consumptive use or Actual Evapotranspiration, (inches).

APPENDIX A

PART II

Simulation of Monthly Streamflow

```

C      COMPUTATION OF MONTHLY FLOW STATISTICS AND MULTIPLE REGRESSION
C      SIMULATED MONTHLY STREAMFLOW GENERATOR FOR SIX STATIONS OR LESS
CCCC1  DIMENSION N(6),AA(6),DC(6),Q(6,19),O6(6,13),S(6,13),G(6,13)
CCCC2  DIMENSION PC(21,13),PP(21,21),PC4(6,12),PP6(6,17),SOAT(12)
CCCC3  DIMENSION SSOAT(12),O6T(12),O7(6),F(6,13),F(6,12),B(7),A(7,8)
CCCC4  DIMENSION AS(6,17),SS(6,12),GS(6,12),AG(14),SA(14),A6S(17),SACS(17)
CCCC5  DIMENSION BR(27,12),D(6,12),XC(6,13),OT(6,12),AX(6,17)
CCCC6  DIMENSION SX(6,17),GA(6),AL(6,17)
CCCC7  DIMENSION MONTH(12),SNAME(6,19),RC(7)
CCCC8  DATA RC/'01','02','03','04','05','06','07'/
CCCC9  DATA MONTH/'OCT ','NOV ','DEC ','JAN ','FEB ','MAR ','APR ','MAY ','
C      1,'JUNE','JULY','AUG ','SEPT'/
C      READ NO. OF PERIODS, LENGTH OF PERIODS YRS., I, AND RW. NO. SEED
CC10   READ(5,2)NP,LP,ISEED
C      READ NO. OF STATIONS AND NO. OF YRS. OF DATA FOR COMPUTATIONS
CC11   READ(5,2)NS,NY
CC12   FORMAT(10I5)
C      READ STATION IDS. AND DESCRIPTIONS
CC13   READ(5,20)(NIL),(SNAME(L,I),I=1,10),L=1,NS)
CC14   FORMAT(15,5X,10A4)
C      READ AVE. ANNUAL FLOW/STATION
CC15   READ(5,22)(AA(L),L=1,NS)
CC16   FORMAT(RE10.0)
C      WRITE HEADINGS AND INPUT INFO.
CC17   WRITE(3,105)
CC18   FORMAT(10X,'S I M U L A T I O N O F M O N T H L Y R U N D F
C      I F F')
CC19   WRITE(3,101)
CC20   FORMAT(//10X,'S I M U L A T I O N O F M O N T H L Y R U N D F')
CC21   WRITE(3,102)NP,LP,ISEED
CC22   102  FORMAT(//15X,'NUMBER OF PERIODS OF SIMULATION',13/15X,'LENGTH OF TH
C      1E PERIOD(YEARS)',13/15X,'INITIAL RANDOM NUMBER SEED',16)
CC23   WRITE(3,103)
CC24   103  FORMAT(//10X,'MONTHLY FLOW STATISTICS AND MULTIPLE REGRESSION')
CC25   WRITE(3,104)NY,NS
CC26   104  FORMAT(//15X,'NUMBER OF YEARS OF DATA USED IN COMPUTATIONS',13/15X,
C      1'NUMBER OF STATIONS USED IN COMPUTATIONS',13)
CC27   WRITE(3,105)
CC28   105  FORMAT(//10X,'STATION IDENTIFICATION'/15X,'STATION ID.',10X,'AVERAG
C      1E ANNUAL FLG',10X,'DESCRIPTION')
CC29   WRITE(3,106)(NIL),AA(L),(SNAME(L,I),I=1,10),L=1,NS)
CC30   106  FORMAT(10X,15,17X,E11.4,10X,10A4)
CC31   NY=NY-1
CC32   AN=NYB
CC33   BN=AN-1.
CC34   ANA=.25/(AN-1.)
CC35   ANH=SOAT(AN)
CC36   ANC=.35/(AN-1.)
CC37   AND=.71/SOAT(AN-2,3)
CC38   ANE=.17/(AN-1.6)
CC39   CN=AN/(AN-2.)
CC40   LX=0
CC41   DO 3 L=1,NS
CC42   LXR=LX
CC43   DC(L)=AA(L)*0.001
CC44   AA(L)=AA(L)*1.012
CC45   READ(5,4)(Q(L,M),M=8,19)
CC46   FORMAT(10X,6E10.0)
    
```

1. a) Simulation of Monthly Streamflow - Program Listing

```

CC47      DO 5 M=1,7
CC48      5      Q(L,M)=Q(L,M+12)+DQ(L)
CC49      Q6(L,1)=0.
CC50      DO 6 M=1,6
CC51      6      D6(L,1)=C6(L,1)+G(L,M)
CC52      DO 7 M=1,13
CC53      S(L,M)=0.
CC54      G(L,M)=0.
CC55      DO 7 L2=1,L
CC56      LX=LXR+L2
CC57      7      PC(LX,M)=0.
CC58      DO 3 M=1,17
CC59      DO 8 L2=1,L
CC60      LX=LXR+L2
CC61      8      PP(LX,M)=0.
CC62      PC6(L,M)=0.
CC63      3      PP6(L,M)=0.
CC64      DO 9 M=1,17
CC65      SG6T(M)=0.
CC66      9      SSG6T(M)=0.
CC67      DO 10 NYC=1,NYB
CC68      DO 11 M=1,17
CC69      11     S6T(M)=0.
CC70      DO 17 L=1,NS
CC71      READ(5,4)(C(L,M),M=8,19)
CC72      DO 13 M=1,12
CC73      Q(L,M+7)=Q(L,M+7)+DQ(L)
CC74      Q6T(M)=Q6T(M)+C6(L,M)/AA(L)
CC75      13     Q6(L,M+1)=C6(L,M)+Q(L,M+6)-C(L,M)
CC76      Q6(L,1)=Q6(L,13)
CC77      DO 14 M=1,6
CC78      14     Q(L,M)=Q(L,M+12)
CC79      12     Q7(L)=C(L,19)
CC80      DO 15 M=1,17
CC81      Q6T(M)=ALOG(Q6T(M))
CC82      SG6T(M)=SG6T(M)+Q6T(M)
CC83      15     SSG6T(M)=SSG6T(M)+Q6T(M)+Q6T(M)
CC84      LX=0
CC85      DO 16 L=1,NS
CC86      LXR=LX
CC87      DO 17 M=1,13
CC88      C(L,M+6)=ALOG(Q(L,M+6))
CC89      S(L,M)=S(L,M)+C(L,M+6)
CC90      G(L,M)=G(L,M)+C(L,M+6)+Q(L,M+6)+Q(L,M+6)
CC91      DO 17 L2=1,L
CC92      LX=LXR+L2
CC93      17     PC(LX,M)=PC(LX,M)+Q(L,M+6)+Q(L2,M+6)
CC94      DO 19 M=1,12
CC95      DO 18 L2=1,L
CC96      LX=LXR+L2
CC97      18     PP(LX,M)=PP(LX,M)+Q(L,M+6)+C(L2,M+7)
CC98      PC6(L,M)=PC6(L,M)+Q6T(M)+Q(L,M+7)
CC99      19     PP6(L,M)=PP6(L,M)+Q6T(M)+C(L,M+6)
C100      10     Q(L,7)=Q7(L)
C101      LX=0
C102      DO 26 L=1,NS
C103      LXR=LX
C104      DO 28 M=1,13

```

```

C105      F(L,M)=S(L,M)/AN
C106      DO 23 L2=1,L
C107      LX=LXR+L2
C108      23  PC(LX,M)=PC(LX,M)-E(L,M)*S(L2,M)
C109      F(L,M)=SQRT(PC(LX,M)/BN)
C110      GT=G(L,M)-F(L,M)*(3.0*PC(LX,M)+F(L,M)*S(L,M))
C111      28  G(L,M)=GN+GT/(F(L,M)+PC(LX,M))
C112      DO 21 M=1,L2
C113      DO 25 L7=1,L
C114      LX=LXR+L7
C115      25  PP(LX,M)=PP(LX,M)-E(L,M)*S(L2,M+1)
C116      PC(L,M)=PC(L,M)-SCGT(M)*E(L,M+1)
C117      21  PP(L,M)=PP(L,M)-SCGT(M)*E(L,M)
C118      DO 26 M=1,L3
C119      E(L,M)=F(L,M)*0.43429
C120      26  F(L,M)=F(L,M)*0.43429
C121      DO 27 M=1,L7
C122      27  SSCGT(M)=SSCGT(M)-SCGT(M)*SCAT(M)/AN
C123      FX=-1
C124      LX=0
C125      DO 31 K=1,NS
C126      LXM=LX
C127      KA=K+1
C128      KAA=K+2
C129      KXR=KX-1
C130      WRITE(3,115)
C131      115  FORMAT(1H1,2X,'MONTHLY RUNOFF STATISTICS AND MULTIPLE REGRESSION'
1)
C132      WRITE(3,116)N(K),ISNAME(K,1),I=1,19)
C133      116  FORMAT(1X,'STATION ID.',I5,' ',I0A4)
C134      WRITE(3,117)IDC(1),I=1,KA)
C135      117  FORMAT(1X1,'MULTIPLE',I21,'MEAN',T42,'SKEW',T51,'DETER.',T11,
1'MONTH LOG(FLOW) STD.DEV.',2(3X,'COEFF.',1,7(6X,A7))
C136      DO 30 M=2,L3
C137      DO 36 J=3,KAA
C138      KX=KXR+J
C139      A(1,J)=PC6(J-2,M-1)
C140      36  A(2,J)=PP(KX,M-1)
C141      A(1,1)=SSCGT(M-1)
C142      A(1,2)=PP6(K,M-1)
C143      A(2,2)=PC(KX,M-1)
C144      KX=0
C145      IF (K-1)37,37,38
C146      38  DO 40 J=3,KA
C147      DO 40 I=3,J
C148      KX=KX+1
C149      40  A(I,J)=PC(KX,M)
C150      DO 41 I=3,KA
C151      KX=KX+1
C152      41  A(I,KAA)=PC(KX,M)
C153      37  DO 45 ID=1,K
C154      IDA=ID+1
C155      DO 42 I=IDA,KA
C156      42  A(I,10)=A(ID,1)
C157      DO 48 J=IDA,KAA
C158      48  A(ID,J)=A(ID,J)/A(ID,10)
C159      DO 45 I=IDA,KA

```

```

C161      45  A(I,J)=A(I,J)-A(I,10)*A(I0,J)
C162      R(KA)=ACKA,KAA)/A(KA,KA)
C163      I=K
C164      53  IA=I+1
C165      B(I)=A(I,KAA)
C166      DO 51 J=IA,KA
C167      51  B(I)=B(I)-B(J)*A(I,J)
C168      I=I-1
C169      IF (I)57,57,53
C170      57  DC=B(I)*PC6(K,M-1)*R(2)*PP(KX+1,M-1)
C171      B(I)=B(I)*SQRT(SSC6T(M-1)/PC(KX+1,M))
C172      B(2)=B(2)*F(K,M-1)/F(K,M)
C173      DO 61 J=1,KA
C174      LX=LXR+J
C175      IF (K-1)61,61,59
C176      59  IF (J-3)61,60,60
C177      60  XX=KXR+J
C178      DC=DC*(R(J)*PC(KX,M))
C179      B(J)=B(J)*F(J-2,M)/F(K,M)
C180      61  DO(LX,M-1)=B(J)
C181      D(K,M-1)=DC/PC(KX+1,M)
C182      30  WRITE(3,1)MONTH(M-1),E(K,M),F(K,M),G(K,M),D(K,M-1),
1      (B(J),J=1,KA)
C183      108  FORMAT(11X,A4,2(1X,F7.3,1X),2(2X,F7.3),3X,7(2X,F6.3))
C184      M1=1
C185      M2=13
C186      DO 32 M=1,2
C187      F(K,M1)=E(K,M2)
C188      F(K,M1)=F(K,M2)
C189      G(K,M1)=C(K,M2)
C190      M1=M1*M2
C191      32  M2=M2-11
C192      DO 31 M=2,13
C193      AS(K,M-1)=(0.84*E(K,M)+0.08*(E(K,M-1)+E(K,M+1)))/2.3726
C194      SST=0.5*F(K,M)+F(K,M)+0.25*(F(K,M-1)+F(K,M-1)+F(K,M+1)+F(K,M+1))
C195      SS(K,M-1)=SQRT(SST)/2.3026
C196      CS(K,M-1)=0.3*G(K,M)+0.15*(G(K,M-1)+G(K,M+1))
C197      31  AL(K,M-1)=SQRT(1.0-D(K,M-1))
C198      DO 62 M=2,13
C199      A6(M)=SQ6T(M-1)/AN=0.43429
C200      62  S6(M)=SQRT(SSC6T(K-1)/BN)+0.43429
C201      M1=1
C202      M2=13
C203      DO 73 M=1,2
C204      A6(M1)=A6(M2)
C205      S6(M1)=S6(M2)
C206      M1=M1*M2
C207      73  M2=M2-11
C208      DO 72 M=2,13
C209      A6S(M-1)=(0.84*A6(M)+0.08*(A6(M-1)+A6(M+1)))/2.3026
C210      S6ST=0.25*(S6(M-1)+S6(M-1)+S6(M+1)+S6(M+1))
C211      72  S6S(M-1)=SQRT(0.5*S6(M)+S6(M)+S6ST)/2.3026
C212      DO 75 IP=1,NP
C213      DO 75 I=1,3
C214      GO TO (71,71,76),I
C215      71  IF (NP-1)75,75,78
C216      78  LPA=3
          TO TO 79

```

```

C218 76 LPA=LP+2
C219 79 GO 80 L=1,NS
C220 DO 77 M=1,7
C221 77 Q(L,M)=FXP(AS(L,M+5))
C222 QAT(L)=0.
C223 DO 81 M=1,6
C224 81 QAT(L)=QAT(L)+Q(L,M)
C225 80 XC(L,1)=0.
C226 DO 99 J=1,LPA
C227 DO 85 M=1,12
C228 QATT=0.
C229 DO 86 L=1,NS
C230 QATT=QATT+QAT(L)/AA(L)
C231 86 QAT(L)=QAT(L)+Q(L,M+6)-Q(L,M)
C232 X6=(ALOG(QATT)-A6S(M))/S6S(M)
C233 LX=0
C234 DO 85 L=1,NS
C235 LB=L-1
C236 CALL GAUSS(ISEED,1.0,D,RY)
C237 XC(L,M+1)=BB(LX+1,M)+X6+BB(LX+2,M)+XC(L,M)+AL(L,M)*RY
C238 LX=LX+2
C239 IF (L-1)87,87,88
C240 88 DO 89 L2=1,LH
C241 LX=LX+1
C242 XC(L,M+1)=XC(L,M+1)+BB(LX,M)+XC(L2,M+1)
C243 87 CT(L,M)=XC(L,M+1)+J.16*GS(L,M)+[XC(L,M+1)+XC(L,M+1)-1.0]
C244 C(L,M+7)=EXP(AS(L,M)+QT(L,M)+SS(L,M))
C245 IF (M-5)89,95,99
C246 90 C(L,M-5)=O(L,M+7)
C247 IF (M-12)85,84,84
C248 84 XC(L,1)=XC(L,13)
C249 85 CONTINUE
C250 GO TO (91,119,94),I
C251 91 IF (J-3)99,95,95
C252 95 DO 96 L=1,NS
C253 DO 96 M=2,13
C254 IF (XC(L,M))97,98,98
C255 97 XT=XC(L,M)*(1.+ANA*XC(L,M)+(XC(L,M)-1.0))
C256 GO TO 96
C257 98 XT=XC(L,M)*(1.0+ANA*XC(L,M)+(XC(L,M)+1.0))
C258 AX(L,M-1)=AS(L,M-1)+XT*SS(L,M-1)/ANA
C259 GO TO 99
C260 IF (J-3)99,120,120
C261 120 DO 121 L=1,NS
C262 DO 121 M=2,13
C263 121 SX(L,M-1)=SS(L,M-1)*EXP(ANC-AND*XC(L,M)+ANE*XC(L,M)+XC(L,M))
C264 GO TO 99
C265 94 IYR=J-2
C266 IF (IYR)99,99,122
C267 122 DO 123 L=1,NS
C268 QAT(L)=0.
C269 DO 124 M=8,19
C270 IF (M-1)129,129,130
C271 129 C(L,M)=O(L,M)-DQ(L)
C272 GO TO 127
C273 130 Q(L,M)=EXP(AX(L,M-7)+CT(L,M-7)+SX(L,M-7))-DQ(L)
C274 127 IF (Q(L,M))125,124,124
C275 125 C(L,M)=0.

