MECHANICS OF MATERIALS WITH HIERARCHICAL FRACTAL STRUCTURE

by

Monica A. Soare

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Examining Committee:

Professor Catalin R. Picu, Thesis Adviser

Professor Antoinette Maniatty, Member

Professor George J. Dvorak, Member

Professor Saroj K. Nayak, Member

Rensselaer Polytechnic Institute

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The main goal of this thesis is to propose a new approach to address mechanics problems for materials with hierarchical multiscale structure. Specifically, the deformation of composite materials with fractal (self-similar) geometry is studied. The problem is of real interest since multi-scale fractal structures were evidenced in many engineering materials such as concrete, sandstone, aerogels, volcanic rock, and biological materials. The following barriers are being addressed:

• *Definition of the geometry*. One must consider "ideal" fractals, which have self-similar geometric properties for an infinite number of scales as well as hierarchical structures having self-similarity properties over a finite range of scales. Both cases are important – the former from a theoretical point of view, the latter from a practical point of view, the majority of real materials having a lower scale cut-off. Distinction is being made between deterministic and stochastic structures.

• *Formulation of boundary value problems*. A series of consistent mechanical boundary value problems are being formulated for domains represented by perfect ("ideal") fractals as well as for domains seen as approximations of these at various scales. The difficulties that must be overcome are related to the fact that the degree of complexity of the geometries increases rapidly with increasing resolution (observation scale), leading to problems hardly amenable to treatment with classical theories. The "ideal" fractals lack translational symmetry and the fields are non-differentiable at an infinite number of points. Therefore, the classical mathematical framework for the solution of deformation problems is not applicable. A new formulation of the governing equations is proposed in which use is made of fractional calculus.

• *Solution*. For deterministic structures or given realizations of stochastic fractals, the solution of the new formulation with the appropriate boundary conditions is sought by using a variational formulation. For deterministic structures, the resulting equations are solved using a finite element procedure with appropriately modified shape functions. For stochastic self-similar structures, the response of the structure is determined by using an appropriate adaptation of the stochastic finite element method. The solutions of several test cases are compared with equivalent solutions obtained by "brute force" finite element simulations.