EFFECTIVE TRANSPORT PROPERTIES OF STOCHASTIC BIOPHYSICAL MODELS

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ABSTRACT

Brownian motors are mathematical idealizations of physical and biological phenomena, like walking proteins such as kinesin, in which interactions with the noisy thermal environment must be taken into account. In this context, a stochastic particle is driven by a periodic potential and some external energy input, modeled by a stochastic forcing term. In the long-time, large-scale limit, the properties of the Brownian motor are Gaussian and so the process is uniquely described by its effective drift and diffusivity. By means of a singular perturbation analysis known as Homogenization theory, these effective transport properties are determined in terms of solutions to differential equations, or cell problems.

In this thesis we develop numerical and analytic methods for solving the cell problems and analyzing the effective dynamics of different models of Brownian motors, such as an inertial particle with constant forcing and a Brownian ratchet with dichotomous multiplicative noise. The coherence of transport, as quantified by the Peclet number, is also studied for different regimes. We will present an approach for studying the continuous dynamics of the Brownian motor in terms of a Continuous Time Random Walk (CTRW) approximation. This will provide a basis for expressing the transport properties of the motor in terms of solutions to exit problems, widely used in the literature. We provide criteria for determining when the CTRW approximation is valid, as well as arguments that explain its breakdown.