```

A-13

A-13

```
C276      124      QA(L)=QA(L)+Q(L,M)
C277      123      CONTINUE
C278      109      WRITE(3,109)IP,IYR,(N(L),L=1,NS)
C279      109      FORMAT(11H//25X,'SIMULATED RUNOFF'//10X,'PERIOD',12//15X,
      109      1'SIMULATED YEAR',13//116,'STATION ID.'/11,'MONTH',8X,6(15,4X))
C280      DO 126 M=8,19
C281      126      WRITE(3,110)MONTH(M-7),(Q(L,M),L=1,NS)
C282      110      FORMAT(11X,A4,6(3X,F7.1,3X))
C283      111      WRITE(3,111)(QA(L),L=1,NS)
C284      111      FORMAT(//10X,'TOTAL'//10X,'ANNUAL'//10X,'FLOW ',6(3X,F7.1,3X))
C285      99      CONTINUE
C286      75      CONTINUE
C287      CALL EXIT
C288      END
```

1. b) Input information

A. Card type 1.

NP, LP, ISEED,

where NP = number of periods desired,

LP = length of the periods in years

and ISEED - seed for random number generator,

FORMAT (10I5).

B. Card type 2.

NS, NY,

where NS = number of stations,

and NY = number of years

FORMAT (10I5).

C. Card type 3.

N(1), SNAME (L,I),

where N(L) = identification number for station L,

and SNAME (L,I) = vector of characters representing
the name for station L

FORMAT (I5,5X,10A4).

D. Card type 4.

AA(L),

where AA(L) = average annual flow for station L

FORMAT (8E10.0).

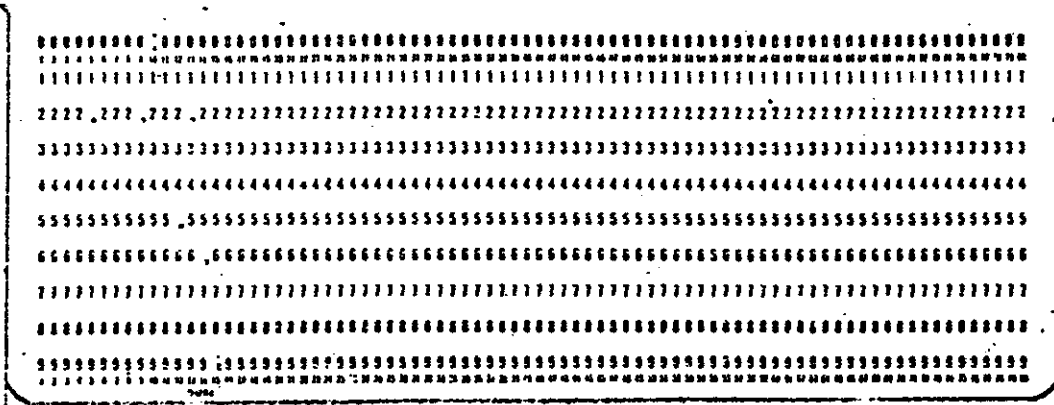
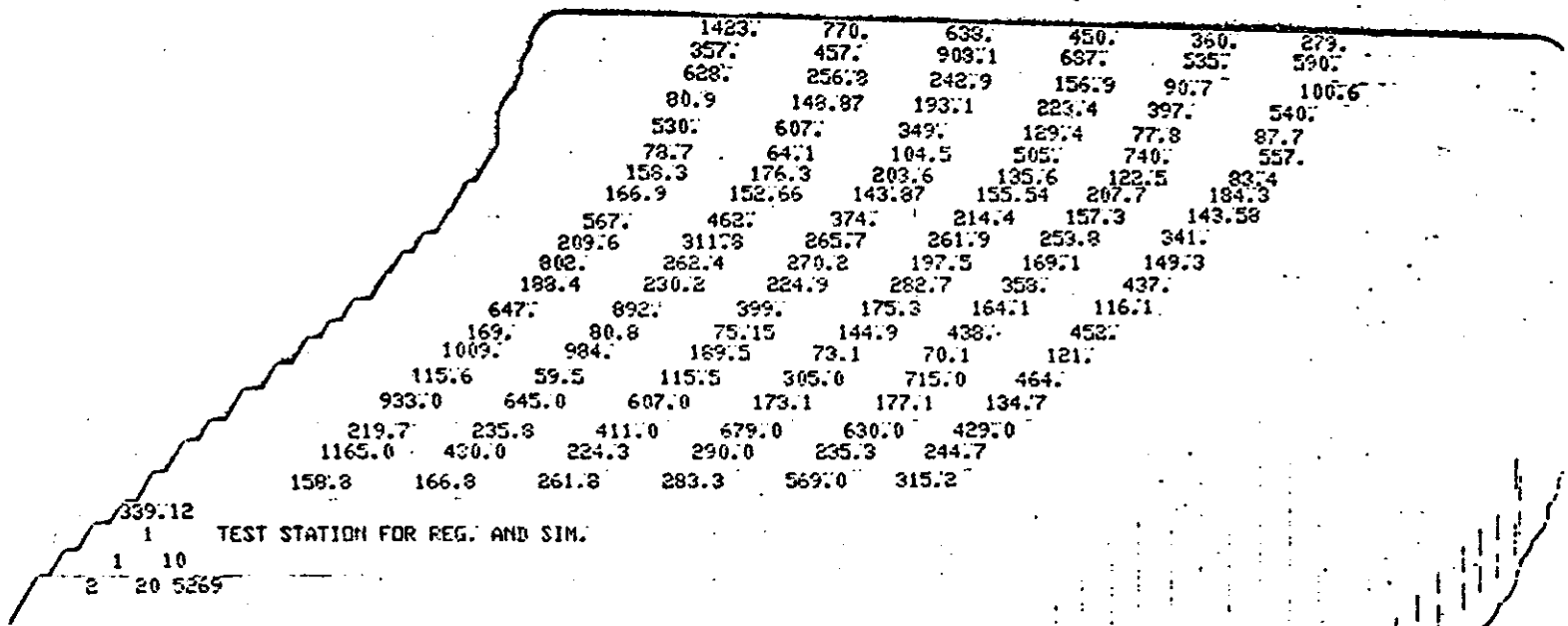
E. Card type 5.

Q(L,M),

where Q(L,M) = observed streamflow at station L
for months 1 to 12

FORMAT (10X,6E10.0).

See sample input for a one station simulation using
ten years of input data on the following page.



SIMULATION OF MONTHLY RUNOFF

SIMULATION OF MONTHLY RUNOFF

NUMBER OF PERIODS OF SIMULATION 2
LENGTH OF THE PERIOD(YEARS) 20
INITIAL RANDOM NUMBER SEED 5269

MONTHLY FLOW STATISTICS AND MULTIPLE REGRESSION

NUMBER OF YEARS OF DATA USED IN COMPUTATIONS 10
NUMBER OF STATIONS USED IN COMPUTATIONS 1

STATION IDENTIFICATION
STATION ID.
1

AVERAGE ANNUAL FLOW
0.3391E 03

DESCRIPTION
TEST STATION FOR REG. AND SIM.

1. c) Sample Output

MONTHLY RUNOFF STATISTICS AND MULTIPLE REGRESSION

STATION ID. 1 TEST STATION FOR REG. AND SIM.

| MONTH | MEAN | | SKEW
COEFF. | MULTIPLE
DETER. | | |
|-------|-----------|----------|----------------|--------------------|--------|-------|
| | LOG(FLOW) | STD.DEV. | | COEFF. | B1 | B2 |
| OCT | 2.702 | 0.212 | -0.130 | 0.221 | 0.137 | 0.383 |
| NOV | 2.194 | 0.311 | -0.973 | 0.638 | -0.194 | 0.836 |
| DEC | 2.306 | 0.332 | 0.826 | 0.807 | -0.088 | 0.899 |
| JAN | 2.493 | 0.251 | 0.230 | 0.480 | -0.240 | 0.843 |
| FEB | 2.641 | 0.194 | -0.467 | 0.792 | -0.739 | 1.077 |
| MAR | 2.627 | 0.153 | -1.869 | 0.595 | 0.066 | 0.758 |
| APR | 2.813 | 0.267 | -1.516 | 0.811 | 0.395 | 0.643 |
| MAY | 2.685 | 0.267 | -0.545 | 0.461 | -0.335 | 0.797 |
| JUNE | 2.524 | 0.190 | 0.291 | 0.435 | 0.594 | 0.097 |
| JULY | 2.230 | 0.210 | 0.450 | 0.590 | -0.282 | 0.908 |
| AUG | 2.137 | 0.219 | 0.481 | 0.842 | 0.044 | 0.936 |
| SEPT | 2.103 | 0.156 | 1.241 | 0.837 | 0.477 | 0.695 |

SIMULATED RUNOFF

PERIOD 1

SIMULATED YEAR 15

STATION ID.

| MONTH | | 1 |
|-------|-------|---|
| OCT | 34.7 | |
| NOV | 71.0 | |
| DEC | 40.9 | |
| JAN | 23.1 | |
| FEB | 31.5 | |
| MAR | 103.3 | |
| APR | 185.1 | |
| MAY | 542.4 | |
| JUNE | 184.9 | |
| JULY | 56.0 | |
| AUG | 93.8 | |
| SEPT | 61.7 | |

TOTAL
ANNUAL
FLOW 1468.5


```

      I
      +-----+
      WRITE
      (3,101)
      +-----+
101  FORMAT(/10X,'SIMULATION OF MONTHLY RUNOFF') **
      I
      +-----+
      WRITE
      (3,102)
      .NP,LP,ISEED
      +-----+
102  FORMAT(/15X,'NUMBER OF PERIODS OF SIMULATION',I3/15X,'LENGTH OF TH**
      **E PERIOD(YEARS)',I3/15X,'INITIAL RANDOM NUMBER SEED',I6)
      I
      +-----+
      WRITE
      (3,103)
      +-----+
103  FORMAT(/10X,'MONTHLY FLOW STATISTICS AND MULTIPLE REGRESSION') **
      I
      +-----+
      WRITE
      (3,104)
      .NY,NS
      +-----+
104  FORMAT(/15X,'NUMBER OF YEARS OF DATA USED IN COMPUTATIONS',I3/15X,**
      **'NUMBER OF STATIONS USED IN COMPUTATIONS',I3)
      I
      +-----+
      WRITE
      (3,105)
      +-----+
105  FORMAT(/10X,'STATION IDENTIFICATION'/15X,'STATION ID',10X,'AVERAG**
      **E ANNUAL FLOW',10X,'DESCRIPTION')
      I
      +-----+
      WRITE
      (3,106)
      .(NIL),AAIL),(SNAME(,
      .L,1),(L=1,10),L=1,NS)
      +-----+
106  FORMAT(18X,I5,17X,E11,4,14X,I0A4) **
      I
      ***   SET UP CONSTANTS
      I
      +-----+
      I   NYB=NY-1
      I   AN=NYB
      I   BN=AN-1
      I   ANA=.25/(AN-1.)
      I   ANB=SQRT(AN)
      I   ANC=.35/(AN-1.)
      I   AND=.71/SQRT(AN-2.3)
      I   ANE=.17/(AN-1.6)
      I   CN=AN/(AN-2.)
      I   LX=0
      +-----+
      I
      ***   READ MONTHLY FLOWS FOR FIRST YEAR
      I
      +-----+
      DO 3   L=1,NS
      +-----+
      I
      I
      I

```



```

      I
-----
* DO 26      L=1+NS      *
-----
      I
      I LXR=LX      I
-----
      I
* DO 28      M=1+13      *
-----
*** COMPUTE MEANS      I
      I
      I E(L,M)=S(L,M)/AN      I
-----
*** ADJUST SUMS OF SQUARES AND X-PROD. TO MEANS      I
      I
* DO 23      L2=1+L      *
-----
      I
      I LX=LXR+L2      I
-----
      I
+++++ 23      +++I PC(LX,M)=PC(LX,M)-E(L,M)*S(L2,M)      I
-----
*** COMPUTE STD. DEVIATIONS AND SKEW COEFFICIENTS      I
      I
      I F(L,M)=SQRT(PC(LX,M)/BN)      I
      I GT=G(L,M)-E(L,M)*(3.0*PC(LX,M)+E(L      I
      I      M)*S(L,M))      I
-----
      I
+++++ 28      +++I G(L,M)=CN*GT/(F(L,M)*PC(LX,M))      I
-----
      I
* DO 21      M=1+12      *
-----
      I
* DO 25      L2=1+L      *
-----
      I
      I LX=LXR+L2      I
-----
      I
+++++ 25      +++I PP(LX,M)=PP(LX,M)-E(L,M)*S(L2,M+1)      I
      I PC6(L,M)=PC6(L,M)-SQ6T(M)*E(L,M+1)      I
-----
      I
+++++ 21      +++I PP6(L,M)=PP6(L,M)-SQ6T(M)*E(L,M)      I
-----
      I
* DO 26      M=1+13      *
-----
      I
      I
      I
      I
      I
      I

```


| STATEMENT NO | PAGE NO | STATEMENT NO | PAGE NO | STATEMENT NO | PAGE NO | STATEMENT NO | PAGE NO | STATEMENT NO | PAGE NO |
|--------------|---------|--------------|---------|--------------|---------|--------------|---------|--------------|---------|
| 2 | 1 | 27 | 7 | 75 | 18 | 102 | 2 | | |
| 3 | 4 | 28 | 6 | 76 | 12 | 103 | 2 | | |
| 4 | 3 | 30 | 10 | 77 | 13 | 104 | 2 | | |
| 5 | 3 | 31 | 11 | 78 | 12 | 105 | 2 | | |
| 6 | 3 | 32 | 11 | 79 | 12 | 106 | 2 | | |
| 7 | 3 | 36 | 8 | 80 | 13 | 108 | 10 | | |
| 8 | 3 | 37 | 8 | 81 | 13 | 109 | 18 | | |
| 9 | 4 | 38 | 8 | 84 | 15 | 110 | 18 | | |
| 10 | 5 | 40 | 8 | 85 | 15 | 111 | 18 | | |
| 11 | 4 | 41 | 8 | 86 | 13 | 113 | 7 | | |
| 12 | 4 | 42 | 9 | 87 | 14 | 116 | 7 | | |
| 13 | 4 | 45 | 9 | 88 | 14 | 117 | 7 | | |
| 14 | 4 | 48 | 9 | 89 | 14 | 119 | 16 | | |
| 15 | 5 | 51 | 9 | 90 | 14 | 120 | 16 | | |
| 17 | 5 | 53 | 9 | 91 | 15 | 121 | 16 | | |
| 18 | 5 | 57 | 9 | 94 | 16 | 122 | 17 | | |
| 19 | 5 | 59 | 10 | 95 | 15 | 123 | 18 | | |
| 20 | 1 | 60 | 10 | 96 | 16 | 124 | 18 | | |
| 21 | 6 | 61 | 10 | 97 | 16 | 125 | 18 | | |
| 22 | 1 | 62 | 11 | 98 | 16 | 126 | 18 | | |
| 23 | 6 | 71 | 12 | 99 | 18 | 127 | 15 | | |
| 25 | 6 | 72 | 12 | 100 | 1 | 129 | 17 | | |
| 26 | 7 | 73 | 11 | 101 | 2 | 130 | 17 | | |

3. Simulation of Monthly Streamflow

- a) This model generates monthly streamflows which have the same statistical characteristics as observed streamflows. A model which relates a monthly streamflow to the streamflow during antecedent months is evaluated using multiple regression analysis. The results of this analysis and other statistics of the observed monthly streamflows are then employed in a Monte Carlo process to generate the monthly streamflows.

