SMOOTHNESS OF SUBDIVISION SCHEMES IN NONLINEAR AND GEOMETRIC SETTINGS

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ABSTRACT

Subdivision schemes are efficient methods for generating curves and surfaces from discrete sets of control points. Subdivision is also a basic block used to build multiscale representations for discrete data. Linear subdivisions have been studied for many years and a rather complete theory is available while nonlinear subdivision studies have just started.

In this thesis, we first study the smoothness of a family of scalar-valued nonlinear but affine invariant subdivisions based on interpolation-imputation of p-means. We propose a linearization principle which, when applied to p-mean subdivisions, leads to a family of linear subdivisions. We conjecture the smoothness of p-mean subdivisions is the same as that of corresponding linear schemes. We prove this conjecture in two special cases when $p = 1$ and $p = \infty$.

We also study manifold-valued nonlinear (not affine invariant) but interpolatory subdivisions based on composing a linear interpolatory subdivision with closest point projection. When the manifold is sphere $\mathbb{S}^{m-1}$ in $\mathbb{R}^m$ or special orthogonal matrices group $SO(m)$, we prove these nonlinear schemes have at least the same smoothness as those corresponding linear schemes by estimating the difference between a nonlinear subdivision and its corresponding linear subdivision.