This model has been adapted from Simulation of Monthly Runoff, Hydrologic Engineering Center, Corps of Engineers, U. S. Army, Nov. 1964. The results of the model will be used to simulate the water level and the volume of water at Lake George.

- b) FORTRAN IV
- c) IBM 360/50
- d) IBM, G compiler, version 18
- e) Algebraic equations are used
- f) (i) The first year's data is used solely to obtain antecedent flow information for the second year's data
- (ii) All data used in the regression analysis is corrected for mean.
- (iii) Gaussian elimination is used to solve for the regression coefficients.
- (iv) Normally distributed random numbers from a $N(0,1)$ are obtained from Subroutine Gauss, part of the IBM 360 Scientific Subroutine Package.
- g) For a particular (i,j);

$$Y_{ij} = \ln F_{ij} .$$

$$X_{ij} = \ln F_{i-1,j} .$$

$$A_{ij} = \ln \sum_{l=1}^j \sum_{k=i-7}^{i-2} F_{k,l} / \Delta A_l .$$

$$M_{ij} = \frac{\sum_{i=1}^n Y_{ij}}{(n-1)} .$$

The beta coefficients for a particular (i,j) are

$$\beta_1 = b_1 \left\{ \frac{\sum_2^n A_{ij}^2 - \left(\frac{\sum_2^n A_{ij}}{(n-1)} \right)^2}{(n-2) S_{ij}^2} \right\}^{1/2},$$

$$\beta_2 = b_2 \left\{ S_{i-1,j} / S_{i,j} \right\},$$

$$\beta_k = b_k \left\{ S_{i,k-2} / S_{i,j} \right\} \quad k=3, \dots, m.$$

Smoothed mean for station j, month i,

$$M'_{ij} = .84 M_{ij} + .08 (M_{i-1,j} + M_{i+1,j}).$$

Smoothed standard deviation for station j, month i,

$$S'_{ij} = \left\{ .5 S_{i,j}^2 + .25 (S_{i-1,j}^2 + S_{i+1,j}^2) \right\}^{1/2}.$$

Smoothed skew coefficient,

$$G'_{ij} = .3 G_{ij} + .15 (G_{i-1,j} + G_{i+1,j}).$$

Correlated standard deviates,

$$K'_{ij} = \beta_1 C_{ij} + \beta_2 K'_{i-1j} + \sum_{\ell=1}^{j-1} \beta (\ell+3) K'_{i,\ell} + RV \sqrt{1-R^2}$$

$$\text{where } C_{ij} = \ln \left\{ \frac{\sum_{\ell=1}^j \sum_{k=i-7}^{i-2} Q_{k\ell} / AA_{\ell} - M'_{ij}}{S'_{ij}} \right\}$$

and RV is normally distributed random number generated from N(0,1).

Generated monthly flow logarithm,

$$GF_{ij} = M'_{ij} + \left\{ K'_{ij} + .16 G'_{ij} \left[(K'_{ij})^2 - 1 \right] \right\} S'_{ij}.$$

Generated monthly flow,

$$Q_{ij} = \text{antilog } GF_{ij}.$$

$$S_{ij} = \frac{\left[\sum_2^n Y_{ij}^2 - M_{ij} \sum_2^n Y_{ij} \right]}{n-2} \quad 1/2$$

$$G_{ij} = \frac{(n-1) \left\{ \sum_2^n Y_{ij}^3 - M_{ij} [3(n-2)S_{ij}^2 + M_{ij} \sum_2^n Y_{ij}] \right\}}{(n-2)(n-3)S_{ij}^3}$$

Postulated regression model for a particular (i,j),

$$Y_{ij} = \beta_{1,i,j} A_{ij} + \sum_{k=1}^j \beta_{k+1,i,j} X_{i,k},$$

or

$$\vec{Y} = Z \vec{b},$$

where \vec{Y} is the vector of observations Y_{ij} ,

Z is the matrix of independent variables,

\vec{b} is the vector of parameters to be estimated, $[\beta_1 \beta_2 \dots \beta_{k+1}]$,

and

$$Z = [\vec{A} \vec{X}_1 \dots \vec{X}_k]$$

\vec{A} is the vector of A_{ij}

\vec{X}_j is the vector of X_{ij} .

The Normal equations are

$$Z^T Z \vec{b} = Z^T \vec{Y}.$$

Solving these, the estimates of the parameters

$$\vec{b} = (Z^T Z)^{-1} Z^T \vec{Y} \text{ are obtained.}$$

The Multiple Determination Coefficient is

$$R^2 = \frac{\vec{b}^T Z^T \vec{Y}}{\vec{Y}^T \vec{Y}}.$$

If more than one period is generated, a set of means and standard deviations for each period are selected from approximated t and Chi-square distributions.

- h) n = number of years of data used in computations.
 m = number of stations.

For each station, $j=1\dots,m$, at which observed streamflow data was recorded, and for months, $i=1,\dots,12$.

F_{ij} = the observed streamflow for month i at station j (inches).

AA_j = the average annual flow at station j (inches).

GF_{ij} = generated flow logarithm for month i at station j .

Q_{ij} = generated flow for month i at station j .

4. This model is still being evaluated as to its effectiveness in predicting monthly runoff to Lake George. Generated streamflow information from this program will be used in predicting the water level and the volume of water at Lake George and will provide input information to the Hydrologic Submodel (HYDRØL).

This program is not yet in the production stage.

APPENDIX B

Nutrient Submodel (NUTRNT)

NUTKNT

```

1. DECLARE RAIN(2,NOYRS,12) DEC(6), VOL(NOYRS,12) , DIS(NOYRS,12) ;
2. DECLARE PLANKT(32) ;
3. PUT LIST('HOW MANY YEARS SIMULATION DESIRED?');
4. GET LIST(NOYRS);
5. PUT LIST('TYPE "1. |" AND WAIT FOR EDIT, THEN TYPE "XEQ START"');
6. STOP ;
7. START: DO ;
8. CALL SKIP(1);
9. PUT LIST('ENTER INITIAL VALUES FOR YEAR(E.G. 1966)');
10. GET LIST(YEAR);
11. CALL SKIP(1);
12. PUT LIST('ENTER INITIAL VALUES FOR WINTER POPLN., WINTER POPLN. SLOPESUMMER POPLN., AND SUMMER POPLN. SLOP');
13. GET LIST(WPOPLN,WPOPCF,SPOPLN,SPOPCF);
14. CALL SKIP(1);
15. PUT LIST('ENTER INITIAL VALUES FOR TRANSIENT POPLN., TRANSIENTPOPLN. SLOP, RURAL POPLN., AND TENT CAMPERS ON IS
16. GET LIST(TPOPLN,TPOPCF,RRLPOP,CAMPER);
17. CALL SKIP(1);
18. PUT LIST('ENTER INITIAL CONC. OF N AND P');
19. GET LIST(INITN,INITP);
20. PREVN=INITN;
21. PREP=INITP;
22. CALL SKIP(1);
23. PUT LIST('ENTER VALUES FOR LAWNS (ACRES), AGRICULTURAL LAND & FORESTS (MI**2)');
24. GET LIST(LAWNS,AGRIC,FOREST);
25. CALL SKIP(1);
26. PUT LIST('ENTER LITERS OF RAIN FOR FOREST AREA OF BASIN AND FOR LAKE AREA');
27. GET LIST(RAIN);
28. CALL SKIP(1);
29. PUT LIST('ENTER LITERS OF DISCHARGE AT TICONDEROGA BY MONTH');
30. GET LIST(DIS);
31. CALL SKIP(1);
32. PUT LIST('ENTER VOLUME OF LAKE (OR CELL) IN LITERS (THIS WILL BE CALLED FROM THE HYDROLOGIC SUBMODEL WHEN THE SJ
33. GET LIST(VOL);
34. PUT LIST('ENTER COEFFS. FOR GAS AS FN. OF POP');
35. GET LIST(GASCF1,GASCF2);
36. CALL SKIP(1);
37. PUT LIST('FRACTION OF PHYTOPLANKTER THAT IS N? THAT IS P?');
38. GET LIST(PCN,PCP);
39. PUT LIST('ENTER VALUES FOR PHYTOPLANKTON IN MICRO-G/L BY .5 M INCREMENTS (THIS WILL BE CALLED FROM PHYTO WHEN S
40. GET LIST(PLANKT);
41. CALL SKIP(5);
42. SEDR,SEDTHK=0;
43. YEARLY: DO ;YEAR=1 TO NOYRS;
44. CALL YFRTLZ;
45. YFRTLZ: PROCEDURE ;
46. LFERT=LAWNS*160/160 LB FERTILIZER/ACRE/YEAR, ALRANY AVE./;
47. LFERTH=LFERT*.1/*LBS OF N/YR, ASSUMING 1/2 REACHES LAKE*/;
48. LFERTP=LFERT*.22/*LBS P205/YR, ASSUMING ALL REACHES LAKE*/;
49. LFERTN=LFERT*.12/*LBS N2O/YR. */;

```

```

61. MNTHLY: DO IMTH=1 TO 12;
62. IF IMTH<=5||IMTH=9 THEN GO TO WINTER;
63. SUMMER: POP=SPOPLN+IYEAR*SPOPCF;
64. TRNPOP=TPOPLN+IYEAR*TPOPCF;
65. PITSWG=CAMPER;
66. CALL WASTE;
67. CALL BOATS;
68. BOATS: PROCEDURE ;
69. GAS=GASCF1*POP+GASCF2*POP**2/*GAL*/;
70. UNRGAS=GAS*57/3*1000/*MICRO-G/MONTH; 57 G GAS IN EXHAUST FROM 1 GAL*/;
71. UNBOIL=GAS*105/3*1000/*MICRO-G/MONTH*/;
72. RETURN ;
73. END BOATS;
74. GO TO FROZEN;
75. WINTER: POP=UPOPLN+IYEAR*WPOPCF;
76. TRNPOP,PITSWG=0;
77. UNRGAS,UNBOIL=0;
78. CALL WASTE;
79. WASTE: PROCEDURE ;
80. TRNSWG=POP*100/*100 GPCD SEWAGE FOR PERMANENT RESIDENTS & SUMMER-HOME RESIDENTS (METCALF&EDDY,1965)*/;
81. RRLSWG=RRLPOP*30/*80 GPCD SEWAGE FOR RURAL POPLN.*/;
82. TRNSWG=TRNPOP*50/*50 GPCD IS ASSUMED FOR TRANSIENTS*/;
83. SEWAGE=(TRNSWG+TRNSWG+RRLSWG)*114/*LITERS/MONTH (3.8*30)*/;
84. SWGP=SEWAGE*10+PITSWG*1500*30/*P IN MG/MONTH 10 MG/L, 1.5 G/DAY *30 DAYS*/;
85. SNOW=SEWAGE*30+PITSWG*16300*30/*N IN MG/MONTH*/;
86. RETURN ;
87. END WASTE;
88. FROZEN: IF IMTH<=6 THEN FERTLN,FERTLP=0; ELSE GO TO FRTLZR;
89. GO TO PRECIP;
90. FRTLZR: FERTLN=1/3*FRTN/*ASSUMING UNIFORM DISTRIBUTION DURING ICE-FREE MONTHS*/;
91. FERTLP=1/8*FERTP;
92. PRECIP: CALL RAINUT;
93. RAINUT: PROCEDURE ;
94. /*DISSOLVED NUTRIENTS IN RAIN & SNOW AS A FN. OF TIME*/;
95. H=4.12/*MG/L*/;
96. P=.75/*MG/L BASED ON MEAN VALUES FOR MAR-JUNE, 1970*/;
97. PCPTN=RAIN(2,IYEAR,IMTH)*N;
98. PCPTP=RAIN(2,IYEAR,IMTH)*P;
99. IF IMTH>5 THEN RETURN ;
100. FRZN=FRZN+PCPTN;
101. FRZP=FRZP+PCPTP;
102. IF IMTH=5 THEN GO TO MELT; ELSE PCPTN,PCPTP=0;
103. RETURN ;
104. MELT: PCPTN=FRZN;
105. PCPTP=FRZP;
106. RETURN ;
107. RETURN ;
108. END RAINUT;
109. CALL RUNOFF;
110. RUNOFF: PROCEDURE ;
111. /*NUTRIENTS CARRIED BY RUNOFF AS A FN. OF TIME & RAIN*/;
112. FORSTN=10*FERTLN/*DUMMY DATA FOR NOW*/;
113. FORSTP=10*FERTLP;
114. RETURN ;
115. END RUNOFF;
116. CALL ADJNUT;
117. ADJNUT: PROCEDURE ;
118. /*PROPORTIONATE ANT. OF NUTRIENTS FROM ADJ. SEGMENT OF LAKE*/;
119. INH,INP=0/*BASED ON PHYSICAL LIMNOLOGIC MODEL*/;
120. RETURN ;
121. END ADJNUT;
122. CALL OUT;
123. OUT: PROCEDURE ;
124. /*CALC. NUTRIENTS LOST TO OUTLET (OR ADJ. SEGMENT) OF LAKE*/;

```

```

125.      OUTH=PREVN*DIS(IYEAR,IMTH);
126.      OUTP=PREVP*DIS(IYEAR,IMTH);
127.      RETURN ;
128.      END OUT;
129.      CALL SEDMNT;
130. SEDMNT: PROCEDURE ;
131.      /*CALC. NUTRIENTS LOST TO SEDIMENTS*/;
132.      DEEPT=31/*SAMPLE 31 IS DEEPEST IN PHYTO AT PRESENT*/;
133.      ARFA=44*2590/*M1**2 TO M**2, AREA OF CELL (L.G. NOW)*/;
134.      SED=PLANKT(DEEPT)*AREA*1000000/*BIOMASS IN MICROG IN DEEPEST M**3*/;
135.      SEDN=SED*PCN;
136.      SEDP=SED*PCP;
137.      SEDVOL=.001/*(DUMMY) THICKNESS OF SED. FROM 1 MG BIOMASS IN 1 M COL. OF H2O*/;
138.      SEDR=SEDP+SED*SEDVOL;
139.      RETURN ;
140.      END SEDMNT;
141.      CALL DECOMP;
142. DECOMP: PROCEDURE ;
143.      /*REGENERATION OF NUTRIENTS AS FN. OF TEMP., SED. RATE, PH, ETC.*/;
144.      REGENN, REGENP=0/*MG (DUMMY DATA FOR NOW)*/;
145.      RETURN ;
146.      END DECOMP;
147.      NITROG=(SWGN+FERTLN+FORSTN+INN)*1000+PCPTN-OUTN-SEDN+REGENN;
148.      PHOS=(SWGP+FERTLP+FORSTP+INP)*1000+PCPTP-OUTP-SEDP+REGENP;
149.      NCONC=NITROG/VOL(IYEAR,IMTH)+PREVN;
150.      PCONC=PHOS/VOL(IYEAR,IMTH)+PREVP;
151.      PREVN=NCONC;
152.      PREVP=PCONC;
153.      PUT IMAGE(IMTH,NCONC,PCONC,SWGN*1000,FERTLN*1000,PCPTN,FORSTN*1000,SEDN,OUTN)(IM3);
154. IM3:   IMAGE;
-----N:-----
155.      PUT IMAGE(SWGP*1000,FERTLP*1000,PCPTP,FORSTP*1000,SEDP,OUTP)(IM4);
156. IM4:   IMAGE;
-----P:-----
157.      END MONTHLY;
158.      PUT LIST('SEDIMENTATION RATE=',SEDR,'MM PER YEAR');
159.      SEDTHK=SEDTHK+SEDR;
160.      END YEARLY;
161.      PUT LIST('-----SIMULATION TERMINATED-----');
162. SKIP:  PROCEDURE (SK);
163. S:     DO J=1 TO SK;
164.        PUT LIST(' ');
165.      END S;
166.      RETURN ;
167.      END SKIP;
168.      END START;

```

1. b) Input Information

All input is keyed in from the remote terminal in free format.
CPS will request input by outputting the variable's name.

See the sample input for this model on the following page.

?NO YES

??YEAR

1966

??WPEPLN

4500

??WPOPCF

180

??SPOPLN

45000

??SPOPCF

8200

??TPOPLN

20000

??KKLPOP

5000

??CAMPER

1000

??INITN

200

??INITP

40

??LAWNS

200

--

??AGRIC

20

??FOREST

215

??RAIN

.25845E16 .37505E11 .42764E11 .13385E11

.40105E11 .33579E11 .3154E11 .5801E11 .65624E11

.40037E11 .5329E11 .24047E11 .5549E10 .832E10

.11054E11 .3335E10 .9524E10 .7666E10 .7161E10

.12967E11 .14725E11 .9383E10 .11589E11 .5531E10

??DIS

8345 5882 12085 12779 14563 2567 2407 1290

3955 4916 4774 4322

??VHL

.215081E13 .215831E13 .222957E13 .220695E13

.214661E13 .207412E13 .202266E13 .200207E13

.195161E13 .197034E13 .198419E13 .2033344E13

??GASCF1

10

??GASCF2

2

??PCN

.005

??PCP

.0005

??PLANKT

106500 105900 103000 101000 98500 96000

93000 90000 86500 83200 79200 75000 70500

65000 59000 52000 44000 35500 27000 20000

15500 12000 9600 7600 6100 5000 4400 3800

3400 3100 3000 3000

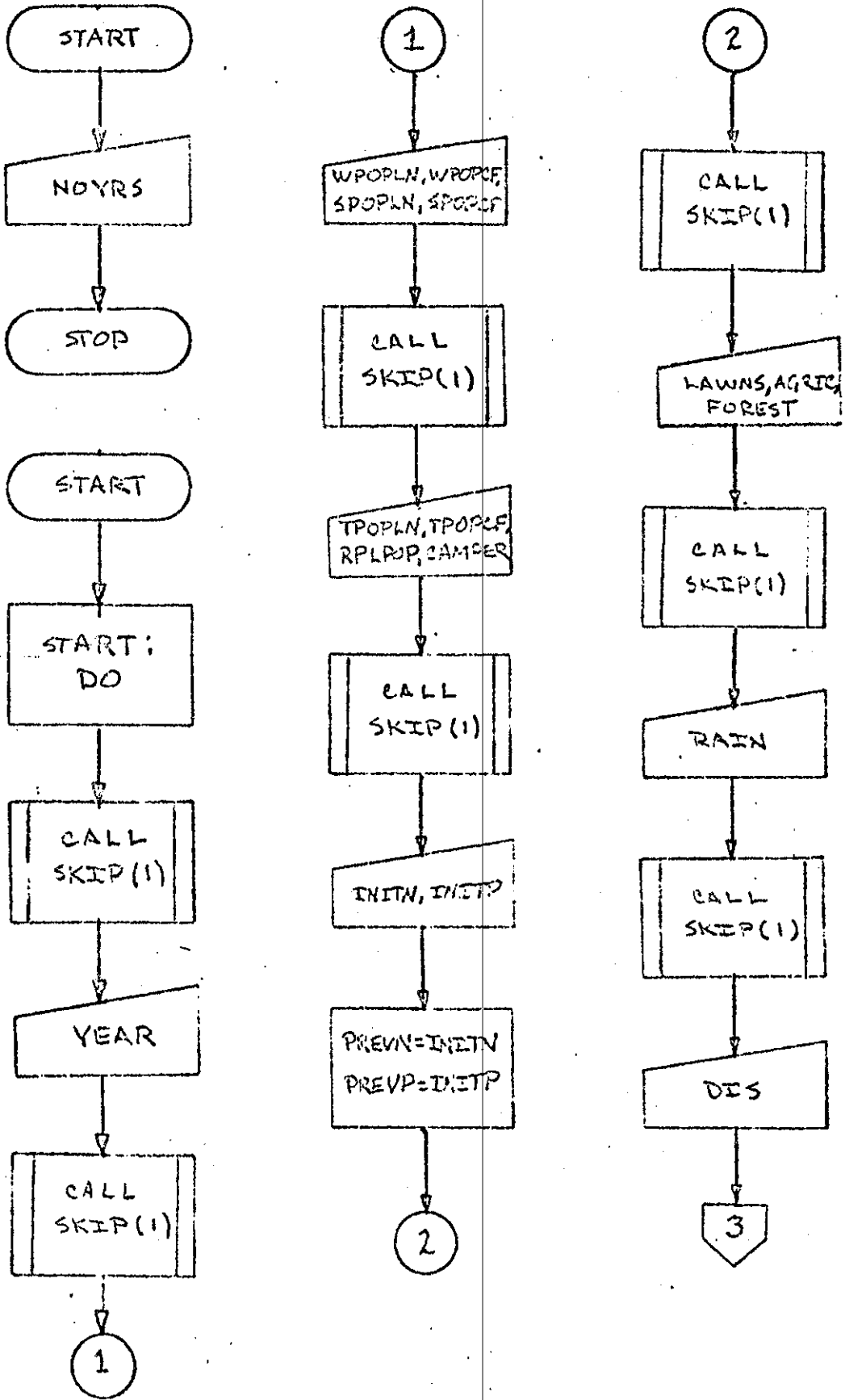
1. c) Sample Output

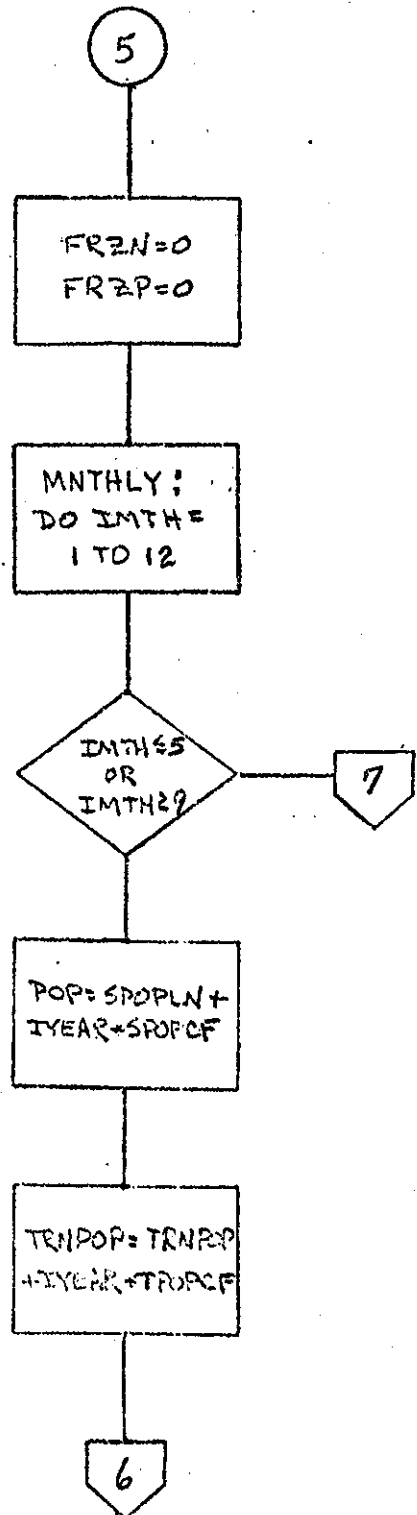
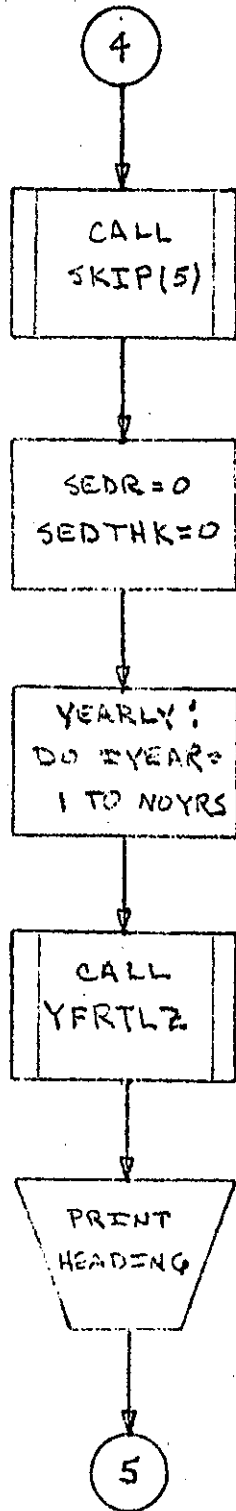
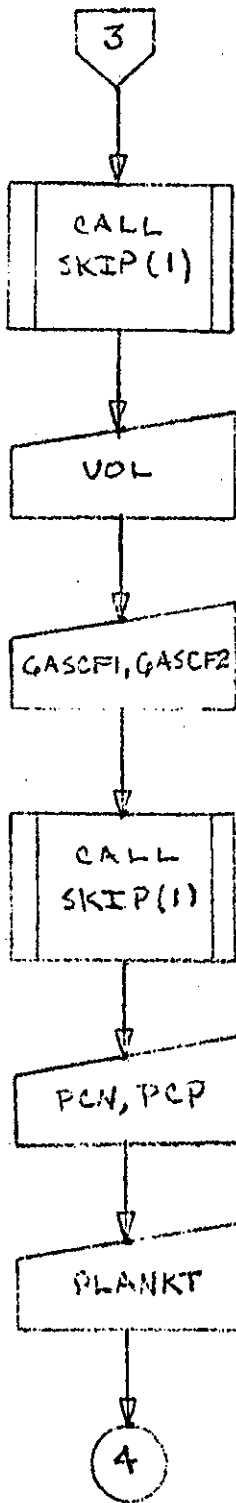
| YEAR | MONTH | N (UG/L) | P (UG/L) | SEWAGE | FERT. | PCPT. | FORESTS | SEDIMENTS | OUTL. |
|------|-------|----------|------------|--------|-------|-------|---------|-----------|--------|
| 1966 | 1 | .3006E03 | .4038E02N: | .30E13 | .0 | .0 | .0 | .17E13 | .25E07 |
| | | | P: | .99E12 | .0 | .0 | .0 | .17E12 | .33E06 |
| | 2 | .3012E03 | .4076E02N: | .30E13 | .0 | .0 | .0 | .17E13 | .18E07 |
| | | | P: | .99E12 | .0 | .0 | .0 | .17E12 | .24E06 |
| | 3 | .3017E03 | .4113E02N: | .30E13 | .0 | .0 | .0 | .17E13 | .36E07 |
| | | | P: | .99E12 | .0 | .0 | .0 | .17E12 | .49E06 |
| | 4 | .3023E03 | .4150E02N: | .30E13 | .0 | .0 | .0 | .17E13 | .39E07 |
| | | | P: | .99E12 | .0 | .0 | .0 | .17E12 | .53E06 |
| | 5 | .3170E03 | .4367E02N: | .30E13 | .3E13 | .2E12 | .27E14 | .17E13 | .50E07 |
| | | | P: | .99E12 | .3E12 | .3E11 | .35E13 | .17E12 | .69E06 |
| | 6 | .3424E03 | .4928E02N: | .24E14 | .3E13 | .3E11 | .27E14 | .17E13 | .27E07 |
| | | | P: | .80E13 | .3E12 | .6E10 | .35E13 | .17E12 | .37E06 |
| | 7 | .3684E03 | .5504E02N: | .24E14 | .3E13 | .3E11 | .27E14 | .17E13 | .62E06 |
| | | | P: | .80E13 | .3E12 | .5E10 | .35E13 | .17E12 | .12E06 |
| | 8 | .3948E03 | .6085E02N: | .24E14 | .3E13 | .5E11 | .27E14 | .17E13 | .48E06 |
| | | | P: | .80E13 | .3E12 | .1E11 | .35E13 | .17E12 | .71E05 |
| | 9 | .4106E03 | .6319E02N: | .30E13 | .3E13 | .6E11 | .27E14 | .17E13 | .12E07 |
| | | | P: | .99E12 | .3E12 | .1E11 | .35E13 | .17E12 | .19E06 |
| | 10 | .4265E03 | .6555E02N: | .30E13 | .3E13 | .4E11 | .27E14 | .17E13 | .20E07 |
| | | | P: | .99E12 | .3E12 | .7E10 | .35E13 | .17E12 | .31E06 |
| | 11 | .4423E03 | .6789E02N: | .30E13 | .3E13 | .5E11 | .27E14 | .17E13 | .20E07 |
| | | | P: | .99E12 | .3E12 | .9E10 | .35E13 | .17E12 | .31E06 |
| | 12 | .4577E03 | .7017E02N: | .30E13 | .3E13 | .2E11 | .27E14 | .17E13 | .19E07 |
| | | | P: | .99E12 | .3E12 | .4E10 | .35E13 | .17E12 | .30E06 |

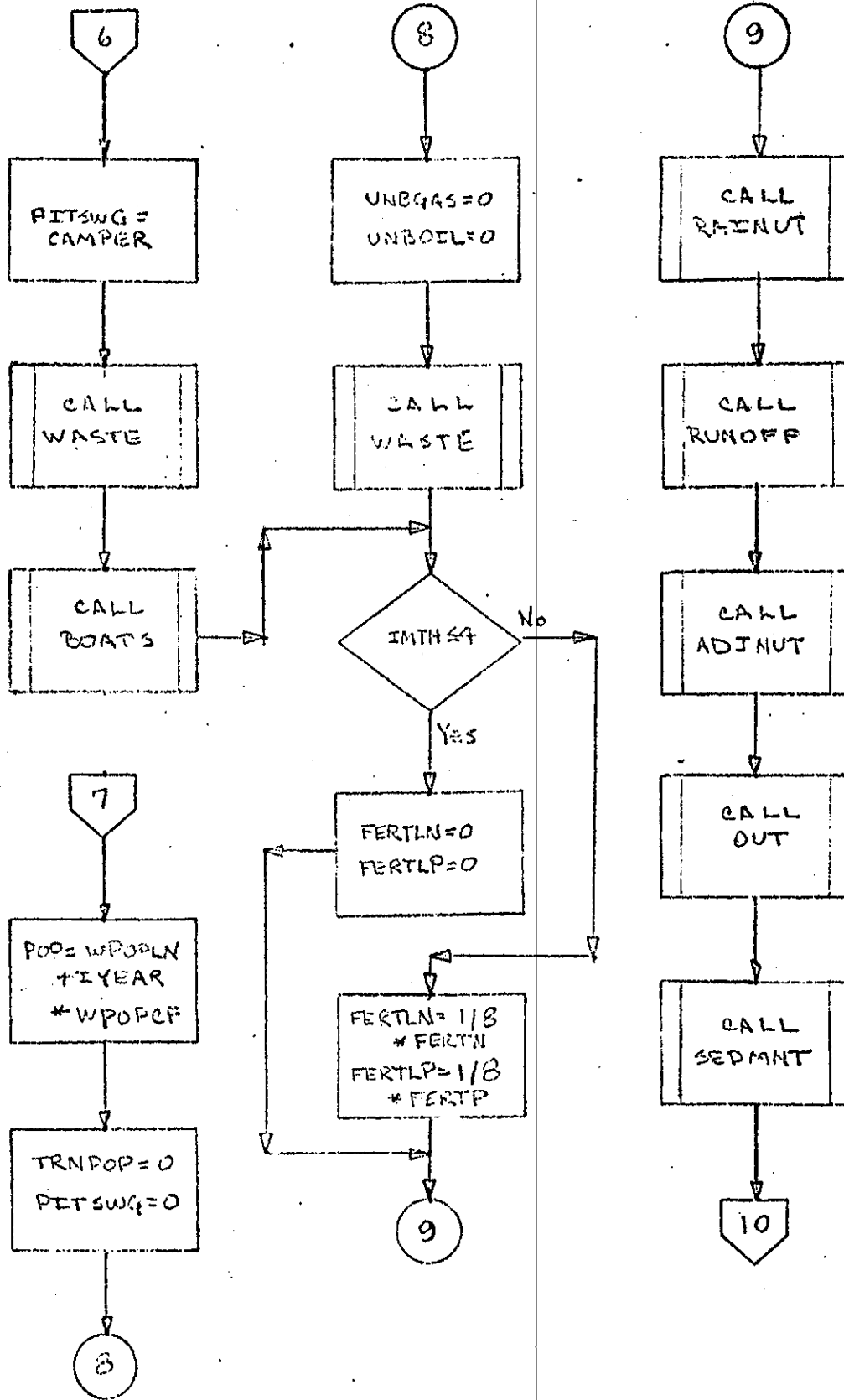
SEDIMENTATION RATE= .410256E13 MM PER YEAR

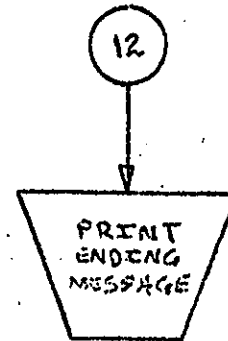
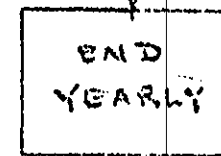
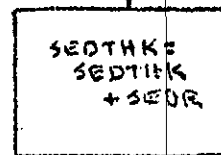
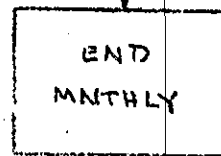
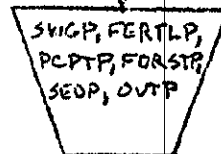
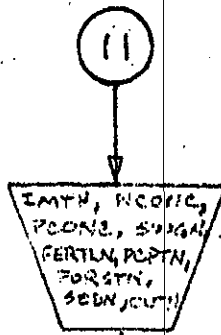
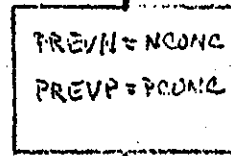
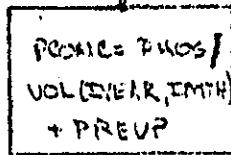
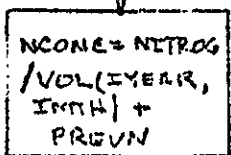
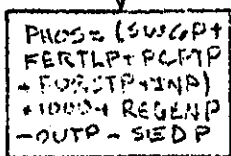
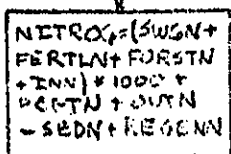
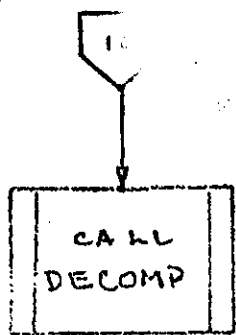
-----SIMULATION TERMINATED-----

2. Nutrient Submodel (NUTRNT) - Detailed Flowchart

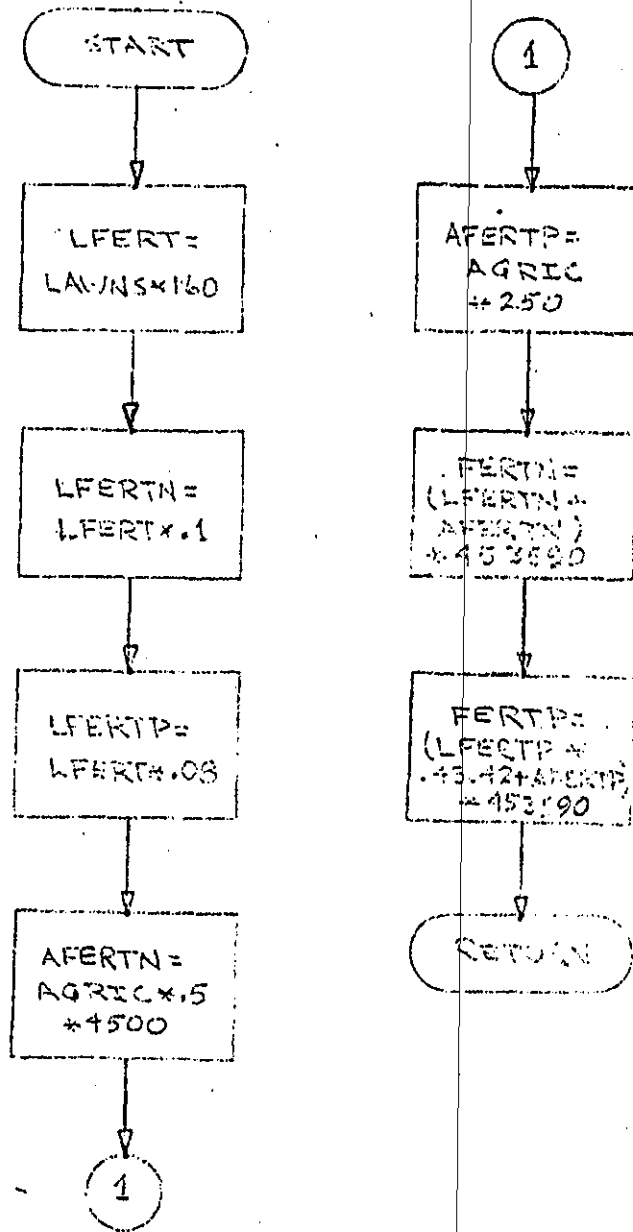








YFRTLZ : PROCEDURE



BOATS: PROCEDURE

START

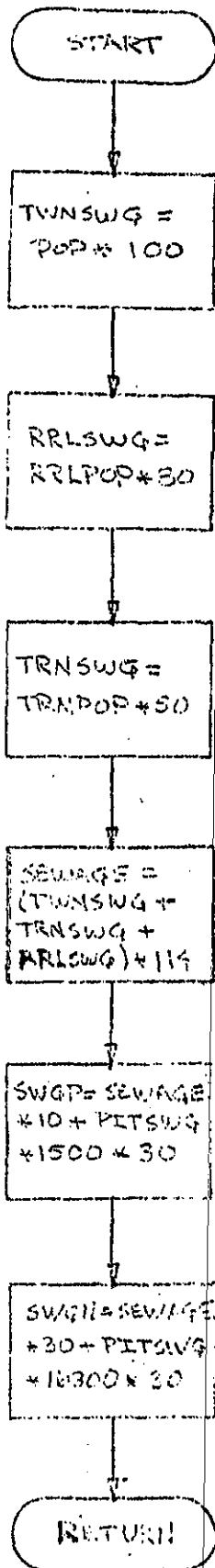
$GAS = GASCF1$
 $+ POP + GASCF2$
 $* POP * v 2$

$UNB GAS =$
 $GAS * 57 / 3$
 $* 1000$

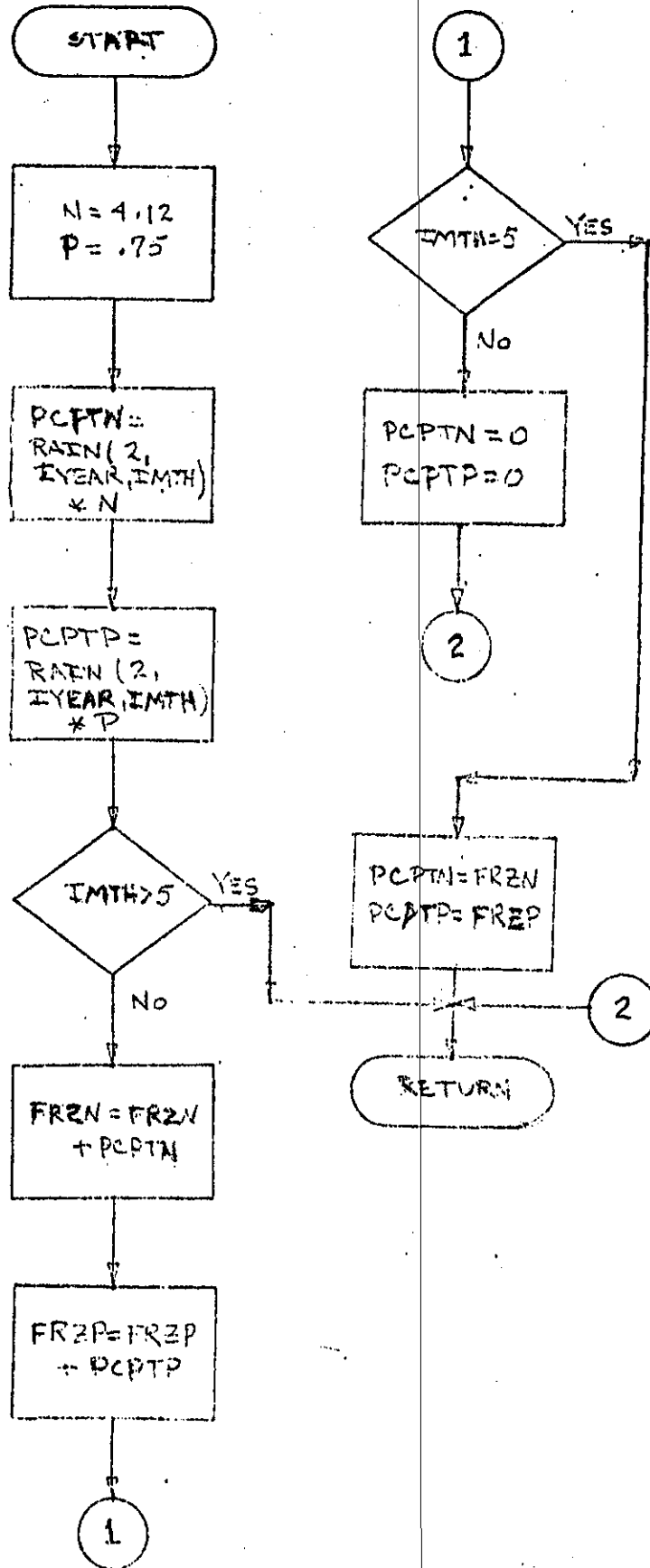
$UNBOILS =$
 $GAS * 105 / 3$
 $* 1000$

RETURN

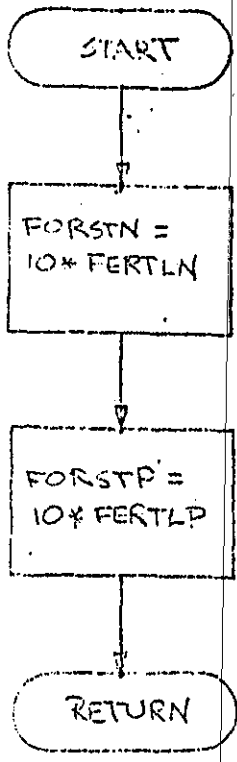
WASTE : PROCEDURE



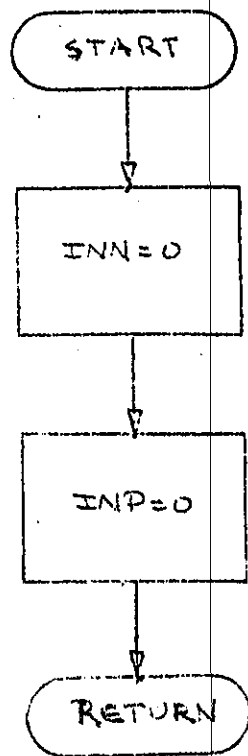
RAINFUT: PROCEDURE



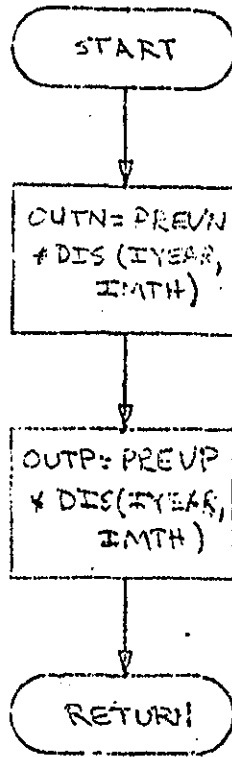
RUNOFF, PROCEDURE



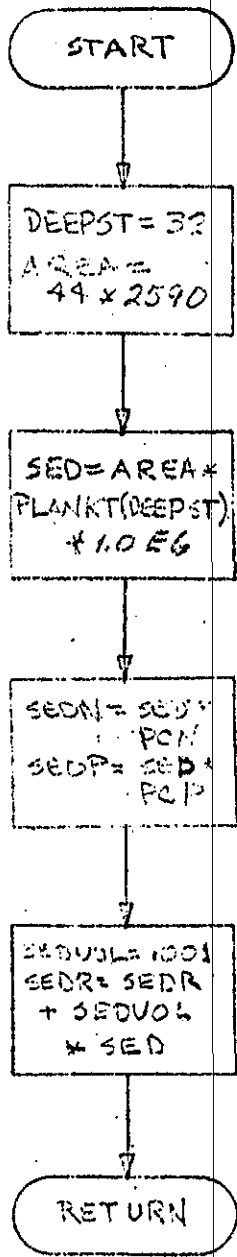
ADJNUT: PROCEDURE



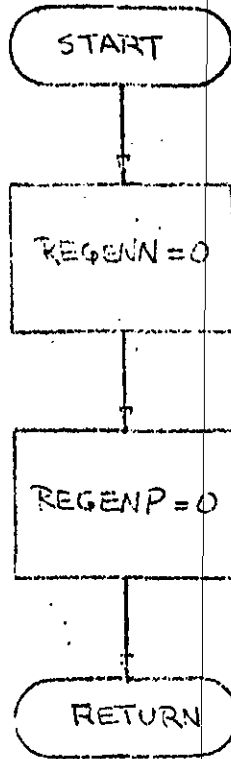
OUT: PROCEDURE



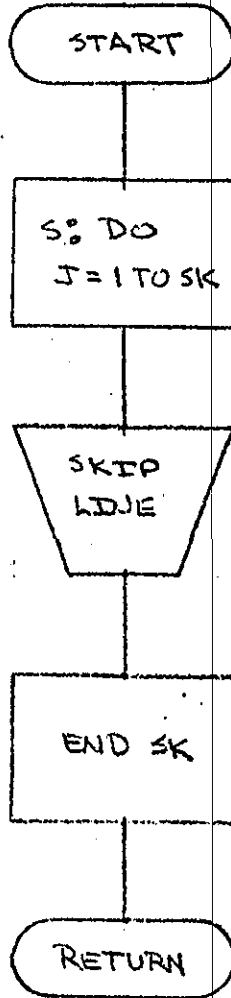
SEDMIT : PROCEDURE



DECOMP: PROCEDURE



SKIP: PROCEDURE (SK)



3. Nutrient Submode (NUTRNT)

- a) This deterministic submodel computes the overall nutrient balance in Lake George on a monthly basis, taking into account all important sources of nutrients and sinks. It is written in modular form as a series of procedures which may be expanded easily as development progresses; the flow is in logical sequence, and all variables have mnemonic names so that it may be used readily by any investigator.
- b) Conversational Programming System, PL/1.
- c) IBM 360/50.
- d) Conversational Programming System, release 4.
- e) Fundamentally two algebraic equations with many variables.
- f) No unusual numerical methods.
- g) See flowchart.

h) Nutrient Submodel (NUTRNT) Variables

- NOYRS - number of years of simulation
- YEAR - initial year
- WPØPLN - initial winter population (permanent residents on the lake)
- WPØPCF - coefficient of winter population growth
- SPØPLN - initial summer population (summer-home residents)
- SPØPCF - coefficient of summer population growth
- TPØPLN - initial transient population (per day.)
- TPØPCF - coefficient of transient population growth
- RRLPØP - rural population (assumed constant for purposes of modeling)
- CAMPER - population of campers per day at camp sites with pit toilets (all other campers are included in transient population)
- INITN - initial concentration of nitrogen in Lake ($\mu\text{g}/\ell$)
- INITP - initial concentration of phosphorus in Lake ($\mu\text{g}/\ell$)
- LAWNS - area of lawns in basin (acres)
- AGRIC - area under cultivation (square miles)
- FOREST - area of land not developed (square miles); land use is not expected to change significantly but these areas may be varied for experimental purposes.
- RAIN - array of values derived from hydrologic submodel giving volume of precipitation (in liters of rain equivalents) per month for basin, excluding lake, and for lake surface
- DIS - array of discharge values at outlet at Ticonderoga (liters/month) from hydrologic submodel
- VØL - array of values of volume of lake (or cell) from hydrologic submodel (liters/month)
- GASCF1,
GASCFZ,
GASCF3 - coefficients for gas as a function of population
- PCN - fraction of phytoplankter that is nitrogen
- PCP - fraction of phytoplankter that is phosphorus
- PLANKT - array of values of phytoplankton biomass by depth ($\mu\text{g}/\ell$) from phytoplankton submodel; when submodels are merged this will be updated at the first of each month in the MNTHLY iteration (at this time only the value for the deepest layer is used in this model)

- IYEAR - internal counter; because of interdependence of submodels, IYEAR (and NØYRS) has been restricted to 1 during model development
- YFRTLZ - procedure for calculating yearly input from fertilizer
- LFERT - pounds of lawn fertilizer per year
- LFERTN - pounds of nitrogen per year from lawn fertilizer, assuming 1/2 reaches lake
- LFERTP - pounds of P_2O_5 per year from lawn fertilizer, assuming all reaches lake
- AFERTN - pounds of nitrogen per year from agricultural fertilizer, assuming 1/2 reaches lake
- AFERTP - pounds of phosphorus per year from agricultural fertilizer, assuming all reaches lake
- FERTN - mg/year nitrogen from fertilizer
- FERTP - mg/year phosphorus from fertilizer
- IMTH - counter in MNTHLY iteration
- PØP - summer or winter population (semi-permanent or permanent, non-rural) at a given time
- TRNPØP - transient population at a given time
- PITSWG - same as CAMPER
- BØATS - procedure to calculate input of unburned gas and oil into the lake; from this input of carbon, lead, phenols, etc. may be calculated
- GAS - gallons of gas used per year as a curvilinear function of population
- UNBGAS,
UNBØIL - unburned gas and oil in grams/month
- WASTE - procedure for input of nitrogen and phosphorus from sewage in mg/month
- TWNSWG - gallons sewage per capita per day from residents
- RRLSWG - gpcd sewage from rural population
- TRNSWG - gpcd sewage from transients
- SEWAGE - total sewage that is subject to treatment, in liters/month
- SWGØP - phosphorus from sewage in mg/month
- SWGØN - nitrogen from sewage in mg/month
- FERTLN - nitrogen input into lake from fertilizer in mg/month for the 8 ice-free months, assumes uniform distribution for those months.

- FERTLP - phosphorus input into lake from fertilizer, mg/month
- RAINUT - procedure for calculating dissolved nutrient input from rain; uses mean values for March-June, 1970, but will be adjusted as more data are available; compensates for build-up while ice cover is present
- N - concentration of nitrogen in rain, $\mu\text{g}/\ell$
- P - concentration of phosphorus in rain, $\mu\text{g}/\ell$
- PCPTN - input of nitrogen in $\mu\text{g}/\text{month}$
- PCPTP - input of phosphorus in $\mu\text{g}/\text{month}$
- FRZN,
FRZP - accumulate N and P while ice cover is present
- RUNOFF - procedure for calculating input of nutrients from basin by runoff; uses dummy values for now, but will calculate input as a function of time and amount and intensity of rain when more data are available
- FØRSTN,
FØRSTP - input of N and P, arbitrarily taken as ten times input of fertilizer for purposes of model development
- ADJNUT - procedure to calculate input of nutrients from adjacent cell in the lake based on physical limnologic model; because the entire lake is being modeled during the development stage this is a null procedure
- INN,INP - variables for input of N and P in mg/month
- ØUT - procedure for calculating loss of nutrients to outlet (or to adjacent segment of lake)
- ØUTN-
ØUTP - nitrogen and phosphorus loss in $\mu\text{g}/\text{month}$
- SEDMNT - procedure for calculating nutrients lost to sediments (as a function of phytoplankton concentration in deepest layer) and for calculating sedimentation rate
- DEEPST - deepest layer of lake simulated in phytoplankton layer; will be used to get proper element of an external array from PHYTØ when the submodels are merged
- AREA - area of cell being modeled (entire lake at this time)
- SED - total biomass in deepest horizon 1 meter thick over the area modeled (μg)
- SEDN,
SEDP - loss of N and P to sediments, $\mu\text{g}/\text{month}$

- SEDVOL - thickness in mm of flocculent organic sediment derived from 1 μg biomass suspended in column of water with unit horizontal cross-sectional area; dummy value is used for now until experimental data can be obtained
- SEDR - accumulative sedimentation rate over a year in mm
- DECØMP - procedure to calculate recycling of nutrients as a function of temperature, sedimentation rate, pH, etc.; this is a nude procedure now, but it will be replaced by a decomposer submodel in the near future
- REGENN,
REGENP - regenerated N and P in $\mu\text{g}/\text{month}$
- NITRØG - total nitrogen balance, excluding previous available nitrogen but including new nitrogen input to lake and nitrogen lost from previous nutrient pool to sinks ($\mu\text{g}/\text{month}$)
- PHØS - total phosphorus balance ($\mu\text{g}/\text{month}$)
- NCØNC,
PCØNC - concentrations of total N and P in the lake water in $\mu\text{g}/\text{l}$ for each month; the concentrations of dissolved N and P will vary on a daily basis as determined by PHYTØ when the submodels are merged
- SEDTK - an accumulative variable to indicate total sediment thickness when simulation proceeds for long periods of symbolic time
- SKIP - a procedure to skip lines on the output device

APPENDIX C

Phytoplankton Submodel (PHYTØ)

DIFFUS.CBN

```

1.      N=1;
2.      DECLARE APHOTC(N) , ANUTFA(N) , ARRAZC(N) , HERRS(N) , TEMP(N) , ADRO(N) , U(N) , C(N,3) , D(N) , R(N) , RA
3.      ZERO:  DX=.5;
4.            DT=1;
5.            THETA=.5;
6.      FIRST: GET LIST(N);
7.            CALL SKIP(2);
8.            PUT LIST('TYPE '2)' THEN TYPE 'XEQ FIRSTA THRU...' );
9.            STOP ;
10.     FIRSTA: GET LIST(U);
11.            PLANKS=U;
12.            GET LIST(TINIT);
13.            CALL SKIP(4);
14.            PUT IMAGE(TINIT)(FORM4);
15.            PUT IMAGE(PLANKS)(FORM3);
16.            TIME=TINIT;
17.     FIRSTC: GET LIST(TSTEP);
18.     FIRSTB: GET LIST(ADRO);
19.            GET LIST(TEMP);
20.            GET LIST(HFRS);
21.            NS=TSTEP/DT;
22.     I01:   DO I=1 TO NS;
23.           TIME=TIME+DT;
24.           CALL PHYTOP;
25.           END I01;
26.           CALL HEADNG;
27.           GO TO FIRSTC;
28.     PHYTOP: PROCEDURE ;
29.           CALL INCRAX;
30.           CALL VERTUX;
31.           GET LIST(APHOTC);
32.           GET LIST(ANUTFA);
33.           GET LIST(ARRAZC);
34.     PHY1:  DO I=1 TO N;
35.           DEPTH=.5*I;
36.           CALL SIGMAX;
37.           CALL ACONSX;
38.           CALL BCONSX;
39.           CALL DIFFUS;
40.           END PHY1;
41.           CALL THOMAS;
42.           U=0;
43.           CALL HEADNG;
44.           RETURN ;
45.     INCRAX: PROCEDURE ;
46.           GET LIST(INCRA);
47.           RETURN ;
48.           END INCRAX;
49.     VERTUX: PROCEDURE ;
50.           GET LIST(EUZONE);
51.           GET LIST(MIXLAY);
52.           VERTUR=EUZONE/MIXLAY;
53.           RETURN ;
54.           END VERTUX;
55.     SIGMAX: PROCEDURE ;
56.           SIGMA=.65*ADRO(I);
57.           RETURN ;
58.           END SIGMAX;
59.     ACONSX: PROCEDURE ;

```

1. a) Phytoplankton Submodel (PHYTO) - Program Listing

```

62.      RETURN ;
63.      END ACONSX;
64. BCONS: PROCEDURE ;
65.      CALL PHOTX;
66.      CALL RESPLX;
67.      CALL GRAZFX;
68.      RCONST=PHOTO-RESPLX-GRAZFT;
69.      RETURN ;
70. PHOTX: PROCEDURE ;
71.      CALL PHOTCX;
72.      CALL EXTCOX;
73.      CALL NUTFAX;
74.      AA=EXTCO-EUZONE;
75.      PHOTO=PHOTCO+INCRAD/AA*(1-EXP(-AA)*NUTFAC*VERTUR);
76.      RETURN ;
77. PHOTCX: PROCEDURE ;
78.      PHOTCO=APHOTC(IPHY1);
79.      RETURN ;
80.      END PHOTCX;
81. EXTCOX: PROCEDURE ;
82.      PCF=.5;
83.      PC=PCF*U(IPHY1);
84.      EXTCO=.04+.0088*PC+.054*PC**.6666666667;
85.      RETURN ;
86.      END EXTCOX;
87. NUTFAX: PROCEDURE ;
88.      NUTFAC=ANUTFA(IPHY1);
89.      RETURN ;
90.      END NUTFAX;
91. PHOTOY: END PHOTX;
92. RESPLX: PROCEDURE ;
93.      RESPLX=.0175*EXP(.069*TEMP(IPHY1));
94.      RETURN ;
95.      END RESPLX;
96. GRAZFX: PROCEDURE ;
97.      GRAZCO=AGRAZC(IPHY1);
98.      GRAZFT=GRAZCO*HERBS(IPHY1);
99.      RETURN ;
100.     END GRAZFX;
101. BCONSY: END BCONS;
102. DIFFUS: PROCEDURE ;
103.      C1=-SIGMA*DT/DX/DX;
104.      C2=C1*(THETA-1);
105.      C1=C1*THETA;
106.      A1=.5*ACONST*DT/DX;
107.      B1=BCONST*DT;
108.      B1=1-2*C1;
109.      LET OF(R,S,T)=(C2-A1)*R*(1+B1-A1-2*C2)*S+(C2*A1)*T;
110.      IF IPHY1=1 THEN GO TO SONE;
111.      IF IPHY1=N THEN GO TO SALL;
112.      C(N,1)=C1;
113.      C(N,2)=B1;
114.      D(N)=DF(U(N-1),U(N),0);
115.      RETURN ;
116. SONE: C(1,2)=B1;
117.      C(1,3)=C1;
118.      D(1)=DF(0,U(1),U(2));
119.      RETURN ;
120. SALL: C(IPHY1,2)=B1;
121.      C(IPHY1,3)=C1;
122.      D(IPHY1)=DF(U(IPHY1-1),U(IPHY1),U(IPHY1+1));
123.      RETURN ;

```

```

124.      END DIFFUS;
125. THOMAS: PROCEDURE ;
126.      B(1)=C(1,2);
127. TH1:   DO I=2 TO N;
128.      B(1)=C(1,2)-C(1,1)*C(1-1,3)/B(1-1);
129.      END TH1;
130.      D(1)=D(1)/C(1,2);
131. TH2:   DO I=2 TO N;
132.      D(1)=(D(1)-C(1,1)*D(1-1))/B(1);
133.      END TH2;
134. TH3:   DO I=N-1 TO 1 BY -1;
135.      D(1)=D(1)-C(1,3)*D(I+1)/B(1);
136.      END TH3;
137.      RETURN ;
138.      END THOMAS;
139. PHYTOY: END PHYTOP;
140. HEADNG: PROCEDURE ;
141.      CALL SKIP(5);
142.      PUT IMAGE(TIME,DT)(FORM2);
143.      CALL SKIP(1);
144.      PUT LIST(' PROFILE AT THIS TIME IS');
145.      PUT IMAGE(U)(FORM3);
146.      RETURN ;
147. FORM2:  IMAGE;
TIME = ----- TIME WAS INCREMENTED BY ----
148. FORM3:  IMAGE;
-----
149.      END HEADNG;
150. FORM4:  IMAGE;
ORIGINAL PROFILE AT INITIAL TIME = ----- IS
151. MAIEND: END ;
152. SKIP:  PROCEDURE (SK);
153. S:     DO I=1 TO SK;
154.      PUT LIST('');
155.      END S;
156.      RETURN ;
157.      END SKIP;

```

1. b) Input Information

All input is keyed in from the remote terminal in free format.
CPS will request input by outputting the variable's name.

See the sample input for this model below.

??N

31

??U

106500 105000 103000 101000 98500 96000
93000 90000 86500 83200 79200 75000 70500
65000 59000 52000 44000 35500 27000 20000
15500 12000 9600 7600 6100 5000 4400 3600
3400 3100 3000

??TINIT

1

?

?TSTEP

1

? ?

ADRG

16 15.3 14.7 14 13.3 12.6 11.9 11.2 10.5
9.7 9 8.3 7.5 6.7 5.9 5.1 4.3 3.5 2.8 2.1
1.7 1.3 1.1 .95 .82 .71 .6 .52 .46 .4 .37

??TEMP

21.02 21 20.98 20.96 20.94 20.92 20.9 20.88
20.86 20.84 20.82 20.8 20.78 20.76 20.74
20.72 20.7 20.68 20.66 20.64 20.62 20.6
20.58 20.56 20.54 20.52 20.4 20.2 19.4 17.5
16.2

--

??HERBS

5 5 6 6 7 8 9 10 11 12 14 16 18 20 20 18
16 14 9 8 8 8 6 5 4 4 3 3 3 2 2

??APHETC

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
18 19 20 22 24 26 28 30 32 34 36 38 40 45

??ANUTFA

9.0909090909091 7.2727272727273 14.545454545455
18.181818181818 21.818181818182 27.272727272727
30.909090909091 27.272727272727 27.272727272727
25.454545454545 23.636363636364 21.818181818182
18.181818181818 18.181818181818 16.363636363636
14.545454545455 12.727272727273 10.909090909091
9.0909090909091 27.272727272727 45.454545454545
63.636363636364 81.818181818182 100 118.18181818182
121.818181818182 123.636363636364 127.272727272727
130.909090909091 136.363636363636 140.

AGRAZC(1)

?

.01 .02 .03 .04 .05 .06 .07 .08 .09 .1 .11 .12 .13 .14 .15 .16
.19 .20 .22 .24 .26 .28 .30 .32 .34 .36 .38 .40 .45 .50

1. c) Sample Output

ORIGINAL PROFILE AT INITIAL TIME = 1.0 IS.

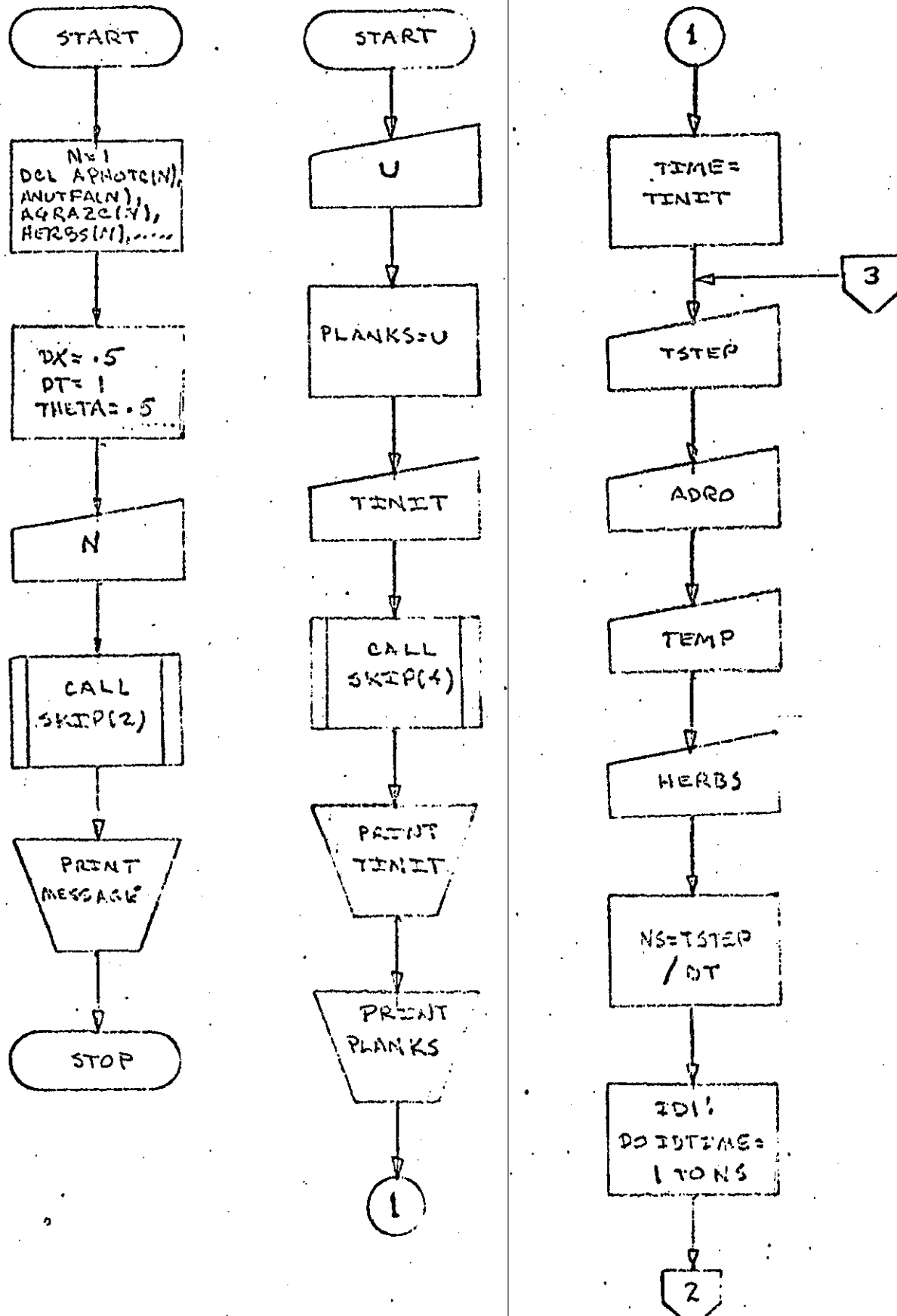
| | | | | | | | | |
|--------|--------|--------|--------|-------|-------|-------|-------|-------|
| 106500 | 105000 | 103000 | 101000 | 98500 | 96000 | 93000 | 90000 | 86500 |
| 83000 | 79200 | 75000 | 70500 | 65000 | 59000 | 52000 | 44000 | 35500 |
| 27000 | 20000 | 15500 | 12000 | 9600 | 7600 | 6100 | 5000 | 4400 |
| 3800 | 3400 | 3100 | 3000 | | | | | |

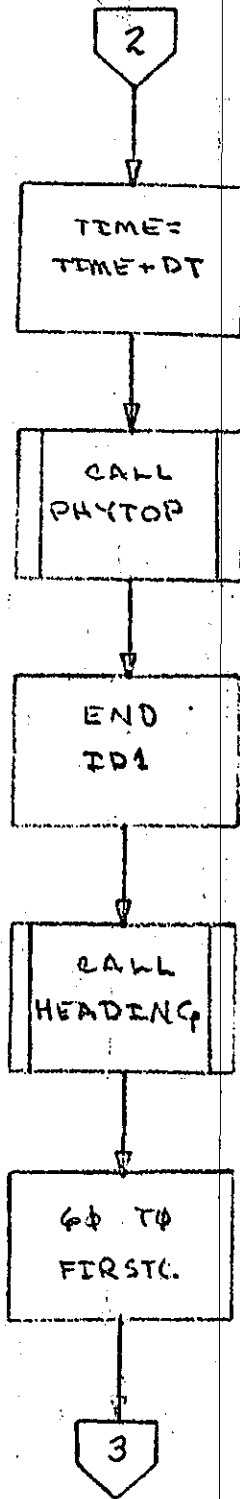
TIME = 2.00 TIME WAS INCREMENTED BY 1.00

PROFILE AT THIS TIME IS

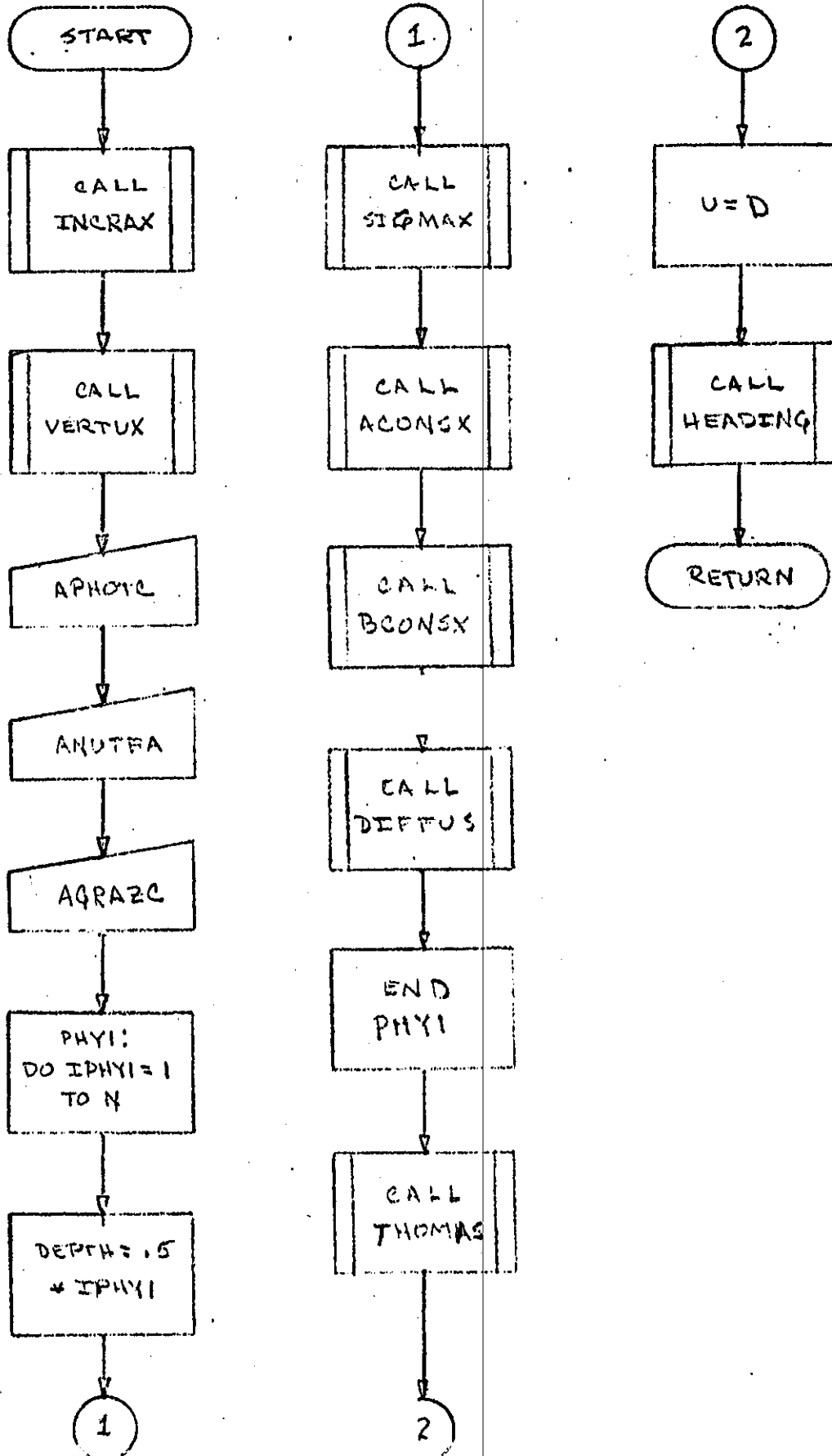
| | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|-------|-------|
| -91380 | -75448 | -60470 | -46908 | -34246 | -22941 | -12450 | -3214 | 5331 |
| 12564 | 19457 | 25661 | 31455 | 37731 | 44218 | 51508 | 59502 | 67593 |
| 75176 | 80431 | 82576 | 83424 | 83101 | 82180 | 80461 | 77844 | 74001 |
| 68687 | 60689 | 48543 | 29284 | | | | | |

2. Phytoplankton Submodel (PHYTØ) - Detailed Flowchart





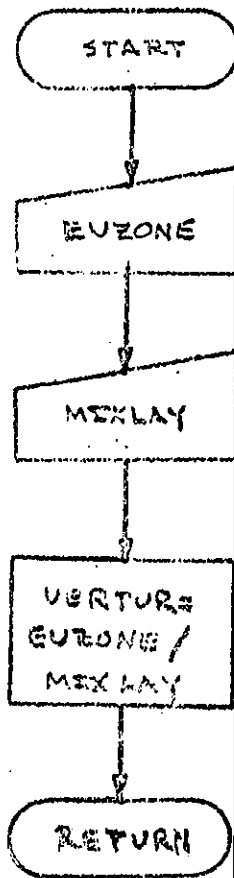
PHYTOP: PROCEDURE



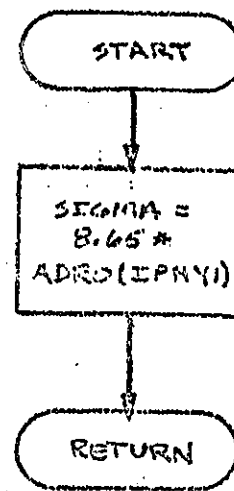
INCRAX:PROCEDURE



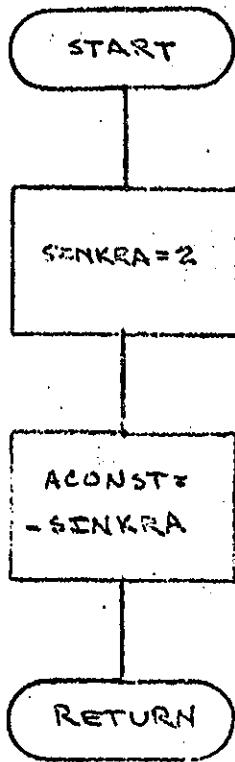
VERTUX:PROCEDURE



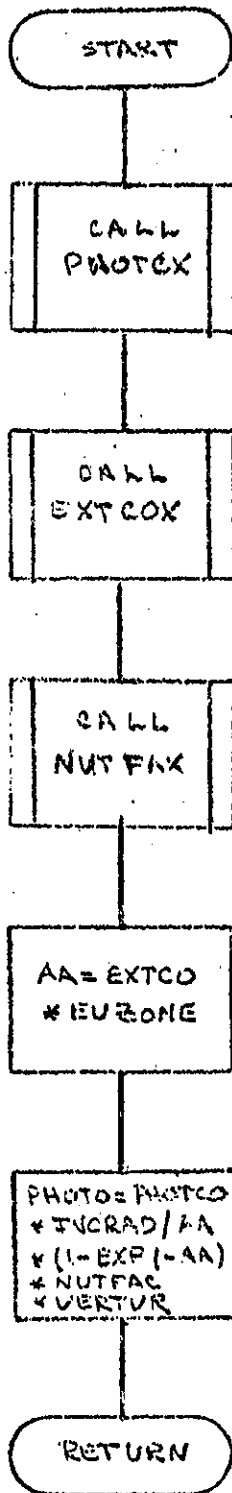
SIGMAX:PROCEDURE



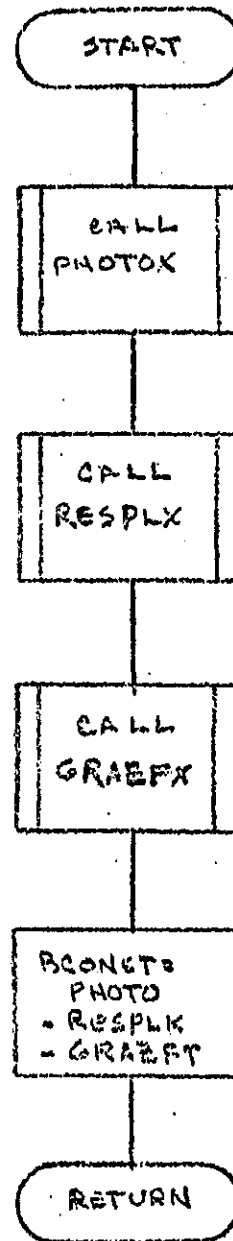
ACONSK: PROCEDURE



PHOTOX: PROCEDURE



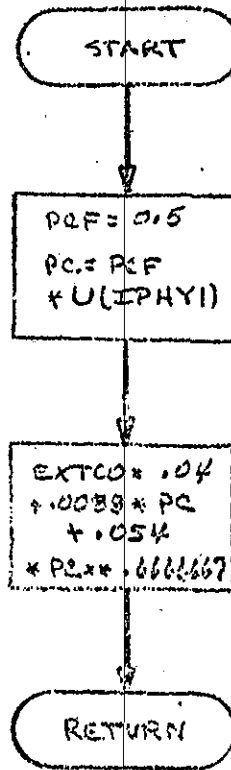
BCONSK: PROCEDURE



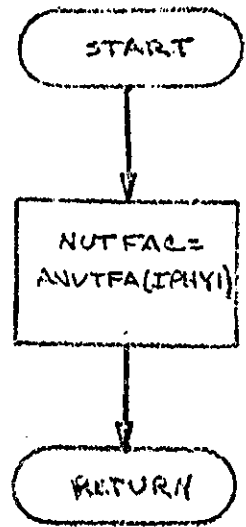
PHOTCX: PROCEDURE



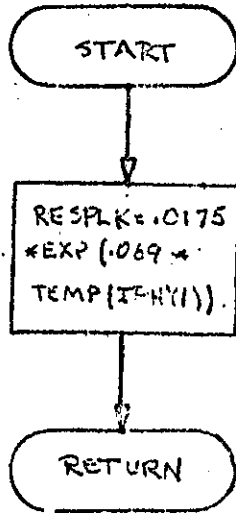
EXTCOX: PROCEDURE



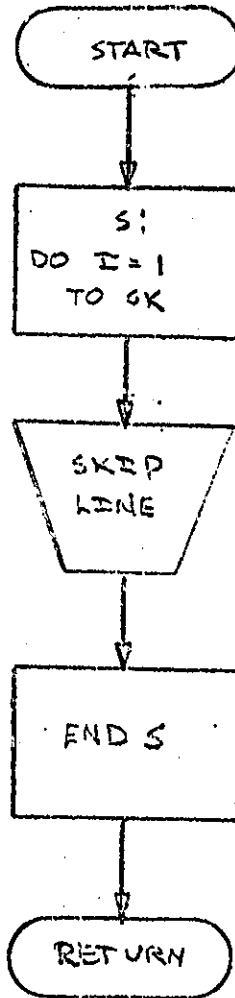
NUTFAX: PROCEDURE



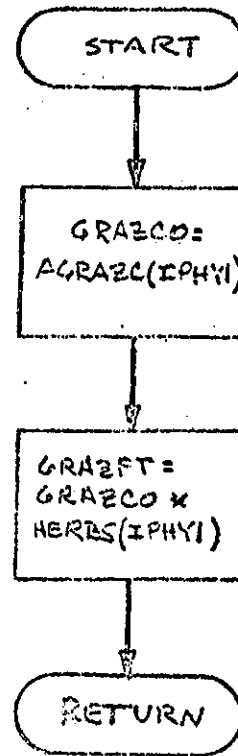
PHOTOY: PROCEDURE



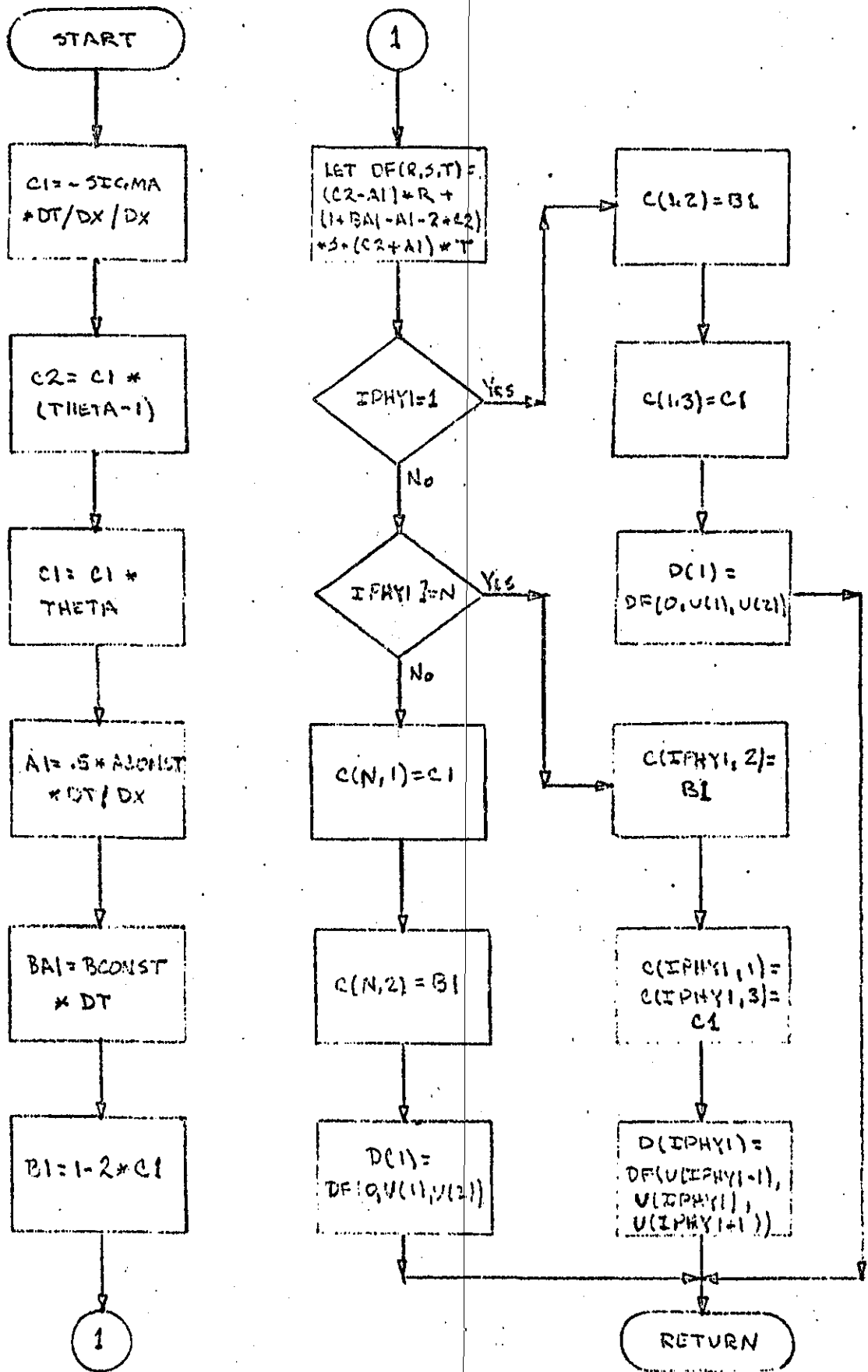
SKIP: PROCEDURE (SK)



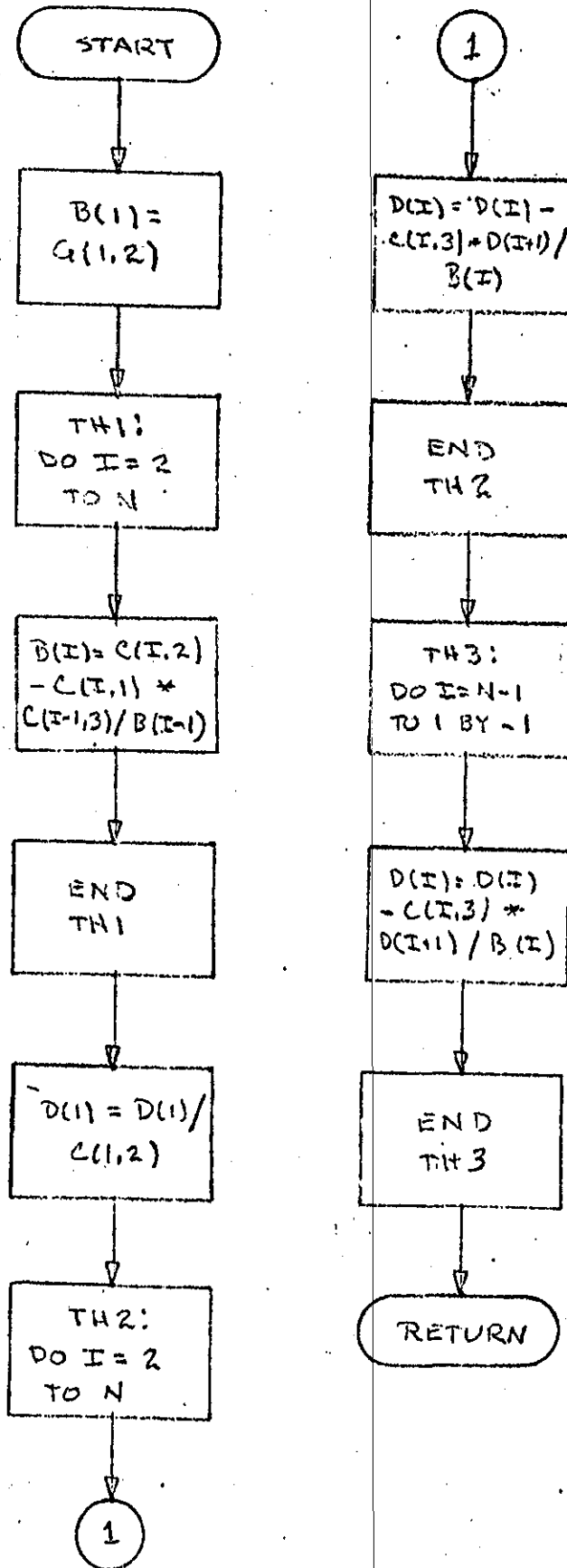
GRAZFX: PROCEDURE



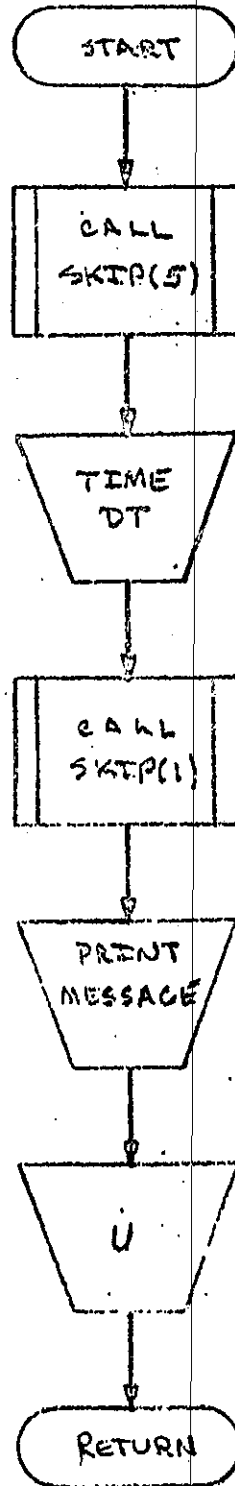
DIFFUS: PROCEDURE



THOMAS: PROCEDURE



HEADING : PROCEDURE



3. Phytoplankton Submodel (PHYTØ)

- a) This model, based primarily on Riley, Stommel and Bumpus (1949), computes the phytoplankton rate of productivity using difference equations involving depth and time.
- b) Conversational Programming System PL/1.
- c) IBM 360/50.
- d) Conversational Programming System, Release 4.
- e) One second-order differential equation.
- f) No unusual numerical methods.
- g) The following model relating plankton (u), depth (x) and time (t) was investigated:

$$\frac{\partial u(x,t)}{\partial t} = \sigma \frac{\partial^2 u(x,t)}{\partial x^2} + a \frac{\partial u(x,t)}{\partial x} + bu(x,t),$$

where σ , a , b are coefficients requiring estimation.

In this instance

$\sigma = A/\rho$, where A = Austausch coefficient of vertical eddy diffusivity and ρ = density,

$a = -v$, where v = sinking rate of phytoplankton,

$b = p_h - r_p - gh$, where p_h = photosynthesis factor

$$= \frac{pI_0}{kz_1} (1 - e^{-kz_1 NV}),$$

p = experimentally determined photosynthesis coefficient,
 I_0 = incident light, k = extinction coefficient, z_1 = depth of euphotic zone, z_2 = depth of mixed layer (maximum depth at which p is no greater than .02 standard deviation units more than surface p).

N = nutrient factor, $V = Z_1/Z_2$ correction for vertical turbulence, r_p = coefficient of phytoplankton respiration
 g = grazing coefficient of herbivores, and h = herbivore population at each depth.

The model has been treated by a method using finite differences as described in "Difference Methods for Initial-Value Problems", R. D. Richtmyer, New York Interscience Publishers, 1957.

h) Phytoplankton Submodel (PHYTØ) Variables

- N: number of samples (equally spaced) in the distance dimension (depth).
(original data may have to be smoothed and interpolated to get a larger number of samples.)
- DX: spacing between samples - constant - (meters).
(value of .5 gives a sufficient number of stations).
- U(i): array for storing number (concentration) of phytoplankton at present time: (mg/l or g/m³).

i = 1,2,...N.
U(1) is the top value (surface).
U(n) is the bottom value (at depth).
- PLANKS (i): array of original concentrations of phytoplankton at each level: (mg/l or g/m³).
- TINIT: initial time for which PLANKS, et al are given (days)
- TSTEP: length of step in time, which will be added to TINIT to determine the time (days) at which a solution is wanted.
- DT: spacing in the time dimension (days). (determines how many iterations must be performed to reach final time).
- NS: number of steps in time which must be taken to reach final time; NS = TSTEP/DT.
- IDTIME: incrementing parameter in DØLOOP ID1
- incremented for each step (iteration).
- TIME: present time = TINIT + ----
- incremented by DT each time IDTIME is incremented (days). (Final time given by TINIT + NS*DT = time after last increment (iteration).
- THETA: parameter induced by finite differences method.
(recommended value = .5, space - centered scheme).
- HERBS (i): care of herbivores model (mg/l or g/m³).
- SIGMA: variable in the diffusion equation - combination of other variables (m²/day).
- ACØNST: variable in the diffusion equation - combination of other variables (m/day).
- BCØNST: variable in the diffusion equation - combination of other variables (day⁻¹).
- ADRØ: A/ρ A = Austausch-coefficient of vertical eddy diffusivity and ρ = density of water (m² day⁻¹).

ADRØ: (values gotten from pages 124-129
cont'd if in units of $\text{cm}^2 \text{sec}^{-1}$ then multiply by 8.65
to get correct units).

SINKRA: sinking rate of phytoplankton.
(should be variable, but currently is set at 2).
(m day^{-1}).

PHØTØ: photosynthesis factor - internally determined.
(day^{-1}).

AA: internal variable.

PHØTCØ: coefficient of photosynthesis - dependent on depth
& time, so is input by means of APHØTC,
($\text{cm}^2 \text{gm}^{-1} \text{cal}^{-1}$).

APHØTC (i): array of PHØTCØ values for given time ($\text{cm}^2 \text{gm}^{-1} \text{cal}^{-1}$).

EXTCØ: extinction coefficient of light (m^{-1}).

PCF: fraction of weight of phytoplankton which is
chlorophyll (dimensionless).

PC: concentration of chlorophyll (mg/ℓ or g/m^3).

NUTFAC: nutrient factor (dimensionless). (dependent on depth
& time ... input by ANUTFA).

ANUTFA: array of NUTFAC values for various depths at given
times (dimensionless).

RESPLK: coefficient of respiration for phytoplankton (days^{-1}).

GRAZCØ: coefficient of grazing of herbivores ($\ell \text{day}^{-1} \text{mg}^{-1}$).
(dependent on time & depth; input by means of AGRAZC;
fraction of phytoplankton concentration consumed per
day by one herbivore).

AGRAZC: array of GRAZCØ values ($\ell \text{day}^{-1} \text{mg}^{-1}$).

GRAZFT: internal variable (day^{-1}).

C1, C2, A1, B1, BA1, R, S, T are all internal variables in procedure
DIFFUS. (this sets up the matrices C(i,3) & D(i)
for the finite differences solution).

C(i,3) $i=1, \dots, N$, holds the non-zero elements of a tri-
diagonal matrix generated by the finite differences
solution.

D(i) $i=1, \dots, N$, holds the constants for the equations in the
finite differences solution and in THØMAS which holds
the solution of the system equations.

B(i) $i=1, \dots, N$, is one array in procedure THØMAS (necessary
to the solution of the system of equations generated
by DIFFUS).

INCRAD: incident radiation for given time ($\text{gm cal cm}^{-2} \text{ day}^{-1}$)

EUZONE: depth of the euphotic zone (m^{-1}).

MIXLAY: depth of the mixed layer (maximum layer where density is not more than .02 standard deviation units greater than the surface value).

VERTUR: internal variable, (dimensionless).

IPHYI: major incrementing variable of program - DO LOOP PHY in PHYTOP.
(calculates SIGMA, ACNST, BCNST, & C(i,3) & D(i) for each station (depth) - IPHYI indexes the station)

TEMP (i): temperature ($^{\circ}\text{C}$).

PROCEDURES

MAIN - increments time, gets initial values.

PHYTOP

INCRAX - calculates incident radiation.
VERTUX - calculates vertical turbulence.
SIGMAX - calculates SIGMA.
ACONSX - calculates ACONST.
BCONSX - calculates BCNST.

PHOTOX - calculates photosynthesis factor PHOTO.

PHOTCX - calculates PHOTC.
EXTCOX - calculates EXTC.
NUTFAX - calculates NUTFAC.

RESPLX - calculates RESPLK.
GRAZFX - calculates GRAZFT.

DIFFUS - calculates $C(i,3)$ & $D(i)$ - system of equations.
THOMAS - solves system of equations, gives solution.

HEADING - outputs solution.

SKIP - skips lines in output.